Layered Media Multicast Control (LMMC): Real-Time Error Control

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***Abstract*— We study the problem of real-time error control in layered and replicated media systems. We formulate an optimization problem aimed at minimizing a cost metric defined over the wasted bandwidth of redundancy in such systems. We also provide an analytical solution to the problem in the context of Layered Media Multicast Control (LMMC) protocol. In doing so, we present closed-form expressions describing the temporally correlated loss pattern of communication networks. Utilizing our closed-form expressions, we rely on an apriori estimate of loss along with a hybrid proactive FEC-ARQ scheme to statistically guarantee the quality of service for the receivers of a media system. We show the effectiveness of our protocol by means of simulating realistic error control scenarios.**

***Index Terms*— Multicast IP Networks, Layered Media, Replicated Media, Error Control, Apriori Estimate of Loss, Statistical Guarantee of QoS.**

I. INTRODUCTION

TRANSMITTING real-time compressed digital media over multicast IP networks has been the subject of heavy research in the recent years as surveyed by Li et al. in [10] and the references cited therein. Replicated media streams approach first presented by Cheung et al. [3] within the context of DSG protocol and layered media streams approach first proposed by Deering et al. [5] in the context of multicast routing and by McCanne et al. [14] in the context of RLM protocol are convincingly the two most important approaches in this area.

Real-time video and audio have limited tolerance for random loss within the compressed digital stream. The quality of decoded media at a receiver is subject to a significant degradation as the result of excessive loss from network congestion or latency. In order to overcome the loss effects, error control techniques can be used. There have been three general error control approaches in the context of multicasting. In Retransmission-based Automatic Repeat reQuest (ARQ), retransmissions occur only if data can be delivered before the real-time deadline. Two of such approaches are the error control aspect of LVMR presented by Li et al. [11] and STORM presented by Xu et al. [25]. In Forward Error Correction (FEC), the source assigns a portion of its bandwidth for proactive transmission of repair packets to the receivers. Among the rich set of articles in the literature, the two most closely related to our work are by Rubenstein et al. [20] in which the idea of using real-time reliable multicast using proactive FEC is proposed and Rhee et al. [18] in which a proactive reliable FEC multicast layering scheme is presented.

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There are also hybrid FEC-ARQ approaches suggesting different alternatives for proactive transmission of redundant packets based on retransmission requests. Towsley et al. [22] and Nonnenmacher et al. [16] analyzed the advantages of hybrid approaches over a stand-alone ARQ and in conjunction with local recovery, respectively. Other related examples of hybrid FEC-ARQ approaches include the works of Maxemchuk et al. [13], Bolot et al. [1], Carle et al. [2], and Chou et al. [4].

Our work in this paper spans over network transport layer. We study the real-time error control aspect of layered and replicated media systems over multicast IP networks. We address some of the related signal processing issues of layered and replicated media systems such as source coding, channel coding, consumed power, distortion, and peak signal-to-noise ratio in our related works of [26] and [28].

We assume the existence of congestion and flow control mechanisms capable of dynamically addressing inter-session fairness and flow control issues. A closely related flow control scheme is given in [27]. Other related examples of flow and congestion control algorithms are given in [23] and [24]. In [29], we address the rate allocation and partitioning aspect of Layered Media Multicast Control (LMMC). In this study, we focus on the real-time error control aspect of LMMC manifesting in dynamic distribution of an available bandwidth among data and redundant traffic portions. For each individual multicast group related to a layered or a replicated media system, LMMC specifies the assignment of data and redundancy bandwidths such that the resulting bandwidth wastage of redundancy is minimized. The main contributions of this paper are in the following areas. First, the paper introduces closed-form expressions identifying the packet loss pattern of an erasure channel under the Gilbert model [7]. Second, the paper proposes a method allowing individual receivers of each multicast group to provide the source with an apriori estimate of their redundancy requirement in order to statistically guarantee the quality of service (QoS). Third, the paper formulates an optimization problem aimed at minimizing the wasted bandwidth under the specific constraints of real-time latency and the impact of feedback implosion. The paper also provides a low complexity analytical solution to the formulation of the problem. The technique proposed in this paper can be independently applied to both replicated and layered media systems.

An outline of the paper follows. In Section II, we adopt the notion of round-based delivery of real-time reliable multicast information for LMMC error control scheme while considering temporally correlated loss for a type I hybrid FEC-ARQ protocol utilized in our study. In this section, we provide an analysis of statistically guaranteeing QoS for different size multicast groups in media systems. In Section III, we formulate and analytically solve an optimization problem aimed at minimizing the wasted bandwidth of individual multicast groups free of feedback implosion effects. In Section IV, we describe LMMC error control protocol relying on the analytical results of Section III. In Section V, we focus on performance evaluation and provide simulation results along with practical considerations. Finally, Section VI concludes the paper.

II. LMMC Analysis of Redundancy

We begin our analysis by providing a brief overview of a media session composition according to our LMMC rate allocation and partitioning work of [29]. Consider a multicast media session with a partitioning of receivers into K data groups. For a media session with N receivers and K data groups, each group $k \in \{1, \cdots, K\}$ consists of N_k receivers such that $N = \sum_{k=1}^{K} N_k$. For such a media session, a set $P = \{G_1 | \cdots | G_K\}$ is called a partitioning of the receiver set $\{1, \cdots, N\}$ if P is a decomposition of the set of receivers into a family of disjoint sets. The term group rate is used to denote the aggregated receiving data rate of a receiver in a group while the term layer rate is used to denote the transmission data rate to a specific layer. For an ordered partitioning of receivers into K data groups with ordered group data rates of $g_1, g_2, \cdots, g_K$ such that $g_1 \leq g_2 \leq \cdots \leq g_K$, the layer data rates of a layered media session are calculated in the form of

$$g_1, g_2 - g_1, g_3 - g_2, \cdots, g_K - g_{K-1} \tag{1}$$

A receiver in data group k subscribes to data layers 1 through k receiving an aggregated data rate of g_k. Interpretation of our formulation in the case of replicated media streams is also straight forward. For an ordered partitioning of the receivers into K data groups $G_1, G_2, \cdots, G_K$ with ordered group data rates of $g_1, g_2, \cdots, g_K$ such that $g_1 \leq g_2 \leq \cdots \leq g_K$, the layer data rates are the same as the group data rates. A receiver in group k only subscribes to layer k receiving a data rate of g_k.

We now turn our focus on the analysis of redundancy for a layered media session. We start by adopting the general notion of round-based delivery of real-time multicast information as proposed in [19] for LMMC error control scheme and continue by making necessary changes to make the original protocol fit into the framework of LMMC. We begin our discussion by providing the definition of a statistical guarantee for QoS in a custom tailored type I hybrid FEC-ARQ scheme utilized in our study. In a such a scheme, a block of $u = v + z$ transmitted packets can be recovered if at least v packets are received. Next, we investigate how our definition is applied to temporally correlated loss relying on the Gilbert model. We also introduce two alternatives appropriate for moderate and large size multicast groups in media systems with negligible NAK traffic.

A round-based hybrid FEC-ARQ error recovery scheme for delivering multicast information appropriately applies to real-time scenarios in which a hard deadline has to be met. This deadline typically has to do with the availability of data at the playback time in a multimedia application. For each receiver, a hard deadline can be expressed in terms of the available number of rounds. Assuming that a hard deadline is given by τ_k time units for a data group G_k and a receiver i in data group G_k measures the average round trip time of a packet from the session source to be RTT_i time unit, the number of available rounds for receiver i is calculated as

$$RD_i = \lfloor \frac{\tau_k}{RTT_i} \rfloor \tag{2}$$

Applying the round-based concept to individual data groups $\{G_1, \cdots, G_K\}$ of a media session, the available number of rounds for data group G_k is defined as

$$\Gamma_k = \min_i RD_i, \qquad \forall i \in G_k \tag{3}$$

In the original round-based protocol of [19], the authors introduce two statistical methods relying on which a receiver can recover a block of data with a given probability, Π. In the first method, Last Round Guarantee (LRG), a receiver guarantees enough repairs are delivered in a last round -should it be necessary- to assure the conditional probability of receiving all packets in the block is greater than the given probability Π. In the second method Block Good Put (BGP), a receiver achieves an overall block good put rate such that the data block is recovered on or before going to the last round with the given probability Π. Since the receiver has to specify the number of packets going to the last round, neither one of these methods are appropriate for error recovery techniques relying on an apriori knowledge or estimate of loss.

In what follows we propose a novel method appropriate for error recovery techniques relying on an apriori estimate of loss. In the first step of our method, we provide an analysis of calculating the number of required redundant packets in order to guarantee recovering a data block with a probability greater than a given probability Π. Considering the fact that the analysis of the first step calculates the number of redundant packets independent of the round-based recovery scheme, we fit the results of the first step into a round-based scheme in the second step.

Prior to proposing new techniques that can be effectively employed for error recovery techniques relying on an apriori estimate of loss, we briefly explain the two-state Gilbert model. As pointed out in [15], [19], and many other articles, Internet packet loss typically undergoes burst loss representing temporally correlated loss. This is related to the fact that many of the routers utilized in the Internet have deployed drop-tail routing. The two-state Gilbert loss model provides an elegant mathematical model to capture the loss behavior of the ever-changing network conditions. In the Gilbert model, packet loss is described by a two-state Markov chain as illustrated in Fig. 1. The first state G known as the GOOD state represents the receipt of a packet while the other state B known as the BAD state represents the loss of a packet. The GOOD state introduces a probability $P_G = \gamma$ of staying in the GOOD state and a probability $1 - P_G$ of transitioning to the BAD state while the BAD state introduces a probability $P_B = \beta$ of staying in the BAD state and a probability $1 - P_B$ of transitioning to the GOOD state. The parameters γ and β can be typically measured from the observed loss rate and burst length. In the theorem below, we introduce a closed-form expression for

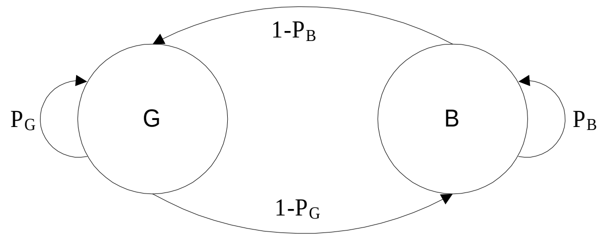

Fig. 1. The two-state Gilbert loss model with the state transition probabilities $1-P_G$ and $1-P_B$ for $P_G=\gamma$ and $P_B=\beta$.

receiving exactly v packets from $v+z$ transmitted packets under the Gilbert loss model.

Theorem 2.1: The closed-form expression for receiving exactly v packets from $v+z$ transmitted packets under the Gilbert loss model is given by

$$P(v+z,v) \;=\; P(v+z,v,G)+P(v+z,v,B) \tag{4}$$

where $P(v+z,v,G)$ the probability of receiving exactly v packets from $v+z$ transmitted packets and winding up in the GOOD state is given by

$$\begin{aligned} &P(v+z,v,G) \;=\\ &\quad \gamma^{v-z}\,(1-\beta)\,(1-\gamma)\,\{\\ &\qquad \textstyle\sum_{i=0}^{z-1}\binom{z-1}{i}\binom{v}{i+1}(\beta\,\gamma)^{z-1-i}\,[(1-\beta)\,(1-\gamma)]^i\\ &\quad \}\,g_{ss} \quad + \quad \gamma^{v-z-1}\,(1-\beta)\,\{\\ &\qquad \textstyle\sum_{i=0}^{z}\binom{z}{i}\binom{v-1}{i}(\beta\,\gamma)^{z-i}\,[(1-\beta)\,(1-\gamma)]^i\\ &\quad \}\,b_{ss} \qquad z\geq 1\,,\; v\geq z+1 \end{aligned} \tag{5}$$

Similarly, $P(v+z,v,B)$ the probability of receiving exactly v packets from $v+z$ transmitted packets and winding up in the BAD state is given by

$$\begin{aligned} &P(v+z,v,B) \;=\\ &\quad \gamma^{v-z+1}\,(1-\gamma)\,\{\\ &\qquad \textstyle\sum_{i=0}^{z-1}\binom{z-1}{i}\binom{v}{i}(\beta\,\gamma)^{z-1-i}\,[(1-\beta)\,(1-\gamma)]^i\\ &\quad \}\,g_{ss} \quad + \quad \gamma^{v-z}\,(1-\beta)\,(1-\gamma)\,\{\\ &\qquad \textstyle\sum_{i=0}^{z-1}\binom{z}{i+1}\binom{v-1}{i}(\beta\,\gamma)^{z-1-i}\,[(1-\beta)\,(1-\gamma)]^i\\ &\quad \}\,b_{ss} \qquad z\geq 1\,,\; v\geq z \end{aligned} \tag{6}$$

for $v,z\in\{1,2,3,\cdots\}$, steady state probability of the GOOD state $g_{ss}=\frac{1-\beta}{2-\gamma-\beta}$, steady state probability of the BAD state $b_{ss}=\frac{1-\gamma}{2-\gamma-\beta}$, and the following initial conditions:

$$\begin{aligned} P(v,0,G) &= 0\\ P(v,v,B) &= 0\\ P(v,v,G) &= \gamma^v\,g_{ss}+(1-\beta)\gamma^{(v-1)}\,b_{ss}\\ P(v,0,B) &= (1-\gamma)\beta^{(v-1)}\,g_{ss}\,+\,\beta^v\,b_{ss} \end{aligned} \tag{7}$$

A formal proof of Theorem (2.1) is provided in Appendix I.

Imposing a practical upper bound of v on the value of z, we introduce the following algebraic placement algorithm with a time complexity of $\mathcal{O}(z\,v)$ to calculate the smallest number of required transmitted packets $u=v+z$ in order to guarantee the receipt of at least v packets with a probability Π or better for a system governed by the Gilbert loss model.

Statistical Guarantee for Packet Arrival Algorithm

- $for \quad (z=1 \quad to \quad v)\,\{$
  - Calculate $P(v+z,v) = P(v+z,v,G)+P(v+z,v,B)$ from Equation (5) and Equation (6).
  - If $\sum_{i=v}^{v+z}P(v+i,i)\geq\Pi$ Break.

  $\}$ /* $for \quad (z=1 \quad to \quad v)$ */
- Report the number of required packets, $u=v+z$.

Taking into consideration specific design issues of LMMC pertaining to combining its rate allocation/partitioning aspects with its error control aspect and considering the above algorithm, we now propose two new alternatives in providing a statistical guarantee within the context of our current discussion. In the first alternative to which we refer as the Dynamic Mode (DM) of requesting redundant packets, we propose that an individual receiver i of a media session data group G_k waits until the last round in order to report its required redundancy by finding u_i from the Gilbert model assuming the receiver is in need of v_i packets going to the last round. An individual receiver i then reports $r_i=\min(u_i,B_k)$ as its redundancy requirement where B_k indicates the block size for data group G_k [1]. We note that DM method is essentially an enhanced version of LRG adopted for layered media systems. The major differences between DM and LRG methods are the utilization of closed-form solutions rather than recursive solutions in the case of Gilbert loss model and considering an upper bound on the number of redundant packets.

In the second alternative to which we refer as the Static Mode (SM) of requesting redundant packets, we propose that an individual receiver i of a media session data group G_k carries out an apriori estimate of loss. In our analysis pertaining to SM alternative, we consider a block recovery probability of Π_k with equal per round probabilities of π_k for the available number of rounds in data group G_k of a media session. We assume that the source of a media session is in synch with the reveivers of the ssession and only initiates a new transmission round for the receivers of data group G_k as the result of receiving at least one NAK from the receivers of the group. Therefore, we can relate the two quantities as

$$\Pi_k = 1-(1-\pi_k)^{\Gamma_k} \tag{8}$$

yielding

$$\pi_k = 1-\sqrt[\Gamma_k]{1-\Pi_k} \tag{9}$$

Hence, given the overall probability of block recovery Π_k for data group G_k, the per round probability of block recovery is calculated from Equation (9). In the second alternative, a receiver obtains an estimate of required redundant packets by assuming that it receives an expected number of packets according to its probability distribution $D(u_i,v_i)$ going from one round to another. Inserting an assurance coefficient ψ in the range of $1\leq\psi<2$ and starting from an initial value of $v_i=\psi\,B_k$ for the first round, the number of requested packets

[1] For practical reasons, we place an upper bound equal to the block size on the redundancy requirement of data group G_k.

u_i requested by receiver i in each round is calculated by deducting the expected number of arrived packets in the previous round from the current value of v_i. Consequently, receiver i of data group G_k calculates the number of packets for round j, u_i^j based on the expected number of required packets for round j, v_i^j as

$$D(u_i^j, v_i^j) \geq \pi_k \tag{10}$$

We note that Equation (10) holds assuming $v_i^j = v_i^{j-1} - \bar{u}_i^{j-1}$ for $v_i^1 = \psi B_k$ and realizing the fact that the term $\bar{u}_i^{j-1}$ indicates the expected number of arrived packets in round $j-1$. We also note that $\bar{u}_i^{j-1} = g_{ss}.u_i^{j-1}$ in the case of utilizing the Gilbert model. The overall required redundancy of receiver i is, then, calculated as

$$r_i = \min(\sum_{j=1}^{\Gamma_k} u_i^j, B_k) \tag{11}$$

The receiver then announces its overall redundancy and per round required redundancy sequence r_i and $\{u_i^1, \cdots, u_i^{\Gamma_k}\}$ to the source.

From a complexity standpoint, our approach introduces a time complexity of $\mathcal{O}(z\ B_k)$ where z is the smallest number chosen in order to statistically guarantee the receipt of at least v packets from $v+z$ transmitted packets. We note that the complexity of our approach matches that of a dynamic programming approach $\mathcal{O}(B_k^2)$ only in its worst case scenario. We note that the main objective in the second alternative is to provide the receivers with an opportunity to recover a block with equal probabilities π_k in each round.

The latter is of special interest from the design standpoint of LMMC in which an apriori estimate of receivers loss is required in order to combine rate allocation and receiver partitioning aspects of a media system with its error control aspect. We discuss the details of integrating rate allocation and partitioning aspects of LMMC with its error control aspect in [30].

III. LMMC Optimal Solution to The Error Control Problem

Having calculated the required redundancy for individual receivers of a multicast group in a media multicast group, we now focus on the formulation of the optimal error control problem and LMMC's analytical solution it. We formulate our layered real-time error control problem in a way similar to Layered Multicast Recovery (LMR) protocol proposed in [17]. However, we make note of the differences in the formulation as well as the solution. First, unlike the formulation of [17] that is intended for reliable multicast, the formulation of our problem is within the context of layered or replicated media systems and is hence subject to real-time constraints applied to media systems. In addition, because of targeting at providing a set of integrated protocols for media systems in conjunction with what was discussed in [29], we rely on an apriori estimate of redundancy. Finally, rather than relying on dynamic programming, we propose a lower complexity analytical solution to the problem within the context of LMMC error control protocol. In our error control model for media systems, we associate ς_k multicast *redundancy* groups with every individual data group G_k. Although we apply a fixed value to parameter ς_k in our formulation, the choice of ς_k is a design parameter with the objective of providing a balance between the bandwidth wastage and the overhead of managing multicast groups.

The sequence of events is as follows. First, the source polls individual receivers about their redundancy requirement with the details of polling mechanism discussed in Section IV. Receivers then respond based on one of DM or SM schemes of Section II indicating the number of redundant packets required to statistically guarantee the recovery of data blocks. We note that the process of collecting redundancy information is subject to feedback implosion and subsequently address the implosion problem.

Assuming a block size of B_k for data group G_k, the source transmits B_k data packets to data group G_k followed by ρ_j redundant packets for $j = 1, \cdots, \varsigma_k$ to ς_k independent redundancy groups. From a layering standpoint, the formulation of the error control problem is similar to the two-phase rate allocation and partitioning problem of our earlier work in [29]. This means that a receiver can subscribe to a redundancy group only if it has already subscribed to all of the previous redundancy groups. However, we note that in this case the collection of redundancy groups $\{1, \cdots, \varsigma_k\}$ combined together are considered to be the error control groups associated with data group G_k in the rate allocation and partitioning problem.

In this analysis, we consider a partitioning of the receivers of data group G_k into ς_k groups according to their redundancy requirement. For data group G_k with N_k receivers, we associate ς_k redundancy groups each redundancy group carrying a portion of redundant traffic. For a partitioning $\Omega_k = \{R_1|\cdots|R_{\varsigma_k}\}$ of data group $G_k = \{1, \cdots, N_k\}$ with ordered group redundancy rates of $\rho_1, \rho_2, \cdots, \rho_{\varsigma_k}$ such that $\rho_1 \leq \rho_2 \leq \cdots \leq \rho_{\varsigma_k}$, the layer redundancy rates of a layered error control scheme are calculated in the form of

$$\rho_1, \rho_2 - \rho_1, \rho_3 - \rho_2, \cdots, \rho_{\varsigma_k} - \rho_{\varsigma_k - 1} \tag{12}$$

A receiver in redundancy group j subscribes to layers 1 through j receiving an aggregated redundancy rate of ρ_j. If required, LMMC error control protocol allows receivers to subscribe to extra redundancy groups only at the beginning of each polling period. This is necessary to control the overhead of multicast group joins and leaves considering real-time constraints of media systems.

In order to formulate a per group error control problem for individual data groups G_k with $k \in \{1, \cdots, K\}$ of a media session while considering the impact of feedback implosion, we observe that for a block size of B_k in group G_k with $k \in \{1, \cdots, K\}$, all of the receivers' reported redundancy numbers are in the range of $[1, B_k]$. The source can, hence, rely on a hierarchical tree-based feedback aggregation protocol similar to the one proposed in [12] or [9] to identify the subgroup of receivers with redundancy requirements matching i redundant packets in the range $[1, B_k]$. By sending individual polling packets to sweep the redundancy range of $[1, B_k]$, the source can effectively eliminate the impact of feedback implosion. Assuming there exists a per data group upper bound on the maximum number of redundant packets in the form of $\max_i r_i = U_k$

where $U_k \leq B_k$, we formulate the optimal error control problem of data group G_k of a media session as

$$\min_{\rho_1,\cdots,\rho_{\varsigma_k}} ECW_k \equiv \min_{\rho_1,\cdots,\rho_{\varsigma_k}} \sum_{j=1}^{\varsigma_k} \sum_{i=1}^{B_k} w_i \left(\rho_j - i\right) \quad (13)$$

$$\text{Subject To:} \qquad \rho_{\varsigma_k} \leq B_k \quad (14)$$

where ς_k with $k \in \{1, \cdots, K\}$ is the number of redundant groups associated with data group G_k and w_i is the weighting function associated with the number of receivers requesting redundancy i with $\sum_{i=1}^{B_k} w_i = N_k$. Further, the function ECW_k is the bandwidth wastage of data group G_k over all of its redundancy groups R_j with $j \in \{1, \cdots, \varsigma_k\}$.

Rather than relying on a dynamic programming approach as suggested in [17], we utilize an analytical approach in solving Equation (13) with Constraint (14). In our approach, we introduce an iterative partitioning scheme that is guaranteed to converge to a local minimum. In our partitioning strategy, it is imperative to assign a receiver i with required redundancy r_i to the redundancy group R_j with the group redundancy rate ρ_j for a set of given group redundancy rates $\{\rho_1, \cdots, \rho\varsigma_k\}$, if the receiver bandwidth wastage $(\rho_j - r_i) \geq 0$ is minimized for the choice of ρ_j. As the result, we make the observation that the optimal receiver partitioning strategy has to assign receiver i with the redundancy rate r_i to the redundancy group R_j with the group redundancy rate ρ_j such that

$$0 \leq (\rho_j - r_i) \leq (\rho_l - r_i) \quad l \in \{1, \cdots, \varsigma_k\} \quad (15)$$

It is proven in **Lemma (II.1)** of [18] that for such a partitioning of the receivers utilized in LMMC formulation, the optimal redundancy rate of each partition is equal to the largest redundancy requirement of the receivers of that specific partition, i.e.,

$$\rho_j^* = \max_{i \in R_j} r_i \quad j \in \{1, \cdots, \varsigma_k\} \quad (16)$$

Let us now pay attention to the implication of the latter result in the case of applying an optimal partitioning strategy to a simple partitioning of the receivers into two redundancy groups. For an ordered partitioning $\Omega_k = \{R_1|R_2\}$ of the receivers $G_k = \{1, \cdots, L_1, L_1{+}1, \cdots, L_2\}$ with L_1 indicating the last receiver of partition R_1 and L_2 indicating the last receiver of partition R_2, we note that a receiver s with redundancy requirement r_s and all of the receivers with greater redundancy requirements in partition R_1 have to move to partition R_2 if

$$L_1(r_{L_2} - r_{L_1}) \; < \; (s-1)(r_{L_2} - r_{s-1}) \quad (17)$$

Likewise, a receiver t with redundancy requirement r_t and all of the receivers with lower redundancy requirements in partition R_2 have to move to partition R_1 if

$$L_1(r_{L_2} - r_{L_1}) \; < \; t(r_{L_2} - r_t) \quad (18)$$

Generalizing these results for an ordered partitioning $\{R_1|\cdots|R_{\varsigma_k}\}$ of the receivers, we propose the following iterative algorithm to solve the optimal error control problem of Equation (13) with Constraint (14).

LMMC Error Control Algorithm: An Iterative Layered Partitioning Approach

- Step 1: Start from an initial ordered partitioning of the receivers by uniformly distributing the receivers among the redundancy groups. In addition, set the initial iteration number $it = 0$ and the maximum number of iterations it_{max}.
- Step 2: Calculate the optimal redundancy rates of each partition R_j with $j \in \{1, \cdots, \varsigma_k\}$ from Equation (16) and the resulting error control cost function ECW_k from Equation (13). Save the previously calculated ECW_k in variable q_1 and the currently calculated ECW_k in variable q_2.
- Step 3: If $\frac{|q_1 - q_2|}{q_1} < \delta$ or $it > it_{max}$ STOP.
- Step 4: $for \quad (j = \varsigma_k \quad downto \quad 2)$ {
  - Repartition groups $j-1$ and j according to Equation (17) and Equation (18).

  } /* $for \quad (j = \varsigma_k \quad downto \quad 2)$ */
- Step 5: Go back to Step 2.

We note that LMMC error control algorithm moves multiple receivers with the same redundancy requirements from one redundancy group to another together. We also note that the time complexity of implementing LMMC error control algorithm is $\mathcal{O}(IB_k)$ where I indicates the number of iterations.

Theorem 3.1: "LMMC Error Control Algorithm" given in this section converges to a local minimum.

A formal proof is given in Appendix II. Intuitively, LMMC algorithm is employing steepest descent optimal control strategy. It is important to note that considering the convergence speed of the proposed LMMC algorithm as proven by steepest descent approach and supported by our simulation results of Section V, the use of LMMC error control algorithm yields fast converging results.

IV. LMMC Error Control Protocol

This section focuses on describing LMMC error control protocol relying on the analytical study of the previous sections. Generally speaking, LMMC error control protocol relies on the source of a media system to solve the error control problem based on the information collected from the receivers of a media system. The information includes the number of available rounds and the redundancy requirement of individual receivers. The source repeats the calculations pertaining to the solution of the combined problem as the result of a significant potential change in the status of the system. A significant potential change in the status of the system can be flagged based on one of the following two events. First, when the source polling period timer goes off and second, when a significant change is reported by a designated receiver in the middle of a polling period. In the event of the first scenario, the calculations are repeated if at least a given percentage of the receivers report a change. The second scenario may have been caused for example by the occurrence of congestion in a segment of the network impacting the receivers of a specific zone. LMMC relies on designated per zone receivers to collect such information and notify the source about the existence of such conditions. The polling frequency is typically few times larger than the largest RTT and few times smaller than the media clip playback time. At the beginning of every new polling period caused by expiration of the source timer or a significant redundancy change of a group of receivers in a local zone, the source probes the receivers for the number of available rounds as well as their redundancy requirement. Individual receivers then rely on the methods of Section II to calculate the number of rounds as well as their redundancy requirement. The source then proceeds with collecting and calculating the bandwidth assignment of data and redundancy traffic following the algorithm of Section III.

The source discovers the available number of rounds for each group by polling the group of receivers through multicasting pilot packets. The receivers then set their own timers with a random value reporting back the result after having an expired timer only if not having seen a value less than or equal to the calculated number of rounds from Equation (2). The number of available rounds Γ_k for group G_k is eventually announced to the group utilizing Equation (3). The source relies on a hierarchical tree-based feedback aggregation protocol similar to RMTP proposed in [12] to poll the receivers for their required redundancy. It first sends a message to data group G_k specifying the block size B_k for the group and continues by multicasting individual polling packets for the number of redundant packets i associated with the natural numbers in the range $[1, B_k]$. As the default rule of practice and considering the fact that the source requires an apriori estimate of the receivers' redundancy over the total number of rounds, receivers rely on the SM method of section II to estimate their redundancy requirement and report it back to the source. In this section, it makes sense to review the issues involved with the implementation of the SM method described in Section II. Noting that per round redundancy requirement of every receiver changes over a polling period according to Equation (10), the source needs to discover the number of available rounds Γ_k and the sequence of per round redundancy of the receivers $\{u_i^1, \cdots, u_i^{\Gamma_k}\}$ for each data group G_k. Having collected the information, the source needs to calculate per round distribution of redundancy groups and rates for every individual round of the polling period separately.

The source then continues by announcing data and redundancy rates of individual multicast groups. Each receiver then has the opportunity of subscribing to the appropriate number of multicast data groups as well as multicast redundancy groups satisfying its redundancy requirement. We note that although LMMC error control protocol allows the receivers to drop any number of layers that they are already subscribed to at any time, it only allows the receivers to subscribe to extra redundancy groups at the beginning of a polling period and after the new redundancy rates have been announced. This is necessary to control the overhead of multicast groups joins and leaves considering real-time constraints of media systems.

During a polling period, the source sends the data packets of a block for individual groups G_k with $k \in \{1, \cdots, K\}$ followed by the first round of redundant packets. At the end of each round, the first receiver not capable of recovering the data block with size B_k multicasts a NAK message to the group notifying the source about the need for initiating the next round. A receiver only sends a NAK message if it has neither been able to recover the block nor seen another NAK message with a sequence number matching the current round. The proposed mechanism effectively eliminates the NAK traffic as the overall number of transmitted NAKs is in the order of number of rounds Γ_k. Going from one round to another, the source only initiates another round if it has received a NAK request from one of the receivers of the group within a certain number of time units past the end of current round.

We also argue that as an alternative to the polling mechanism and only for moderate size group of receivers in which the overhead of dynamically calculating the number of redundant packets is acceptable, the source can rely on DM method of Section II to dynamically adjust the data and redundancy rates without relying on an apriori estimate of overall redundancy. In this scenario, each receiver calculates the number of required packets only going to the last round and reports the result to the source.

Before we conclude this section, it is in order to provide a discussion of LMMC error control protocol practicality for real-time media systems. Perhaps the most important concern pertains to explaining why the latency of joining and/or leaving multicast trees does not make the protocol overhead prohibitive. We argue that LMMC error control protocol is custom tailored for media systems according to the following reasons. First, we note that having a reduced loss rate resulting in dropping redundant groups is not a problem as a receiver is not concerned with the delay of multicast tree topology changes in this case. This is of special importance in the case of the SM approach of Section II in which a receiver needs a lower number of redundant packets going from one round to another. Second, calculation of the bandwidth for individual redundant groups is done considering redundancy requirements of individual receivers at the beginning. Third, the built-in polling mechanism of LMMC counts for adjusting the number of redundant packets according to the current loss condition of individual receivers so that the receivers do not have to subscribe to extra redundancy groups often. Considering the above factors, we do not anticipate having frequent changes in multicast tree memberships and LMMC error control protocol can be hence effectively deployed in real-time media systems.

V. Numerical Performance Analysis

In this section, we present the numerical results of applying LMMC error control algorithm to a number of layered media scenarios. First, we compare LMMC results with the results of optimal LMR (OLMR) utilizing dynamic programming and heuristic LMR (HLMR) algorithms of [17]. In our comparisons, we review the performance of the approaches from the standpoint of tracking the minimum value of the bandwidth wastage, time complexity indicated by experiment runtime, and space complexity indicated by memory allocation. Additionally, we review the scalability of the techniques by covering a relatively broad range of multicast group sizes ranging from hundreds to thousands of receivers. We remind that per group time complexity of LMMC error control algorithm is $\mathcal{O}(IB_k)$

and that of OLMR algorithm is $\mathcal{O}(\varsigma_k B_k^2)$. In addition, per round space complexity of the LMMC error control algorithm in our implementation is $\mathcal{O}(B_k)$ where as that of OLMR is $\mathcal{O}(B_k^2)$ assuming block size B_k indicates an upper bound on the maximum required redundancy. In our simulations, we ran in excess of $20,000$ experiments with different number of groups K, different group sizes N_k with $k \in \{1, \cdots, K\}$, and different receiver redundancy requirements. For each combination of the parameters, the results of our experiments were consistent with a confidence level of 98%.

Fig. 2 through Fig. 4 compare the sample results of LMMC algorithm with those of OLMR and HLMR algorithms for some individual data groups. Consistent with real network traces reported in [17], we have relied on a normal random number generator simulating receiver loss rates within the range of $[1\%, 10\%]$ in each experiment with confidence intervals of 99.8%. Different figures have been obtained for different choices of different parameters of interest. Different parameters of interest include the block size indicated by B and the number of redundancy groups ς associated with an individual data group. In our simulations B is set at 64, 128, and 256 packets; ς is set at 2, 3, and 4. The x-axis of each curve is always in logarithmic scale indicating different values of the group size from the set $\{100, 300, 1000, 3000, 10000, 30000, 100000\}$. Each figure consists of two pairs of curves. The first set of curves

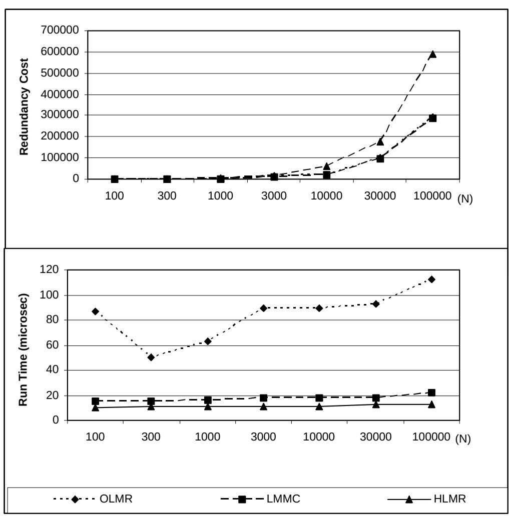

Fig. 2. Redundancy cost and runtime comparison of LMMC, OLMR, and HLMR methods vs number of receivers (N) for ς=2 and block size of $B = 64$.

compare the bandwidth wastage or redundancy cost in bps of the three techniques. While LMMC and OLMR keep a close bandwidth wastage across the board, we observe that for group sizes of 1000 or more the bandwidth wastage of HLMR departs from the other two. Considering the results, we note that HLMR can be effectively used only if the distribution of the redundancy is not highly skewed and the group size is not very large. Additionally for about $20,000$ experiments made by us, we observed a maximum 6% cost advantage of OLMR over LMMC. Considering the fact that a dynamic programming approach identifies a global optimum where as a gradient-based approach identifies a local optimum, our experiments indicate impressive convergent behavior of LMMC. The second pair of

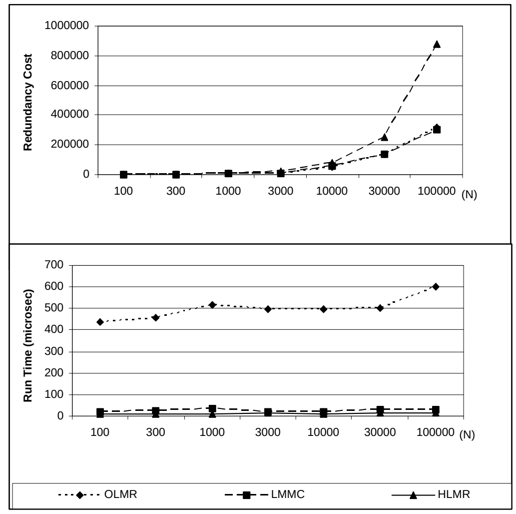

Fig. 3. Redundancy cost and runtime comparison of LMMC, OLMR, and HLMR methods vs number of receivers (N) for ς=3 and block size of $B = 128$.

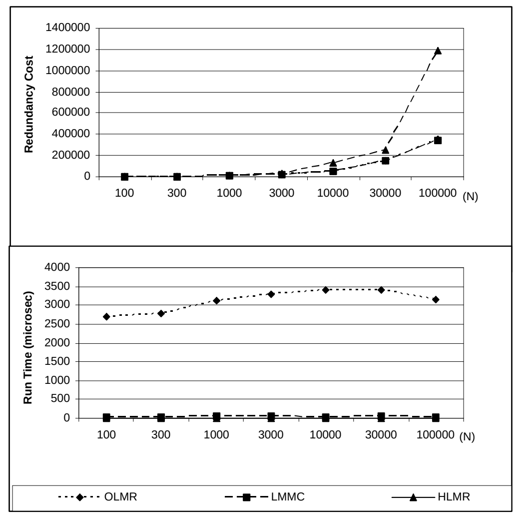

Fig. 4. Redundancy cost and runtime comparison of LMMC, OLMR, and HLMR methods vs number of receivers (N) for ς=4 and block size of $B = 256$.

curves display the runtime of the experiments as an indicator of the time complexity of the three techniques. To our expectation, the complexity of HLMR for a small size group is the lowest among the three considering its negligible overhead of computation. In this area, a review of the results reveals closeness of LMMC results to those of HLMR. The review also reveals great performance advantage of LMMC over OLMR consistent with the time complexity analysis reporting a linear and a quadratic dependency on the value of B in the runtime of LMMC and OLMR, respectively.

In the rest of this section, we qualitatively discuss some of the practical findings of our experimentations pertaining to the comparison and combination of LMMC error control technique applied as a reliable multicast technique [2] with FEC-based and ARQ-based techniques. Due to lack of space, we do not provide quantitative results of our experiments. In our experiments, we

[2] We have investigated such a scenario by relaxing real-time constraints and calculating receiver redundancies based on the probability of recovery.

looked at the impact of utilizing LMMC in conjunction with ARQ-based SRM recovery [6], as well as hierarchical scoping techniques such as scoped SRM [21], and SHARQFEC [8]. The following summarizes our findings.

First, we have observed that utilizing LMMC error control relying on proactive FEC-based recovery greatly reduces the overall amount of redundant traffic compared to reactive ARQ-based recovery utilized in single-scoped SRM. Second, we have seen that utilizing layered recovery in a hybrid technique resulting from the combination of LMMC with SRM significantly reduces the overall amount of redundant traffic. In our experiments, we have also observed that increasing the number of recovery layers has led to lower amount of redundant traffic at the expense of higher protocol overhead. Despite the fact that we did not see the threshold point in our experiments with up to 5 groups, we expect that increasing the number of redundant groups beyond a certain threshold point is not justified considering the extra amount of multicast joins and leaves overhead. As a cautionary step, our implementation of LMMC error control relies on utilizing up to four redundant groups associated with each individual data group Third, we have been able to achieve great repair locality by combining LMMC layering technique with a hierarchical technique such as scoped SRM or SHARQFEC. We note that most of our findings are consistent with the results reported in [17].

VI. Conclusion

In this paper, we studied the problem of real-time error control for layered and replicated media systems over multicast IP networks. Assuming the existence of congestion and flow control mechanisms as well as a rate allocation and partitioning scheme, we proposed our Layered Media Multicast Control (LMMC) real-time error recovery framework. Our framework aimed at providing an analytical solution to a formulation of the problem by minimizing the bandwidth wastage of individual multicast groups. Our framework was capable of effectively eliminating the impact of feedback implosion and providing a statistical guarantee for the quality of service of each receiver. We evaluated the performance and scalability of our LMMC solution and illustrated its applicability in realistic network topologies through the use of simulations. We are currently investigating the effects of network topology in the effectiveness of our error recovery scheme. We are also working on the fine tuning of LMMC framework for accommodating hybrid wired and wireless media systems.

Appendix I
Proof of Theorem 2.1

Relying on a set of lemmas and a proposition, we provide a formal proof of Theorem 2.1 in this appendix. We start by providing the recursive equations of the Gilbert loss model describing the probability of receiving v packets from u transmitted packets $P(u,v)$ as

$$\begin{aligned} P(u,v,G) &= P(u-1,v-1,G)\gamma \\ &\quad + P(u-1,v-1,B)(1-\beta) \\ P(u,v,B) &= P(u-1,v,G)(1-\gamma) + P(u-1,v,B)\beta \\ P(u,v) &= P(u,v,G) + P(u,v,B) \end{aligned} \tag{19}$$

where $P(u,v,G)$ and $P(u,v,B)$ are the probabilities of receiving v packets from u transmitted packets and winding up in the GOOD state and the BAD state respectively. Next with the assumption that $u = v + z$, we use the set of iterative equations of (19) to derive the following lemmas.

Lemma 1:
(a) The closed-form of equation $P(v+1,v,G)$ is given by

$$\begin{aligned} P(v+1,v,G) &= v\gamma^{v-1}(1-\beta)(1-\gamma)\, g_{ss} \\ &+ \gamma^{v-2}(1-\beta)[\beta\gamma + (v-1)(1-\beta)(1-\gamma)]\, b_{ss} \end{aligned} \tag{20}$$

(b) The closed-form of equation $P(v+1,v,B)$ is given by

$$\begin{aligned} P(v+1,v,B) &= \gamma^{v}\,(1-\gamma)\, g_{ss} \\ &+ \gamma^{v-1}\,(1-\beta)(1-\gamma)\, b_{ss} \end{aligned} \tag{21}$$

Lemma 2:
(a) The closed-form of equation $P(v+1,v-1,G)$ is given by

$$\begin{aligned} &P(v+1,v-1,G) = \\ &\gamma^{v-3}\,(1-\beta)\,(1-\gamma)\,\{ \\ &\quad \textstyle\sum_{i=0}^{1} \binom{1}{i}\binom{v-1}{i+1}(\beta\,\gamma)^{1-i}\,[(1-\beta)\,(1-\gamma)]^{i} \\ &\}\, g_{ss} \quad + \quad \gamma^{v-4}\,(1-\beta)\,\{ \\ &\quad \textstyle\sum_{i=0}^{2} \binom{2}{i}\binom{v-2}{i}(\beta\,\gamma)^{2-i}\,[(1-\beta)\,(1-\gamma)]^{i} \\ &\}\, b_{ss} \end{aligned} \tag{22}$$

(b) The closed-form of equation $P(v+1,v-1,B)$ is given by

$$\begin{aligned} &P(v+1,v-1,B) = \\ &\gamma^{v-2}\,(1-\gamma)\,\{ \\ &\quad \textstyle\sum_{i=0}^{1} \binom{1}{i}\binom{v-1}{i}(\beta\,\gamma)^{1-i}\,[(1-\beta)\,(1-\gamma)]^{i} \\ &\}\, g_{ss} \quad + \quad \gamma^{v-3}\,(1-\beta)\,(1-\gamma)\,\{ \\ &\quad \textstyle\sum_{i=0}^{1} \binom{2}{i+1}\binom{v-2}{i}(\beta\,\gamma)^{1-i}\,[(1-\beta)\,(1-\gamma)]^{i} \\ &\}\, b_{ss} \end{aligned} \tag{23}$$

Lemma 3:
(a) The closed-form of equation $P(v+2,v,G)$ is given by

$$\begin{aligned} &P(v+2,v,G) = \\ &\gamma^{v-2}\,(1-\beta)\,(1-\gamma)\,\{ \\ &\quad \textstyle\sum_{i=0}^{1} \binom{1}{i}\binom{v}{i+1}(\beta\,\gamma)^{1-i}\,[(1-\beta)\,(1-\gamma)]^{i} \\ &\}\, g_{ss} \quad + \quad \gamma^{v-3}\,(1-\beta)\,\{ \\ &\quad \textstyle\sum_{i=0}^{2} \binom{2}{i}\binom{v-1}{i}(\beta\,\gamma)^{2-i}\,[(1-\beta)\,(1-\gamma)]^{i} \\ &\}\, b_{ss} \end{aligned} \tag{24}$$

(b) The closed-form of equation $P(v+2,v,B)$ is given by

$$\begin{aligned} &P(v+2,v,B) = \\ &\gamma^{v-1}\,(1-\gamma)\,\{ \\ &\quad \textstyle\sum_{i=0}^{1} \binom{1}{i}\binom{v}{i}(\beta\,\gamma)^{1-i}\,[(1-\beta)\,(1-\gamma)]^{i} \\ &\}\, g_{ss} \quad + \quad \gamma^{v-2}\,(1-\beta)\,(1-\gamma)\,\{ \\ &\quad \textstyle\sum_{i=0}^{1} \binom{2}{i+1}\binom{v-1}{i}(\beta\,\gamma)^{1-i}\,[(1-\beta)\,(1-\gamma)]^{i} \\ &\}\, b_{ss} \end{aligned} \tag{25}$$

Proof: The proof of all of the lemmas is based on mathematical induction utilizing recursive equations of the Gilbert model. In the case of Lemma 1.(a), we verify that $P(v+1,v,G)$ holds assuming $P(v,v-1,G)$ and $P(v,v-1,B)$ hold. In the case of Lemma 1.(b), we verify that $P(v+1,v,B)$ holds given the initial conditions $P(v,v,G)$ and $P(v,v,B)$. In the case of Lemma $2.(a)$, we verify that $P(v+1,v-1,G)$ holds assuming $P(v,v-2,G)$ and $P(v,v-2,B)$ hold. In the case of Lemma $2.(b)$, we verify that $P(v+1,v-1,B)$ holds given the results of Lemma $1.(a)$ and $1.(b)$. In the case of Lemma $3.(a)$, we verify that $P(v+2,v,G)$ holds assuming $P(v+1,v-1,G)$ and $P(v+1,v-1,B)$ hold. In the case of Lemma $3.(b)$, we verify that $P(v+2,v,B)$ holds given the results of the Lemma $2.(a)$ and $2.(b)$. The verification process is as follows.

Expanding summation terms of the right hand side of equation set (20), (21), (22), (23), (24), and (25) while relying on algebraic properties $\binom{j}{i}+\binom{j}{i+1}=\binom{j+1}{i+1}$ and $\binom{j}{0}=\binom{j}{j}=1$, we observe that the closed-form expressions for the right hand side of equation sets (20), (21), (22), (23), (24), and (25) are reduced to their left hand side counterparts. **QED**

We now generalize Lemma 3 for a fixed z as

Lemma $\mathbf{z+1}$:
(a) The closed-form of equation $P(v+z,v,G)$ is given by Equation (5), and (b) The closed-form of equation $P(v+z,v,B)$ is given by Equation (6).

Proof: Again, the proof is based on mathematical induction. First, we verify that $P(v+z,v,G)$ satisfies the following equality assuming $P(v+z-1,v-1,G)$ and $P(v+z-1,v-1,B)$ hold.

$$\begin{aligned} P(v+z,v,G) = \quad & P(v+z-1,v-1,G)\,\gamma \\ & + P(v+z-1,v-1,B)\,(1-\beta) \end{aligned} \tag{26}$$

Expanding summation terms of the right hand side of Equation (26) while relying on algebraic properties $\binom{j}{i}+\binom{j}{i+1}=\binom{j+1}{i+1}$ and $\binom{j}{0}=\binom{j}{j}=1$, we observe that the closed-form expressions for the right hand side of Equation (26) can be reduced to its left hand side as shown below.

$$\begin{aligned} & P(v+z-1,v-1,G)\,\gamma \\ & \quad + P(v+z-1,v-1,B)\,(1-\beta) \;= \\ & \quad\quad \gamma^{v-z}\,(1-\beta)\,(1-\gamma)\,\{ \\ & \quad\quad\quad \textstyle\sum_{i=0}^{z-1}\binom{z-1}{i}[\binom{v-1}{i}+\binom{v-1}{i+1}] \\ & \quad\quad\quad\quad (\beta\,\gamma)^{z-1-i}\,[(1-\beta)\,(1-\gamma)]^{i} \\ & \quad\quad \}\,g_{ss} \quad + \quad \gamma^{v-z-1}\,(1-\beta)\,\{ \\ & \quad\quad\quad \binom{z}{0}\binom{v-1}{0}(\beta\,\gamma)^{z} \\ & \quad\quad\quad + \textstyle\sum_{i=1}^{z}\binom{z}{i}[\binom{v-2}{i-1}+\binom{v-2}{i}] \\ & \quad\quad\quad\quad (\beta\,\gamma)^{z-i}\,[(1-\beta)\,(1-\gamma)]^{i} \\ & \quad\quad \}\,b_{ss} \\ & = \; P(v+z,v,G) \qquad\qquad z\geq 1\,,\; v\geq z+2 \end{aligned} \tag{27}$$

Next, we verify that $P(v+z,v,B)$ satisfies the following equality given the expressions of $P(v+z-1,v,G)$ and $P(v+z-1,v,B)$ from Lemma z defined in a similar way as previous lemmas.

$$\begin{aligned} P(v+z,v,B) = \quad & P(v+z-1,v,G)\,(1-\gamma) \\ & + P(v+z-1,v,B)\,\beta \end{aligned} \tag{28}$$

Expanding summation terms of the right hand side of Equation (28) while relying on algebraic properties $\binom{j}{i}+\binom{j}{i+1}=\binom{j+1}{i+1}$ and $\binom{j}{0}=\binom{j}{j}=1$, we observe that the closed-form expressions for the right hand side of Equation (28) can be reduced to its left hand side as shown below.

$$\begin{aligned} & P(v+z-1,v,G)\,(1-\gamma) \\ & \quad + P(v+z-1,v,B)\,\beta \;= \\ & \quad\quad \gamma^{v-z+1}\,(1-\gamma)\,\{ \\ & \quad\quad\quad \binom{z-1}{0}\binom{v}{0}(\beta\,\gamma)^{z-1} \\ & \quad\quad\quad + \textstyle\sum_{i=1}^{z-2}\binom{v}{i}[\binom{z-2}{i-1}+\binom{z-2}{i}] \\ & \quad\quad\quad\quad (\beta\,\gamma)^{z-1-i}\,[(1-\beta)\,(1-\gamma)]^{i} \\ & \quad\quad\quad + \binom{v}{z-1}[(1-\beta)\,(1-\gamma)]^{z-1} \\ & \quad\quad \}\,g_{ss} \quad + \quad \gamma^{v-z}\,(1-\beta)\,(1-\gamma)\,\{ \\ & \quad\quad\quad \textstyle\sum_{i=0}^{z-2}\binom{v-1}{i}[\binom{z-1}{i}+\binom{z-1}{i+1}] \\ & \quad\quad\quad\quad (\beta\,\gamma)^{z-1-i}\,[(1-\beta)\,(1-\gamma)]^{i} \\ & \quad\quad\quad + \binom{v-1}{z-1}\binom{z-1}{z-1}[(1-\beta)\,(1-\gamma)]^{z-1} \\ & \quad\quad \}\,b_{ss} \\ & = \; P(v+z,v,B) \qquad\qquad z\geq 2\,,\; v\geq z \end{aligned} \tag{29}$$

This concludes the proof. **QED**

Having proven Lemma $z+1$, we now state our main proposition.

Proposition 1:
For $\forall v,z\in\{1,2,3,\cdots\}$
(a) the closed-form expression for receiving exactly v packets from $v+z$ transmitted packets and winding up in the GOOD state under the Gilbert loss model is given by Equation (5), and
(b) the closed-form expression for receiving exactly v packets from $v+z$ transmitted packets and winding up in the BAD state under the Gilbert loss model is given by (6).

Proof: First we note that Theorem 1 is generalizing Lemma $z+1$ by claiming the accuracy of Equation (5) and Equation (6) for variables v and z rather than a variable v and a fixed parameter z. The proof, hence, has to investigate two cases and is based on mathematical induction. In both cases, the objective is to prove that Equation (26) and Equation (28) hold.

The first case considers the proof for a fixed z, showing that Equation (26) and Equation (28) hold for v assuming they hold for $v-1$. We note that the entire proof of this case matches the proof of Lemma $z+1$.

The second case considers the proof for a fixed v, showing that Equation (26) and Equation (28) hold for z assuming they hold for $z-1$. We note that the closed-form expression for $P(v+z-1,v-1,G)$ and $P(v+z-1,v-1,B)$ can be reached considering the proof of the first case above. The closed-form expression for $P(v+z-1,v-1,G)$ can be obtained as the result of replacing v by $v-1$ in Equation (5). Likewise, the closed-form expression for $P(v+z-1,v-1,B)$ can be obtained as the result of replacing v by $v-1$ in Equation (6). We also note that the closed-form expression for $P(v+z-1,v,G)$ and $P(v+z-1,v,B)$ can be reached considering the induction assumption. The closed-form expression for $P(v+z-1,v,G)$ can be obtained as the result of replacing z by $z-1$ in Equation (5). Likewise, the closed-form expression for $P(v+z-1,v,B)$ can be obtained as the result of replacing z by $z-1$ in Equation

(6). Having explained the reasoning based on which the closed-form expressions of $P(v+z-1, v-1, G)$, $P(v+z-1, v-1, B)$, $P(v+z-1, v, G)$, and $P(v+z, v-1, B)$ can be extracted, reaching the left hand side of Equation (26) and Equation (28) from the right hand side counterparts is the same way described in Equation (27) and Equation (29) as a part of the proof of Lemma $z+1$. **QED**

Appendix II
Proof of Theorem 3.1

Let us make note of the fact that the cost function of Equation (13) consists of a finite number of functions, one for each receiver. These functions are all positive, with a minimum value of zero. Consequently, the positive cost function of Equation (13) has a lower bound. Next, we observe that the cost function of Equation (13) can only decrease in each step considering the operating mechanism of the individual phases of LMMC error control algorithm. Therefore, the sequence of cost function values at each step of the algorithm is a non-increasing sequence with a lower bound. We also note that any non-increasing sequence with a lower bound would converge to a finite number also known as a fixed point. In the case of our optimization problem, converging to a fixed point is equivalent to satisfying the necessary condition for optimality defined below.

For a data group G_k of a media session with N_k receivers, ς_k redundancy groups, partitioning $\Omega_k = \{R_1|\cdots|R_{\varsigma_k}\}$ and redundancy group rate set $\rho = \{\rho_1, \cdots, \rho_{\varsigma_k}\}$, the necessary condition for optimality is defined over partitioning Ω^* and redundancy rate set ρ^* in two steps considering the impact of LMMC iterative error control approach. First, for a fixed partitioning Ω_k^{fixed} and the redundancy group rate set ρ^* such that

$$ECW_k(\Omega_k^{fixed}, \rho^*) \leq ECW_k(\Omega_k^{fixed}, \rho) \qquad (30)$$

for every $\rho \neq \rho^*$. Second, for a fixed redundancy group rate set ρ^{fixed} and the partitioning Ω_k^* such that

$$ECW_k(\Omega_k^*, \rho^{fixed}) \leq ECW_k(\Omega_k, \rho^{fixed}) \qquad (31)$$

for every $\Omega_k \neq \Omega_k^*$.

Since, the two-step necessary condition for optimality holds in each individual step of LMMC error control algorithm, we conclude that LMMC iterative algorithm converges to a local minimum. **QED**

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