Rank Minimization Designs for Underlay MIMO Cognitive Radio Networks with Completely Unknown Primary CSI

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Abstract—This paper studies a novel underlay MIMO cognitive radio (CR) network where the instantaneous or statistical channel state information (CSI) of the interfering channels to the primary receivers (PRs) is completely unknown to the CR. We first show that low-rank CR interference is preferable for improving the throughput of the PRs compared with spreading less power over more transmit dimensions. Based on this observation, we then propose a rank minimization CR transmission strategy assuming a minimum information rate must be guaranteed on the CR main channel. We propose a simple solution referred to as frugal waterfilling (FWF) that uses the least amount of power required to achieve the rate constraint with a minimum-rank transmit covariance matrix. We also present two heuristic approaches that have been used in prior work to transform rank minimization problems into convex optimization problems. We demonstrate that the direct FWF solution leads to higher PR throughput even though it has higher interference “temperature” (IT) compared with the heuristic methods. This calls into question the use of IT as a metric for CR interference.

I. INTRODUCTION

Cognitive radios (CRs) have gained prominence as an efficient method of improving spectrum utilization by allowing coexistence with licensed networks, an idea referred to as dynamic spectrum access (DSA). One popular variant of DSA is known as spectrum underlay, where underlay cognitive transmitters (UCTs) operate simultaneously with licensed (primary) users, but adapt their transmission parameters so as to confine the interference perceived at the primary receivers (PRs) to a pre-specified threshold [1]. Therefore, a fundamental challenge for the CR is to balance between maximizing its own transmit rate and minimizing the interference it causes to the PRs. In CR networks with single-antenna nodes, this is usually achieved by exploiting some knowledge of the interfering cross-channels to the PRs at the UCT and performing some admission control algorithms with power control [2].

If UCTs are equipped with multiple antennas, the available spatial degrees of freedom can be used to mitigate interference to the PRs during transmission to the underlay cognitive receivers (UCRs). Multi-antenna CR networks have recently received extensive attention, assuming some knowledge of the interfering cross-channels to the PRs at the UCT, either perfect PR cross-channel state information (CSI) [3], [4], perturbed PR CSI [5], [6], or statistical PR CSI [7], [8]. However, the UCT may not have the luxury of knowing the CSI of the cross

links to the PRs, as the primary system would not deliberately coordinate the collection of CSI for the CR system.

In this work, we consider the novel scenario where both the realizations and distribution of the PR cross-channels are *completely unknown* at the UCT, thereby precluding the overwhelming majority of existing spectrum underlay schemes in the literature [3], [4]. Specifically, we propose a rank minimization transmission strategy for the UCT while maintaining a minimum information rate on the CR link, and we present a simple solution referred to as frugal waterfilling (FWF) that uses the least amount of power required to achieve the rate constraint with a minimum-rank covariance matrix. In the context of MIMO interference channels (for which the CR underlay network is a special case), rank-minimization has been shown to be a reinterpretation of interference alignment [9], but this approach requires knowledge of interfering cross-channels and treats the overall system sum rate or degrees-of-freedom as the performance metric, assumptions which are both markedly different from the underlay CR scenario we consider.

We also describe two heuristic approaches that have been used in prior work to transform rank minimization problems (RMP) into problems that can be solved via convex optimization. These approaches approximate the rank objective function with two relaxations, one based on the nuclear norm [17], and the other on a log-determinant function [18]. We show theoretically and via numerical simulation that minimizing the rank of the UCT spatial covariance matrix leads to the highest PR throughput in general Rayleigh-fading channels, compared with spreading the transmit power over more dimensions. Furthermore, our simulations indicate that FWF provides a higher PR throughput than the nuclear-norm and log-det heuristic solutions, even though FWF has a higher interference “temperature” (IT). This suggests that the commonly used IT metric does not accurately capture the impact of the CR interference on PR performance. Instead, we propose a metric based on interference leakage (IL) rate that more accurately reflects the influence of the CR interference.

This paper is organized as follows. The underlay system model is introduced in Section II. A rank minimization UCT transmit strategy for the underlay MIMO CR system with completely unknown PR CSI is proposed in Section III. In

Section IV, the FWF and heuristic solutions for the rank minimization design problem are presented. Section V presents several numerical simulations, and we conclude in Section VI.

II. SYSTEM MODEL

Fig. 1. MIMO cognitive radio network where a primary system and an underlay CR system share the same spectral band.

A generic underlay MIMO CR network is shown in Fig. 1, where a primary system and an underlay CR system share the same spectral band. Since the UCT transmit strategy we proposed in the paper is independent of the cross-channels to primary users, the number of PRs and primary transmitters (PTs) and their array sizes can be made arbitrary; however to simplify notation we will consider a solitary multi-antenna PT-PR pair. To introduce the problem, we consider a scenario with a single UCR, and mention that our work can be generalized to the case of multiple UCRs, i.e., an underlay MIMO CR downlink network, by adopting a modified block-diagonalization precoding strategy [10].

We consider a multi-antenna UCT equipped with N_a antennas, which transmits a signal vector $\mathbf{x}_s \in \mathbb{C}^{N_a \times 1}$ to its N_s -antenna underlay cognitive receiver. The PT is equipped with N_p antennas and transmits signal vector $\mathbf{x}_p \in \mathbb{C}^{N_p \times 1}$ to the N_r -antenna PR. Thus, the UCR observes

$$\mathbf{y}_s = \mathbf{G}_1 \mathbf{x}_s + \mathbf{G}_2 \mathbf{x}_p + \mathbf{n}_s, \quad (1)$$

where $\mathbf{G}_1 \in \mathbb{C}^{N_s \times N_a}$, $\mathbf{G}_2 \in \mathbb{C}^{N_s \times N_p}$ are the complex MIMO channels from the UCT and PT, and $\mathbf{n}_s \sim \mathcal{CN}(\mathbf{0}, \sigma_s^2 \mathbf{I})$ is complex additive white Gaussian noise. We assume Gaussian signaling with zero mean and second-order statistics $\mathcal{E}\{\mathbf{x}_s \mathbf{x}_s^H\} = \mathbf{Q}_s$, and we assume that the average UCT transmit power is bounded:

$$\text{Tr}(\mathbf{Q}_s) \leq P_s. \quad (2)$$

The signal at the PR is given by

$$\mathbf{y}_p = \mathbf{H}_1 \mathbf{x}_p + \mathbf{H}_2 \mathbf{x}_s + \mathbf{n}_p, \quad (3)$$

where $\mathbf{H}_1 \in \mathbb{C}^{N_r \times N_p}$, $\mathbf{H}_2 \in \mathbb{C}^{N_r \times N_a}$ are the channels from the PT and UCT (assumed to be full-rank), and $\mathbf{n}_p \sim \mathcal{CN}(\mathbf{0}, \sigma_p^2 \mathbf{I})$ is complex additive white Gaussian noise. The primary signal is also modeled as a zero-mean complex Gaussian signal with covariance matrix $\mathbf{Q}_p$ and average power constraint $\text{Tr}(\mathbf{Q}_p) \leq P_p$. We will assume $\mathbf{Q}_p$ is fixed and

the channels are mutually independent and each composed of i.i.d. zero-mean circularly symmetric complex Gaussian entries, then focus our attention on the design of the UCT transmit signal.

We assume there is no cooperation between the PT and UCT during transmission, and that both receivers treat interfering signals as noise. The network is essentially an asymmetric 2-user MIMO interference channel, where the UCT attempts to minimize the interference to the PR, but no such reciprocal gesture is made by the PT. The interference covariance matrix at the PR is

$$\mathbf{K}_p = \mathbf{H}_2 \mathbf{Q}_s \mathbf{H}_2^H. \quad (4)$$

Define the interference temperature at the PR as [3]-[8]

$$T_p(\mathbf{Q}_s) = \text{Tr}(\mathbf{K}_p), \quad (5)$$

and the interference leakage rate from UCT to the PR as

$$R_l(\mathbf{Q}_s) = \log_2 |\mathbf{K}_p + \sigma_p^2 \mathbf{I}|. \quad (6)$$

Without knowledge of $\mathbf{H}_2$ or its distribution, the UCT cannot directly optimize the PR interference temperature or outage probability as in existing underlay proposals [3]-[8]. To our best knowledge, precoding strategies for MIMO underlay systems with completely unknown primary CSI have not been presented in the literature thus far. In addition to [3]-[8] not being applicable, the blind interference alignment method for the 2-user MIMO interference channel [11] is also precluded since it requires knowledge of the cross-channel coherence intervals, which we assume are also unknown.

The PT achieves the following rate on its link:

$$R_p(\mathbf{Q}_s) = \log_2 |\mathbf{I} + \mathbf{H}_1 \mathbf{Q}_p \mathbf{H}_1^H (\mathbf{K}_p + \sigma_p^2 \mathbf{I})^{-1}|. \quad (7)$$

Similarly, the achievable rate on the CR link is

$$R_s(\mathbf{Q}_s) = \log_2 |\mathbf{I} + \mathbf{G}_1 \mathbf{Q}_s \mathbf{G}_1^H (\mathbf{K}_s + \sigma_s^2 \mathbf{I})^{-1}|, \quad (8)$$

where $\mathbf{K}_s = \mathbf{G}_2 \mathbf{Q}_p \mathbf{G}_2^H$ represents the interference from the PT.

III. A RANK MINIMIZATION STRATEGY FOR CSI-UNAWARE UNDERLAY TRANSMISSION

We now outline the fundamental motivation underlying the UCT transmission strategy proposed in this work. As we have seen, due to a lack of knowledge of $\mathbf{H}_2$ or its distribution, the UCT cannot directly optimize the PR interference temperature. Hence, we propose an alternative transmission strategy where the UCT tries to minimize a measure of the interference caused to the PR in a “best-effort” sense, while achieving a target information rate on the CR link. We first show that in the clairvoyant case where the UCT has some knowledge of the cross channel to the PR, a rank-1 UCT covariance matrix $\mathbf{Q}_s$ on average causes the least interference to the primary link, as described in the following proposition.

Proposition 1: The optimal UCT transmit covariance matrix, denoted by $\mathbf{Q}_s^*$, that maximizes the PR rate $\mathcal{E}\{R_p(\mathbf{Q}_s)\}$ under an average power constraint $\text{Tr}(\mathbf{Q}_s) = P_s$ is of rank one, i.e., $\text{rank}(\mathbf{Q}_s^*) = 1$.

Proof: Since $\mathbf{Q}_s = \mathbf{Q}_s^H \geq \mathbf{0}$, it can be expressed as $\mathbf{Q}_s = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, where $\mathbf{\Lambda}$ is the diagonal matrix of eigenvalues of $\mathbf{Q}_s$ and $\mathbf{U}$ is the unitary matrix with columns consisting of the eigenvectors of $\mathbf{Q}_s$. Defining $\tilde{\mathbf{H}}_2 = \mathbf{H}_2\mathbf{U}$, it follow from Lemma 5 in [12] that the distribution of $\tilde{\mathbf{H}}_2$ is the same as that of $\mathbf{H}_2$. As a result, the average PR rate can be expressed as

$$\begin{aligned} \mathcal{E}\{R_p(\mathbf{Q}_s)\} &= \Phi(\mathbf{\Lambda}) \\ &= \mathcal{E}\left\{\log_2 \left[\det \left(\mathbf{I} + \mathbf{H}_1 \mathbf{Q}_p \mathbf{H}_1^H \left(\tilde{\mathbf{H}}_2 \mathbf{\Lambda} \tilde{\mathbf{H}}_2^H + \sigma_p^2 \mathbf{I} \right)^{-1} \right) \right] \right\} \end{aligned}$$

Thus, the problem we consider is equivalent to determining the diagonal matrix $\mathbf{\Lambda}$ with real nonnegative entries to maximize $\Phi(\mathbf{\Lambda})$ under the constraint $\text{Tr}(\mathbf{\Lambda}) = P_s$.

From [13, Lemma 3], we have that $\Phi(\mathbf{\Lambda})$ is a convex function of $\mathbf{\Lambda}$. Note that given any permutation matrix $\mathbf{\Pi}$, we see that, again using Lemma 5 in [12], that

$$\Phi(\mathbf{\Pi}\mathbf{\Lambda}\mathbf{\Pi}^H) = \Phi(\mathbf{\Lambda}). \quad (9)$$

From convexity, we have

$$\Phi \left(\frac{1}{N_a!} \sum_{\mathbf{\Pi}} \mathbf{\Pi}\mathbf{\Lambda}\mathbf{\Pi}^H \right) \leq \frac{1}{N_a!} \sum_{\mathbf{\Pi}} \Phi(\mathbf{\Pi}\mathbf{\Lambda}\mathbf{\Pi}^H) = \Phi(\mathbf{\Lambda})$$

where we have used Jensen's inequality. From the transmit power constraint, we have $\frac{1}{N_a!} \sum_{\mathbf{\Pi}} \mathbf{\Pi}\mathbf{\Lambda}\mathbf{\Pi}^H = (P_s/N_a)\mathbf{I}_{N_a}$. Thus, we have proved that the lowest PR rate is obtained by $\mathbf{\Lambda} = (P_s/N_a)\mathbf{I}_{N_a}$. Further, due to convexity, we can argue that the largest PR rate is obtained by a point farthest away from $\mathbf{\Lambda} = (P_s/N_a)\mathbf{I}_{N_a}$. Thus, we want $\mathbf{\Lambda}^* = \text{diag}(\lambda_1^*, \dots, \lambda_{N_a}^*)$ that satisfies [14]

$$\max \sum_{i=1}^{N_a} \left(\lambda_i - \frac{P_s}{N_a} \right)^2 \quad \text{s. t.} \quad \sum_{i=1}^{N_a} \lambda_i = P_s \quad (10)$$

Expanding the square leads to

$$\begin{aligned} \sum_{i=1}^{N_a} \left(\lambda_i - \frac{P_s}{N_a} \right)^2 &= \sum_{i=1}^{N_a} \lambda_i^2 - 2 \frac{P_s}{N_a} \sum_{i=1}^{N_a} \lambda_i + \frac{P_s^2}{N_a} \\ &= P_s^2 \left(\sum_{i=1}^{N_a} \left(\frac{\lambda_i}{P_s} \right)^2 - \frac{1}{N_a} \right) \\ &\leq P_s^2 \left(\sum_{i=1}^{N_a} \frac{\lambda_i}{P_s} - \frac{1}{N_a} \right) \\ &= P_s^2 \left(1 - \frac{1}{N_a} \right) \end{aligned} \quad (11)$$

where the inequality is due to the fact that $\sum_{i=1}^{N_a} \left(\frac{\lambda_i}{P_s} \right)^2 \leq \sum_{i=1}^{N_a} \frac{\lambda_i}{P_s} = 1$. Equality in (11) is attained when $(\lambda_1^*, \dots, \lambda_{N_a}^*)$ has all zeros except for one nonzero entry. Hence, we conclude that $\text{rank}(\mathbf{Q}_s^*) = \text{rank}(\mathbf{\Lambda}^*) = 1$. ■

Therefore, in the clairvoyant case where the UCT has some knowledge of the primary CSI, a rank-1 $\mathbf{Q}_s$ causes the least

interference on average to the primary link, while a full-rank $\mathbf{Q}_s$ causes the most interference¹. Of course, the optimal $\mathbf{Q}_s$ will depend on the PR CSI, which we have assumed is unavailable. Still, the result motivates the use of a low-rank transmit covariance at the UCT. It is evident that the UCT does not require knowledge of the PR CSI to minimize the rank of $\mathbf{Q}_s$ needed to achieve a rate target R_b on the CR link. To exploit this observation, we henceforth pose the UCT precoder design problem when the PR CSI is completely unknown as

$$\begin{aligned} (\text{P0}) : \quad & \min_{\mathbf{Q}_s} \quad \text{rank}(\mathbf{Q}_s) \\ & \text{s.t.} \quad R_s(\mathbf{Q}_s) = R_b \\ & \quad \text{Tr}(\mathbf{Q}_s) \leq P_s \\ & \quad \mathbf{Q}_s \succ \mathbf{0}. \end{aligned} \quad (12)$$

This is an RMP, and in general is computationally hard to solve since the rank function is quasi-concave and not convex. As explained below, however, in this case a simple waterfilling solution can be obtained.

IV. SOLUTIONS FOR THE RANK MINIMIZATION DESIGN

Notice that the design problem (P0) is ill-posed in the sense that there are potentially an infinite number of solutions. Suppose that we find one minimum-rank solution to (P0) such that $\mathbf{Q}_s = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ for some unitary matrix $\mathbf{U}$ and diagonal matrix $\mathbf{\Lambda}$ satisfies $R_s(\mathbf{Q}_s) = R_b$, $\text{Tr}(\mathbf{\Lambda}) \leq P_s$. If the required power $\text{Tr}(\mathbf{\Lambda})$ is strictly smaller than P_s , then we could find an infinite number of solutions by making small perturbations to $\mathbf{U}$, which while leading to a higher power requirement, still would require less power than P_s . Obviously, the solution with least power is more desirable for our underlay CR system in order to minimize the interference caused to the PR, and this solution can easily be found using the *frugal waterfilling* (FWF) approach described next.

A. FWF Approach

The FWF solution seeks to find the least amount of power required to achieve the CR rate target of R_b with the minimum rank transmit covariance $\mathbf{Q}_s$. The optimization problem can be solved using a combination of the classic waterfilling (CWF) algorithm and a simple bisection line search. The description for FWF is outlined as Algorithm IV-A.1 below. In brief, FWF cycles through the possible number of transmit dimensions in ascending order starting with a rank-one $\mathbf{Q}_s$, and at each step computes the transmit power required to meet the rate constraint R_b based on CWF. This requires a simple line search over the transmit power for each step. Once a solution is found that satisfies the transmit power constraint, the algorithm terminates. If no feasible solution is found for all N_a transmit dimensions, the CR link will be in outage.

While FWF finds an efficient solution to (P0), in general rank minimization problems are difficult to solve and often require exponential-time complexity. Consequently, heuristic approximations to the matrix rank have been proposed as

¹This notion has been echoed in prior work on MIMO interference channels [15], [16]

Algorithm IV-A.1 Frugal Waterfilling Algorithm for the UCT Rank Minimization Design [20]

Require: $P_s > 0, R_b > 0$

set $r = \text{rank}(\mathbf{G}_1)$

for $M = 1$ to r **do**

Solve:

$$p(M) = \min \text{Tr}(\mathbf{Q}_s)$$

$$\text{s. t. } \log_2 \left| \mathbf{I} + \mathbf{G}_1 \mathbf{Q}_s \mathbf{G}_1^H (\mathbf{K}_s + \sigma_s^2 \mathbf{I})^{-1} \right| = R_b .$$

if $p(M) \leq P_s$ **then**

$\mathbf{Q}_s$ determined by waterfilling $p(M)$ over M largest singular values of $(\mathbf{K}_s + \sigma_s^2 \mathbf{I})^{-1/2} \mathbf{G}_1$, **break**.

end if

end for

if $p(r) > P_s$ **then**

Declare outage

end if

alternatives in order to yield simpler optimization problems. In particular, the nuclear norm [17] and log-determinant [18] heuristics have been proposed in order to convexify RMP problems like (P0) and provide approximate solutions with polynomial-time complexity. In the discussion below, we show how these approximations can be applied to the RMP we consider in this paper.

B. Nuclear Norm and Log-det Heuristic

The *nuclear norm heuristic* is based on the fact the nuclear norm (sum of the singular values of a matrix) is the convex envelope of the rank function on the unit ball. When the matrix is positive semidefinite, the nuclear norm is the same as the trace function. As a result, the design problem (P0) can be formulated as follows:

$$\begin{aligned} \text{(P1): } \min \quad & \text{Tr}(\mathbf{Q}_s) \\ \text{s.t.} \quad & R_s(\mathbf{Q}_s) = R_b \\ & \text{Tr}(\mathbf{Q}_s) \leq P_s \\ & \mathbf{Q}_s \succeq \mathbf{0} . \end{aligned} \quad (13)$$

The nuclear norm heuristic (P1) is a convex optimization problem and can be solved using the CWF algorithm together with a bisection line search (similar to FWF). It is well known that under the CWF algorithm, the lowest transmit power is achieved when $\text{rank}(\mathbf{Q}_s)$ is chosen as large as possible (up to $\text{rank}(\mathbf{G}_1)$). This is clearly contrary to the rank-minimization design formulation, which indicates that the nuclear norm approach for this problem is a poor approximation.

Using the function $\log \det(\mathbf{Q}_s + \delta \mathbf{I})$ as a smooth surrogate for $\text{rank}(\mathbf{Q}_s)$, the *log-det heuristic* can be described as follows:

$$\begin{aligned} \text{(P2): } \min \quad & \log \det(\mathbf{Q}_s + \delta \mathbf{I}) \\ \text{s.t.} \quad & R_s(\mathbf{Q}_s) = R_b \\ & \text{Tr}(\mathbf{Q}_s) \leq P_s \\ & \mathbf{Q}_s \succeq \mathbf{0}, \end{aligned} \quad (14)$$

where $\delta \geq 0$ can be interpreted as a small regularization constant (we choose $\delta = 10^{-6}$ for numerical examples). Since the surrogate function $\log \det(\mathbf{Q}_s + \delta \mathbf{I})$ is smooth on the positive definite cone, it can be minimized using a local minimization method. We use iterative linearization to find a local minimum to the optimization problem (P2) [18]. Let $\mathbf{Q}_s^{(k)}$ denote the k th iteration of the optimization variable $\mathbf{Q}_s$. The first-order Taylor series expansion of $\log \det(\mathbf{Q}_s + \delta \mathbf{I})$ about $\mathbf{Q}_s^{(k)}$ is given by

$$\begin{aligned} \log \det(\mathbf{Q}_s + \delta \mathbf{I}) \approx & \log \det(\mathbf{Q}_s^{(k)} + \delta \mathbf{I}) + \\ & \text{Tr}[(\mathbf{Q}_s^{(k)} + \delta \mathbf{I})^{-1}(\mathbf{Q}_s - \mathbf{Q}_s^{(k)})] , \end{aligned} \quad (15)$$

where we have used the rule that $\partial(\ln(\det(\mathbf{X}))) = \text{Tr}(\mathbf{X}^{-1} \partial \mathbf{X})$. Hence, we could attempt to minimize $\log \det(\mathbf{Q}_s + \delta \mathbf{I})$ by iteratively minimizing the local linearization (15). This leads to

$$\mathbf{Q}_s^{(k+1)} = \arg \min \text{Tr}[(\mathbf{Q}_s^{(k)} + \delta \mathbf{I})^{-1} \mathbf{Q}_s] . \quad (16)$$

If we choose $\mathbf{Q}_s^{(0)} = \mathbf{I}$, the first iteration of (16) is equivalent to minimizing the trace of $\mathbf{Q}_s$. Therefore, this heuristic can be viewed as a refinement of the nuclear norm heuristic. Therefore, we always pick $\mathbf{Q}_s^{(0)} = \mathbf{I}$, so that $\mathbf{Q}_s^{(1)}$ is the result of the trace heuristic, and the iterations that follow try to reduce the rank of $\mathbf{Q}_s^{(1)}$ further.

Note that at each iteration we will solve a weighted trace minimization problem, which is equivalent to the following optimization problem

$$\begin{aligned} \text{(P2-1): } \min \quad & \text{Tr}(\mathbf{F}^H \mathbf{A} \mathbf{F}) \\ \text{s.t.} \quad & \log_2 \det(\mathbf{I} + \mathbf{F}^H \mathbf{R} \mathbf{F}) = R_b \\ & \text{Tr}(\mathbf{F}^H \mathbf{F}) \leq P_s , \end{aligned} \quad (17)$$

where $\mathbf{A} = (\mathbf{Q}_s^{(k)} + \delta \mathbf{I})^{-1}$ and $\mathbf{R} = \mathbf{G}_1^H (\mathbf{K}_s + \sigma_s^2 \mathbf{I})^{-1} \mathbf{G}_1$. This is a Schur-concave optimization problem with multiple trace/log-det constraints. From Theorem 1 in [19], the optimal solution to problem (P2-1) is $\mathbf{F}^* = \mathbf{A}^{-1/2} \mathbf{U} \mathbf{\Sigma}$, and $\mathbf{Q}_s^{(k+1)} = \mathbf{F}^* \mathbf{F}^{*H}$ is an optimal solution to (16), where $\mathbf{A}^{-1/2} = \mathbf{U}_A \mathbf{\Lambda}_A^{-1/2} \mathbf{U}_A^H$, $\mathbf{U}_A$ and $\mathbf{\Lambda}_A$ are defined in the eigen-decomposition $\mathbf{A} = \mathbf{U}_A \mathbf{\Lambda}_A \mathbf{U}_A^H$, $\mathbf{U}$ is a unitary matrix, and $\mathbf{\Sigma} = \text{diag}(\sqrt{\mathbf{p}})$ is a rectangular diagonal matrix containing the square-root of the power allocation vector $\mathbf{p}$.

Substituting the optimal solution structure $\mathbf{F}^*$ into (P2-1), we have the following equivalent problem

$$\begin{aligned} \text{(P2-2): } \min \quad & \text{Tr}(\mathbf{\Sigma} \mathbf{\Sigma}^H) \\ \text{s.t.} \quad & \log_2 \det(\mathbf{I} + \mathbf{\Sigma}^H \mathbf{U}^H \tilde{\mathbf{R}} \mathbf{U} \mathbf{\Sigma}) = R_b \\ & \text{Tr}(\mathbf{U}^H \mathbf{A}^{-1} \mathbf{U} \mathbf{\Sigma} \mathbf{\Sigma}^H) \leq P_s \end{aligned} \quad (18)$$

where $\tilde{\mathbf{R}} = \mathbf{A}^{-1/2} \mathbf{R} \mathbf{A}^{-1/2}$. It is found that the equivalent problem (P2-2) is essentially equivalent to the converse formulation

$$\begin{aligned} \text{(P2-3): } \max \quad & \log_2 \det(\mathbf{I} + \mathbf{\Sigma}^H \mathbf{U}^H \tilde{\mathbf{R}} \mathbf{U} \mathbf{\Sigma}) \\ \text{s.t.} \quad & \text{Tr}(\mathbf{\Sigma} \mathbf{\Sigma}^H) = P_0 \\ & \text{Tr}(\mathbf{U}^H \mathbf{A}^{-1} \mathbf{U} \mathbf{\Sigma} \mathbf{\Sigma}^H) \leq P_s . \end{aligned} \quad (19)$$

This is because both formulations describe the same tradeoff curve of performance versus power. Therefore, the quality-constrained problem (P2-2) can be numerically solved by iteratively solving the power-constrained problem (P2-3), combined with the bisection method.

The problem formulation in (P2-3) is a Schur-convex optimization problem with two trace constraints. Using Theorem 1 in [19] again, if we let $\tilde{\mathbf{R}} = \mathbf{U}_{\tilde{\mathbf{R}}} \mathbf{\Lambda}_{\tilde{\mathbf{R}}} \mathbf{U}_{\tilde{\mathbf{R}}}^H$ denote the eigen-decomposition of $\tilde{\mathbf{R}}$, then the optimal unitary matrix $\mathbf{U}$ will be chosen as $\mathbf{U}_{\tilde{\mathbf{R}}}$. Denoting $\mathbf{a} = \text{diag}(\mathbf{U}^H \mathbf{A}^{-1} \mathbf{U})$ and letting $\lambda_{\tilde{\mathbf{R}},1} \geq \lambda_{\tilde{\mathbf{R}},2} \geq \dots \geq \lambda_{\tilde{\mathbf{R}},N_a}$ represent the diagonal elements of $\mathbf{\Lambda}_{\tilde{\mathbf{R}}}$, the optimal power allocation can be shown to have the form of a *multilevel* waterfilling solution:

$$p_i = \left(\frac{1}{\mu + a_i \nu} - \frac{1}{\lambda_i} \right)^+, \quad i = 1, \dots, N_a \quad (20)$$

where a_i is the i th element of $\mathbf{a}$, and μ, ν can be shown to be the nonnegative Lagrange multipliers associated with the two power constraints. The algorithmic description for the *log-det heuristic* approach is outlined in Algorithm IV-B.1.

Algorithm IV-B.1 Iterative Log-det Heuristic Algorithm for the UCT Rank Minimization Design

Require: $P_s > 0, R_b > 0$,

set $\delta = 10^{-6}, \Delta = 10^{-3}, k = 0$,

$\mathbf{Q}_s^{(0)} = \mathbf{I}, \mathbf{R} = \mathbf{G}_1^H (\mathbf{K}_s + \sigma_s^2 \mathbf{I})^{-1} \mathbf{G}_1$.

repeat

$\mathbf{A} = (\mathbf{Q}_s^{(k)} + \delta \mathbf{I})^{-1}, \tilde{\mathbf{R}} = \mathbf{A}^{-1/2} \mathbf{R} \mathbf{A}^{-1/2}$,

$\text{eig}(\tilde{\mathbf{R}}) = \mathbf{U}_{\tilde{\mathbf{R}}} \mathbf{\Lambda}_{\tilde{\mathbf{R}}} \mathbf{U}_{\tilde{\mathbf{R}}}^H$.

set $\mathbf{U} = \mathbf{U}_{\tilde{\mathbf{R}}}, \mathbf{a} = \text{diag}(\mathbf{U}^H \mathbf{A}^{-1} \mathbf{U})$.

Solve:

$$\begin{aligned} \min \quad & \mathbf{1}^T \mathbf{p} \\ \text{s. t.} \quad & \sum_i \left(\log_2(1 + p_i \lambda_{\tilde{\mathbf{R}},i}) \right) = R_b \\ & \mathbf{a}^T \mathbf{p} \leq P_s. \end{aligned}$$

$\Sigma_p = \text{diag}(\sqrt{\mathbf{p}})$

$\mathbf{F} = \mathbf{A}^{-1/2} \mathbf{U} \Sigma_p$

$\mathbf{Q}_s^{(k+1)} = \mathbf{F} \mathbf{F}^H$

until $\log_2(\det(\mathbf{Q}_s^{(k)} + \delta \mathbf{I})) - \log_2(\det(\mathbf{Q}_s^{(k+1)} + \delta \mathbf{I})) < \Delta$

if $\mathbf{a}^T \mathbf{p} > P_s$ **then**

Declare outage

else

$\mathbf{Q}_s = \mathbf{Q}_s^{(k+1)}$

end if

V. NUMERICAL RESULTS

In this section, we present some numerical examples to demonstrate the performance of the proposed rank-minimization UCT transmit covariance designs in MIMO cognitive radio networks. We assume that each node in the CR network is equipped with 6 antennas, i.e., $N_p = N_r = N_a = N_s = 6$. In all simulations, the channel matrices and background noise samples were assumed to be composed of

independent, zero-mean Gaussian random variables with unit variance. In situations where the desired rate for the UCR cannot be achieved with the given P_s , rather than indicate an outage, we simply assign all power for the UCT to transmit signals. The performance is evaluated by averaging over 1000 independent channel realizations.

Fig. 2. Power and dimension allocation versus UCR desired rate.

Fig. 2 illustrates the average fractional power ρ and the average number of subchannels N required to achieve the UCR desired rates by FWF, CWF and the log-det heuristic algorithms, when $P_s = 100$ and $P_p = 10$. It is shown that the trace and rank of the UCT transmit covariance matrix $\mathbf{Q}_s$ are two competing objectives, and any scheme which requires more power occupies fewer spatial dimensions. Among the three methods, the FWF approach offers the smallest feasible number of transmit dimensions, while CWF demands the largest spatial footprint.

Fig. 3. Achieved PR rate versus UCR desired rate.

The achieved average primary user data rates for all the

three methods are depicted in Fig. 3. As expected, the lower-rank UCT transmit covariance causes less degradation on average to the primary communication link, thus resulting in a higher PR rate in accordance with Proposition 1. Compared to CWF, either of the proposed modified waterfilling algorithm or the log-det heuristic lead to more desirable PR rates, with the advantage of FWF being more pronounced as R_b increases.

Fig. 4. Two metrics of PR Interference versus UCR desired rate.

Fig. 4 compares two PR interference metrics for the three algorithms; one is the newly defined UCT-to-PR leakage rate (6), and the other one is the commonly-used interference temperature (5). Interestingly, the two metrics show opposite trends; the algorithm with the highest leakage rate has the lowest interference temperature, and vice versa. More importantly, the order of algorithm performance based on leakage rate is aligned with the achievable rate of primary link, while interference temperature is not. This indicates that the commonly used interference temperature may not be a good metric for gauging the impact of the CR interference on the primary link.

VI. CONCLUSION

This paper has proposed a rank minimization precoding strategy for underlay MIMO CR systems with completely unknown primary CSI, assuming a minimum information rate must be guaranteed on the CR main channel. We presented a simple waterfilling approach can be used to find the minimum rank transmit covariance that achieves the desired CR rate with minimum power. We also presented two alternatives to FWF that are based on convex approximations to the min-rank criterion, one that leads to conventional waterfilling for our CR problem, and another based on a log-det heuristic. The CWF approach turns out to be a poor approximation to the min-rank objective, while the log-det approach provides performance similar to FWF, although FWF consistently leads to the highest throughput for the primary link. We also observed that reducing the interference temperature metric is

surprisingly not consistent with improving the PR throughput; in particular, FWF has the highest interference temperature of the algorithms studied, but also leads to the highest PR rate. As an alternative, we proposed an interference leakage metric that turns out to be a better indicator of the impact of the CR on the primary link.

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