Scheduling for MIMO Networks with Rate-Constrained Connectivity Requirements

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Abstract—In this paper, we present a novel definition of connectivity for ad hoc networks, and based on this definition, we propose a scheduling algorithm that can optimize the connectivity of the network through scheduling the active links into different time slots. We also propose an adaptive beamforming algorithm that reduces the impact of the co-channel interference on the links that transmit simultaneously. Simulation results demonstrate that the choice of the number of time slots is critical in optimizing the connectivity of the network.

I. INTRODUCTION

Recently the problem of optimizing network connectivity has been widely studied, especially in the context of ad hoc networks. In [1], different types of connectivity for ad hoc networks are investigated and a variety of approaches are explored to improve the connectivity with the deployment of unmanned air vehicles (UAV). In [2], an algorithm that improves the network connectivity through deployment of relays is proposed. Both [1] and [2] model the network as an undirected graph without considering the interference between the nodes. The interference between nodes is considered in [3], where the authors propose the construction of the link-directional interference graph for the ad hoc network to account for the directional traffic over each link. They investigate a coloring algorithm with two colors on the link-directional interference graph to schedule transmissions in ad hoc networks employing time division multiple access (TDMA) or frequency division multiple access (FDMA). In [4], a heuristic algorithm is proposed based on the generalized power based interference graph.

All of the work mentioned above only focus on single antenna systems. It is well known that link capacity can be significantly increased when the transmitter and receiver are both equipped with multiple antennas. The simulation results in [5] demonstrate that the capacity gain achieved by the deployment of the multiple antennas can also significantly enhance the connectivity of the ad hoc network, which is measured as the number of links that meet a given requirement on the outage capacity or the bit error rate. The drawback of this work is that it only focuses on the physical layer and does not consider the effect of scheduling on the connectivity of the network.

In this paper, we consider the problem of network connectivity in the context of wireless multicast over ad hoc networks. Specifically, for a given source node, we study how it can successfully send its messages to all other nodes in the network. We approach this connectivity problem from the perspective of cross-layer design. The main contributions of this paper are:

- 1) With the assumption that the messages are transmitted over a minimum spanning tree (MST) starting from the source node, we propose a novel definition of network connectivity for ad hoc networks.
- 2) Because we allow different nodes to transmit messages simultaneously, it is more appropriate to model the physical layer problem as an interference channel. To this end, we propose an adaptive beamforming approach that tries to maximize the network connectivity.
- 3) The active links on the MST are operated in TDMA mode. Based on the interference graph and the adaptive beamforming algorithm, we propose a scheduling algorithm to allocate the links into different time slots in order to mitigate the interference between different links.

II. SYSTEM MODEL

In this section, we provide our model for the network configuration and the physical layer channel.

A. Network Configuration

We consider a multi-hop wireless network with a set of N nodes where each node is equipped with M antennas. We assume a scenario in which a given source node wants to share a message with all other nodes through the MST of the network. Fig. 1 depicts the transmission graph of a sample network with $N = 8$ nodes and the directed edges representing the directional links in the spanning tree. For N nodes, there are $N_L = N - 1$ links in the MST. We assume that the network nodes are stationary, and that the channel state information for all links in the MST is known at the source node. Using this information, the source node schedules transmissions in the network and broadcasts its message to all nodes. For simplicity, we assume that in any given time slot, a node can only communicate with

Fig. 1. Transmission graph based on the MST of a network with 8 nodes in which node 1 is the source node.

one other node using a single data stream (*i.e.*, no spatial multiplexing), and the maximum transmit power of each node is limited to $P_{\max}$.

B. Interference Channel Model

When active links in the MST are transmitting at the same time, there will be co-channel interference between the links and the data received by the receiving node of the k th active link can be expressed as

$$\mathbf{y}_k = \sqrt{\frac{P_k}{d_{kk}^\alpha}} \mathbf{H}_{kk} \mathbf{t}_k x_k + \sum_{i=1}^{N_k} \sqrt{\frac{P_i}{d_{ik}^\alpha}} \mathbf{H}_{ik} \mathbf{t}_i x_i + \mathbf{n}, \quad (1)$$

where $\mathbf{H}_{kk} \in \mathbb{C}^{M \times M}$ represents the channel matrix between the transmitter and the receiver of the k th link, $\mathbf{t}_k \in \mathbb{C}^{M \times 1}$ is the beamforming vector at the transmitter of the k th link, the transmitted symbol x_k is a unit-magnitude complex scalar, $P_k \leq P_{\max}$ is the actual transmit power of the k th link, d_{kk} is the distance between the transmitter and receiver of the k th link, α is the pathloss exponent, N_k denotes the number of links that will interfere with link k , $\mathbf{H}_{ik} \in \mathbb{C}^{M \times M}$ represents the channel between the interfering transmitter of the i th link and the receiver of the k th link, d_{ik} is the distance between the transmitter of the i th link and receiver of the k th link, $\mathbf{n}$ is an additive white Gaussian noise vector, $E[\mathbf{n}\mathbf{n}^H] = \sigma^2 \mathbf{I}$ and $\mathbf{I}$ is the $M \times M$ identity matrix.

At the receiver of the k th link, the received signal $\mathbf{y}_k$ is weighted by a receive beamforming vector $\mathbf{w}_k \in \mathbb{C}^{M \times 1}$. The optimal $\mathbf{w}_k$ that maximize the signal to interference plus noise ratio (SINR) of link k is given by

$$\mathbf{w}_k = \mathbf{Q}_k^{-1} \mathbf{H}_{kk} \mathbf{t}_k, \quad (2)$$

where $\mathbf{Q}_k = \sum_{i=1}^{N_k} \frac{P_i}{d_{ik}^\alpha} \mathbf{H}_{ik} \mathbf{t}_i (\mathbf{H}_{ik} \mathbf{t}_i)^H + \sigma^2 \mathbf{I}$. The optimal SINR of link k can be expressed as

$$\text{SINR}_k = \frac{P_k}{d_{kk}^\alpha} (\mathbf{H}_{kk} \mathbf{t}_k)^H \mathbf{Q}_k^{-1} \mathbf{H}_{kk} \mathbf{t}_k. \quad (3)$$

III. CONNECTIVITY OF THE AD HOC NETWORK

A. QoS Requirement of the System

The QoS requirement is represented by the spectral efficiency of the link. For a given frame, we assume that each active link will be allocated a single time slot, and during the time slot each transmitter can occupy the entire available system bandwidth to send data packets to the intended receiver. Define the QoS rate requirement as C_{req} and let the number of slots in a frame be N_s . For link k , to meet the QoS requirement, the SINR_k should satisfy the following condition:

$$\text{SINR}_k \geq \text{SINR}_t, \quad (4)$$

where $\text{SINR}_t = 2^{N_s C_{req}} - 1$. Due to fading and co-channel interference, the actual SINR_k may be smaller than SINR_t . A link is said to experience an outage when the SINR at the receiver is smaller than the threshold SINR_t . For link k , the outage probability can be expressed as

$$P_{out,k} = \Pr\{\text{SINR}_k < \text{SINR}_t\}. \quad (5)$$

B. Definition of the Network Connectivity

Qualitatively, it is usually assumed that a network is connected if there exists a path from a source node to all other nodes. Here we wish to *quantify* network connectivity, so that we might improve it and increase the robustness of the network. A reasonable metric for connectivity would be the product of the probabilities of successful transmission across all links in the network's spanning tree:

$$U = \prod_{k=1}^{N_L} (1 - P_{out,k}). \quad (6)$$

Note that if the SINR of the links in the MST are independent, this product in (6) actually measures the probability that a message can be successfully received at all network nodes. In practice, however, the performance of different links in the MST is correlated due to the interference among the different nodes. Nevertheless, this quantity can still be considered to capture how well connected the network is.

Given the channel gains, SINR_k can be expressed as a function of the beamformer and the transmit power of the active links, the slot number in a frame and the scheduling scheme:

$$\text{SINR}_k = f(\mathbf{t}_1, \dots, \mathbf{t}_{N_L}, P_1, \dots, P_{N_L}, N_s, I^S | \mathbf{H}), \quad (7)$$

where $\mathbf{H} = \{\mathbf{H}_{ik} | i, k = 1, \dots, N_L\}$ represents the set of channels between the transmitter of link i and receiver of link k , $I^S = \{I_1^S, \dots, I_{N_L}^S\}$ represents the link scheduling scheme and I_k^S is the slot number that is allocated to link

k , where $I_k^S \in \{1, \dots, N_s\}$, for $k = 1, \dots, N_L$. Based on (7), the connectivity can be further expressed as

$$U = \prod_{k=1}^{N_L} E[I(SINR_k)] = \prod_{k=1}^{N_L} \int I(SINR_k | \mathbf{H}) p(\mathbf{H}) d\mathbf{H}, \quad (8)$$

where $p(\mathbf{H})$ denotes the probability density function (PDF) of $\mathbf{H}$ and $I(SINR_k)$ is the indicator function defined as

$$I(SINR_k) = \begin{cases} 1, & SINR_k \geq SINR_t \\ 0, & SINR_k < SINR_t. \end{cases} \quad (9)$$

To maximize the connectivity of the network, the optimization problem can be formulated as follows:

$$\begin{aligned} \max_{P_i, \mathbf{t}_i, N_s, I^S} \quad & U \\ \text{subject to} \quad & \|\mathbf{t}_i\| = 1, \\ & P_i \leq P_{max}. \end{aligned} \quad (10)$$

A direct solution to the problem in (10) is very complicated to find. In this paper, we focus on the related optimization problem

$$\begin{aligned} \max_{P_i, \mathbf{t}_i, N_s, I^S} \quad & \sum_{i=1}^{N_L} I(SINR_i) \\ \text{subject to} \quad & \|\mathbf{t}_i\| = 1, \\ & P_i \leq P_{max}. \end{aligned} \quad (11)$$

The relationship between this simplified optimization and the original optimization is analogous to the relationship between the arithmetic and geometric means. In section V, we use coloring from graph theory to optimize the problem in (11).

IV. ADAPTIVE BEAMFORMING FOR THE INTERFERENCE CHANNEL

Consider a scenario in which K links are transmitting simultaneously. In this case, the number of links that can meet the QoS requirement depends on the beamformer and transmit power that each link adopts. For our application, the problem of finding the optimal beamformer and transmit power can be stated as

$$\begin{aligned} \max_{P_i, \mathbf{t}_i} \quad & \sum_{k=1}^K I(SINR_k) \\ \text{subject to} \quad & \|\mathbf{t}_k\| = 1, \\ & P_k \leq P_{max}. \end{aligned} \quad (12)$$

The indicator function in problem (12) is not continuous and the problem is difficult to solve with existing optimization algorithms. Through the following approximation [6], the problem (12) can be converted to finding the maximum value of a constrained nonlinear multivariable function

$$\tilde{I}(SINR_k) = \frac{1}{1 + e^{-\beta(SINR_k - SINR_t)}}, \quad (13)$$

where $\beta > 0$ is an approximation parameter. By replacing $I(SINR_k)$ with $\tilde{I}(SINR_k)$, the problem in (11) can be approximated as

$$\begin{aligned} \max_{P_i, \mathbf{t}_i} \quad & \sum_{k=1}^K \frac{1}{1 + e^{-\beta(SINR_k - SINR_t)}} \\ \text{subject to} \quad & \|\mathbf{t}_k\| = 1, \\ & P_k \leq P_{max}. \end{aligned} \quad (14)$$

Note that when β is small, the sigmoid function $\tilde{I}(SINR_k)$ is smooth. As β goes to infinity, the sigmoid function $\tilde{I}(SINR_k)$ approaches the indicator function. The idea behind the use of this approximation is that by starting with relatively small β values and then increasing β to be larger and larger, the solution involving the indicator function can be found. For each fixed value of β , the problem in (14) can be solved numerically using the trust-region-reflective method[7], [8]. The complete algorithm is described as follows:

Adaptive Beamforming Algorithm

Let N_β be the total number of iterations. We have a predefined parameter set $\beta = \{\beta_1, \dots, \beta_{N_\beta}\}$, where $0 < \beta_1 < \dots < \beta_{N_\beta}$.

Step 0: Initialize the transmit power vector $P^{(0)} = \{P_1^{(0)}, \dots, P_K^{(0)}\}$. Each $P_i^{(0)}$ is a random variable uniformly distributed over $[0, P_{max}]$. Initialize the beamforming vector $\mathbf{T}^{(0)} = \{\mathbf{t}_1^{(0)}, \dots, \mathbf{t}_K^{(0)}\}$. Each element of $\mathbf{T}^{(0)}$ is a random vector uniformly distributed on the unit sphere. Set $n = 1$.

Step 1: Use $P^{(n-1)}$ and $\mathbf{T}^{(n-1)}$ as the initial value. Set $\beta = \beta_n$ in (13). Use the trust-region-reflective method to maximize (12). Denote the solution as $P^{(n)}$ and $\mathbf{T}^{(n)}$.

Step 2: Set $n = n + 1$. If $n \leq N_\beta$, go back to step 1. Otherwise, stop.

V. SCHEDULING FOR MAX CONNECTIVITY

In this section, we propose a scheduling algorithm using the concept of coloring from graph theory. The algorithm is based on the use of the interference and collision graph of the network.

A. The Interference Graph and Greedy Coloring

We use an interference and collision graph (ICG) to model the relationships between the active links. In the ICG each vertex represents a directional link in the transmission graph. We define two types of neighbors in the ICG. The first are *interfering* neighbors and the second are *colliding* neighbors. For interfering neighbors, the simultaneous activation of the corresponding links will cause co-channel interference between the links. For colliding neighbors, the simultaneous activation of the corresponding links will produce collisions between the link transmissions. In the following paragraphs, we give several

definitions that are necessary to describe the scheduling algorithm.

Interference and collision graph: the interference and collision graph G_I can be defined based on the transmission graph G_T . Link k in G_T is represented by a vertex v_k in G_I . Consider link k where the transmitter and the receiver are node q and node w , respectively, and link i , where the transmitter and the receiver are node u and node v , respectively. If any of the following is true: $q = u$ or $w = u$ or $v = q$, v_k and v_i are colliding neighbors. If the simultaneous transmission of link k and i will cause co-channel interference, then v_k and v_i are interfering neighbors. There is an edge $e_{k,i}$ between v_k and v_i if they are colliding or interfering neighbors. The ICG indicates which links will have mutual interference and which links will produce collisions. It may be possible for interfering neighbors to occupy the same time slot, provided that the resulting $SINR$ of the link does not drop below the threshold $SINR_t$. However, colliding neighbors are always assigned to different time slots.

Coloring: Given a set of “colors” in the discrete set $\mathcal{C}$ (colors can be considered as distinct non-negative integers), a coloring of a graph G is an assignment of the elements (or colors) in $\mathcal{C}$ to the vertices of G , one color for each vertex, such that no adjacent vertices occupy the same color. A greedy coloring is a coloring of a graph that enumerates the vertices in a specific order $v_1, \dots, v_n$ and colors v_i with the smallest color that is not occupied by the neighbors of v_i among $v_1, \dots, v_{i-1}$. The vertices can be ordered according to their edge degree, which is the number of edges incident to the vertex [9, chap. 5]. In our application, the colors refer to different time slots that the links (nodes in the interference graph) will be assigned to. To apply coloring to the ICG, we need to define an order for the vertices. Before we proceed to the scheduling order, some related definitions are necessary.

Scheduling freedom: for a link k , the scheduling freedom F_k is the number of available colors that can be allocated to this link when scheduling link k . The higher the value of F_k , the higher the possibility that link k will be allocated to a “good” color (one that leads to low interference).

Collision degree: given a link k , the collision degree C_k^D is the number of its colliding neighbors in the ICG.

Constraint and free color sets: for link k , the interfering color set D_k^I includes colors occupied by the neighbors that interfere with v_k . The unavailable color set D_k^U includes the colors occupied by the neighbors that collide with v_k . The constraint color set is defined as $\mathcal{D}_k^C = \mathcal{D}_k^I - \mathcal{D}_k^I \cap \mathcal{D}_k^U$ and the free color set is defined as $\mathcal{D}_k^F = \mathcal{C} - \mathcal{D}_k^I \cup \mathcal{D}_k^U$.

The higher the value of C_k^D or the smaller the channel gain $h_k = \frac{\lambda_k^2}{d_{kk}^2}$, where λ_k denotes the largest singular value of $\mathbf{H}_{kk}$, the smaller the number of colors that link k can be allocated to, and thus the smaller the value of F_k . To guarantee the links with low scheduling freedom can be

allocated a good color, the scheduling priority of link k can be defined as

$$S_k = C_k^D \cdot W + \frac{1}{h_k}, \quad (15)$$

where W is a constant which is larger than $\max_k \frac{1}{h_k}$.

B. Scheduling Algorithm for Connectivity

Based on the above definitions and analysis, we propose here a scheduling algorithm for optimizing connectivity. The basic idea of the scheduling algorithm is to increase the connectivity of the network through letting several active links share the same time slot when the co-channel interference between these links is small. We extend the idea of greedy coloring and map different time slots to different colors. Through use of the coloring algorithm, different colors can be allocated to adjacent vertices in the ICG, and thus the co-channel interference between the links can be effectively reduced and collisions between the links can be completely avoided. The algorithm is outlined below.

- Step 0: Initialize the active link set $\mathcal{A} = \{l | SINR_l = \frac{P_{max}\lambda_l^2}{d_{ll}^2} \geq SINR_t, l = 1, \dots, N_L\}$, the beamformer set $\mathbf{T} = \{\mathbf{v}_l | l \in \mathcal{A}\}$, where $\mathbf{v}_l$ is the principal singular vector of $\mathbf{H}_{ll}$, the transmit power vector $P = \{P_l = P_{max} | l \in \mathcal{A}\}$, the color set $\mathcal{C} = \{1, 2, \dots, N_s\}$ and the scheduling scheme $I^S = \{I_l^S = 0 | l \in \mathcal{A}\}$. Build the ICG based on the links in $\mathcal{A}$.
- Step 1: Compute the scheduling priority S_l of the vertices $v_l, l \in \mathcal{A}$ in the ICG.
- Step 2: Select the vertex v_m with the highest scheduling priority, construct the free color set D_m^F and the constraint color set $\mathcal{D}_m^C$ for link m . For $i \in \mathcal{D}_m^C$, construct the link set $\mathcal{L}_{m,i}$ which includes the links already assigned to color i .
- Step 3: For each $i \in \mathcal{D}_m^C$, use the adaptive beamforming algorithm of Section IV to find the beamformer and transmit power sets for links in set $\mathcal{L}_{m,i} \cup m$, determine the number of links that can not meet the $SINR_t$ due to the additional interference from link m and denote it as $\Delta N_{m,i}$.
- Step 4: Denote the minimum value among $\Delta N_{m,i}$, for $i \in \mathcal{D}_m^C$ as $\Delta N_{m,j}$. If $\Delta N_{m,j} = 0$, allocate the color j to vertex m , else allocate the smallest color in D_m^F to vertex m . Update the corresponding elements in $\mathbf{T}$, P and I^S .
- Step 5: Set $S_m = 0$, repeat step 2 until each vertex $v_l, l \in \mathcal{A}$ is allocated a color.

The active links will transmit data according to the scheduling result I^S using the beamformers in $\mathbf{T}$ and the transmit power in P .

VI. SIMULATION RESULTS

For our simulations, we consider a network with $N = 30$ nodes, where each node is configured with $M = 2$

Fig. 2. The topology of the network, with $N = 30$.

antennas. The nodes are uniformly distributed in a $25m \times 25m$ area and the pathloss exponent is set to $\alpha = 2$. The topology of the network is shown in Fig. 2, where the node represented by a square is assumed to be the source node, and the edges that connect the nodes represent the links in the MST. There are $N_L = 29$ links in the MST. The QoS requirement is assumed to be $C_{req} = 0.5bps/Hz$. We assume noise with unit power and a maximum transmit power of $P_{max} = 25dB$ for each node. For the adaptive beamforming algorithm, we adopt $\beta_1 < 1$. The frame length is set to unity and the scheduling period is one frame. The connectivity performance is averaged over 300 random channel realizations and the performance for different N_s are provided.

Fig. 3 shows the connectivity performance of the scheduling algorithm. It can be observed that the connectivity is maximized when $N_s = 5$. Increasing N_s will reduce the co-channel interference, but the $SINR_t$ will also increase since each link must achieve a higher rate in the shorter time slots. For values of N_s less than 5, $SINR_t$ is reduced, but now the network becomes interference limited due to the increase in the number of links that must be simultaneously active. Our analysis clearly indicates the trade-off between rate and interference. Fig. 3 also shows the connectivity that would be achieved by the network in the ideal case where the links did not produce any interference to one another. In such a situation, each transmitter would simply use the principal singular vector of the link channel matrix as the transmit beamformer, and would transmit with full power. This curve effectively represents an upper bound on connectivity performance. Note that for values of $N_s \geq 5$, the performance of the algorithm is essentially equal to the bound.

VII. CONCLUSION

We have investigated the problem of scheduling in a MIMO network with rate-constrained connectivity require-

Fig. 3. Connectivity performance and upper bound of the network with $N = 30$, $M = 2$, $C_{req} = 0.5bps/Hz$ and $\frac{P_{max}}{\sigma^2} = 25dB$.

ments. Based on a novel definition of network connectivity, a scheduling algorithm was proposed using the concept of greedy coloring from graph theory. The scheduling algorithm also relies on an adaptive interference network beamforming algorithm to reduce the impact of the co-channel interference. The simulation results show that there exists a certain number of time slots that can maximize the connectivity of the network, and that the performance at this optimal value is close to the performance that would be achieved if the network experienced no interference.

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