Adjustable Phase Shift Pilots for Sparse Massive MIMO-OFDM Channels

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Abstract—We propose adjustable phase shift pilots (APSPs) for wideband massive multiple-input multiple-output (MIMO) systems employing orthogonal frequency division multiplexing (OFDM) to reduce the pilot overhead. Based on a physically motivated channel model, we first establish a relationship between channel space-frequency correlations and the channel power angle-delay (AD) spectrum in the massive antenna array regime, which reveals the channel sparsity in massive MIMO-OFDM. With this channel model, we then investigate channel estimation (CE) for massive MIMO-OFDM with APSPs. We show that CE performance in terms of sum mean square error can be minimized if the user terminals' channel power distributions in the AD domain are non-overlapping with proper phase shift scheduling. The Simulation results demonstrate that the proposed APSP approach can provide substantial performance gains in terms of achievable spectral efficiency over the conventional phase shift orthogonal pilot approach in typical high mobility scenarios.

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) systems, which deploy unprecedented numbers of antennas at the base stations (BSs) to simultaneously serve a relatively large number of user terminals (UTs) [1], are believed to be one of the key candidate technologies for forthcoming 5G wireless networks [2]–[4]. Orthogonal frequency division multiplexing (OFDM) combined with massive MIMO is a promising technique for wideband massive MIMO transmission [1]. As in conventional MIMO-OFDM, the performance of massive MIMO-OFDM is highly dependant on the quality of channel estimation (CE). Pilot design and CE for massive MIMO-OFDM is of great practical importance.

Optimal pilot design and CE for conventional MIMO-OFDM has been extensively investigated in the literature. The most common approach is to estimate the channels in the delay domain, and optimal pilots sent from different antennas are typically assumed to satisfy the phase shift orthogonality condition [5]–[7]. However, when such approaches are directly adopted in time-division duplex (TDD) massive MIMO-OFDM for uplink (UL) CE, the corresponding pilot overhead

is proportional to the sum of the number of UT antennas, and this becomes the system bottleneck, especially in high mobility scenarios.

In this paper, we propose *adjustable phase shift pilots (APSPs)* for massive MIMO-OFDM to reduce the pilot overhead. For APSPs, one sequence along with different phase shifted versions of itself in the frequency domain are adopted as pilots for different UTs, where the associated phase shifts are adjustable. The proposed APSPs are different from conventional phase shift orthogonal pilots (PSOPs) [5]–[7], where phase shifts for different pilots are fixed, and phase shift differences between different pilots are no less than the maximum channel delay (divided by the system sampling duration) of all the UTs. As the phase shifts for different pilots are adjustable in our approach, more pilots are available compared with the PSOPs, which leads to significantly reduced pilot overhead.

II. MASSIVE MIMO-OFDM CHANNEL MODEL

We consider a single-cell TDD massive MIMO wireless transmission which consists of one BS equipped with M antennas and K single-antenna UTs. We assume that the BS is equipped with a uniform linear array (ULA),¹ and the antennas are separated by one-half wavelength. Then the BS array response vector corresponding to the incidence angle θ with respect to the perpendicular to the array is given by [9]

$$[\mathbf{v}_{M,\theta}]_m = \exp(-j\pi m \sin(\theta)), \quad 0 \leq m \leq M-1. \quad (1)$$

We assume that the signals seen at the BS are constrained to lie in the angle interval $[-\pi/2, \pi/2]$, which can be achieved through the use of directional antennas at the BS [8].

We consider OFDM modulation with N_c subcarriers, and the guard interval is of length N_g samples. We assume that the guard interval length $T_g = N_g T_s$ is longer than the maximum channel delay of all the UTs [10], and T_s is the system sampling duration.

We assume that the channels remain constant during one OFDM symbol, and evolve from symbol to symbol. With the physical channel modeling approach (see, e.g., [9], [11]), the

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¹We adopt the ULA model in this paper for clarity, although our work can be readily extended to more general array models using the techniques in [8].

channel response vector between UT k and the BS over OFDM symbol ℓ and subcarrier n can be modeled as

$$\mathbf{g}_{k,\ell,n} = \sum_{q=0}^{N_g-1} \int_{-\infty}^{\infty} \int_{-\pi/2}^{\pi/2} \mathbf{v}_{M,\theta} \cdot \exp\left(-j2\pi \frac{n}{N_c} q\right) \cdot \exp(j2\pi \nu \ell) \cdot g_k(\theta, qT_s, \nu) d\theta d\nu \in \mathbb{C}^{M \times 1} \quad (2)$$

where $\mathbf{v}_{M,\theta}$ is given in (1), $g_k(\theta, \tau, \nu)$ is the complex-valued joint angle-delay-Doppler channel gain function of UT k corresponding to the incidence angle θ , delay τ , and normalized Doppler frequency ν . We write the k th UT's channel at OFDM symbol ℓ over all subcarriers as

$$\mathbf{G}_{k,\ell} = [\mathbf{g}_{k,\ell,0} \quad \mathbf{g}_{k,\ell,1} \quad \dots \quad \mathbf{g}_{k,\ell,N_c-1}] \in \mathbb{C}^{M \times N_c} \quad (3)$$

which will be referred to as the space-frequency (SF) domain channel response matrix (SFCRM). It is not hard to show that

$$\text{vec}\{\mathbf{G}_{k,\ell}\} = \sum_{q=0}^{N_g-1} \int_{-\infty}^{\infty} \int_{-\pi/2}^{\pi/2} [\mathbf{f}_{N_c,q} \otimes \mathbf{v}_{M,\theta}] \cdot \exp(j2\pi \nu \ell) \cdot g_k(\theta, qT_s, \nu) d\theta d\nu \quad (4)$$

where $\text{vec}\{\cdot\}$ and $\otimes$ denote the vectorization operation and Kronecker product, respectively, $\mathbf{f}_{N_c,q}$ denotes the q th column of the matrix $\sqrt{N_c} \mathbf{F}_{N_c}$, and $\mathbf{F}_{N_c}$ is the N_c -dimensional unitary discrete Fourier transform (DFT) matrix.

We assume that channels with different incidence angles, delays, and/or Doppler frequencies are uncorrelated [9]. We also assume that the temporal correlations and joint SF domain correlations of the channels can be separated [10], [11], i.e.,

$$\begin{aligned} & \mathbb{E}\{g_k(\theta, \tau, \nu) g_k^*(\theta', \tau', \nu')\} \\ &= \mathbf{S}_k^{\text{AD}}(\theta, \tau) \mathbf{S}_k^{\text{Dop}}(\nu) \cdot \delta(\theta - \theta') \delta(\tau - \tau') \delta(\nu - \nu') \end{aligned} \quad (5)$$

where $\mathbf{S}_k^{\text{AD}}(\theta, \tau)$ and $\mathbf{S}_k^{\text{Dop}}(\nu)$ represent the power angle-delay spectrum (PADS) and power Doppler spectrum (PDS) of UT k , respectively [9]. Then

$$\mathbb{E}\{\text{vec}\{\mathbf{G}_{k,\ell+\Delta_\ell}\} \text{vec}^H\{\mathbf{G}_{k,\ell}\}\} = \varrho_k(\Delta_\ell) \cdot \mathbf{R}_k \quad (6)$$

where $\varrho_k(\Delta_\ell) = \int_{-\infty}^{\infty} \exp(j2\pi \nu \Delta_\ell) \cdot \mathbf{S}_k^{\text{Dop}}(\nu) d\nu$ is the channel temporal correlation function (TCF), and $\mathbf{R}_k$ is the SF domain channel covariance matrix (SFCCM) given by

$$\begin{aligned} \mathbf{R}_k &\triangleq \sum_{q=0}^{N_g-1} \int_{-\pi/2}^{\pi/2} [\mathbf{f}_{N_c,q} \otimes \mathbf{v}_{M,\theta}] [\mathbf{f}_{N_c,q} \otimes \mathbf{v}_{M,\theta}]^H \\ &\quad \cdot \mathbf{S}_k^{\text{AD}}(\theta, qT_s) d\theta \in \mathbb{C}^{MN_c \times MN_c}. \end{aligned} \quad (7)$$

In this work, we consider the widely accepted Clarke-Jakes PDS, and the corresponding channel TCF is given by [11]

$$\varrho_k(\Delta_\ell) = J_0(2\pi \nu_k \Delta_\ell) \quad (8)$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, and ν_k is the normalized Doppler frequency of UT k . Also, we assume that the channel elements exhibit a joint

Gaussian distribution, i.e., $\text{vec}\{\mathbf{G}_{k,\ell}\} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_k)$.

We present a relationship between the SFCCM and the PADS for massive MIMO-OFDM channels in the following proposition.

Proposition 1: Define $\mathbf{V}_M \in \mathbb{C}^{M \times M}$ as $[\mathbf{V}_M]_{i,j} \triangleq \frac{1}{\sqrt{M}} \cdot \exp\left(-j2\pi \frac{i(j-M/2)}{M}\right)$, and $\mathbf{\Omega}_k \in \mathbb{C}^{M \times N_g}$ as

$$[\mathbf{\Omega}_k]_{i,j} \triangleq MN_c (\theta_{i+1} - \theta_i) \cdot \mathbf{S}_k^{\text{AD}}(\theta_i, \tau_j) \quad (9)$$

where $\theta_m \triangleq \arcsin(2m/M - 1)$, and $\tau_n \triangleq nT_s$. Then when the number of antennas $M \rightarrow \infty$, the SFCCM $\mathbf{R}_k$ tends to $(\mathbf{F}_{N_c \times N_g} \otimes \mathbf{V}_M) \text{diag}\{\text{vec}\{\mathbf{\Omega}_k\}\} (\mathbf{F}_{N_c \times N_g} \otimes \mathbf{V}_M)^H$ in the sense that, for fixed non-negative integers i and j ,

$$\lim_{M \rightarrow \infty} \left[\mathbf{R}_k - (\mathbf{F}_{N_c \times N_g} \otimes \mathbf{V}_M) \text{diag}\{\text{vec}\{\mathbf{\Omega}_k\}\} \cdot (\mathbf{F}_{N_c \times N_g} \otimes \mathbf{V}_M)^H \right]_{i,j} = 0 \quad (10)$$

where $\mathbf{F}_{N_c \times N_g}$ denotes the first N_g columns of $\mathbf{F}_{N_c}$, and $\text{diag}\{\mathbf{x}\}$ denotes the matrix with $\mathbf{x}$ along its main diagonal.

Proposition 1 indicates that, for massive MIMO-OFDM channels, the SFCCM can be well approximated by

$$\begin{aligned} \mathbf{R}_k &\approx (\mathbf{F}_{N_c \times N_g} \otimes \mathbf{V}_M) \text{diag}\{\text{vec}\{\mathbf{\Omega}_k\}\} \\ &\quad \cdot (\mathbf{F}_{N_c \times N_g} \otimes \mathbf{V}_M)^H. \end{aligned} \quad (11)$$

Note that the approximation in (11) is consistent with existing results in the literature. For frequency-selective single-input single-output channels, (11) agrees with the results in [10]. For frequency-flat massive MIMO channels, (11) reduces to the results in [8], [12], [13]. Since the SFCCM model given in (11) is a good approximation to the more complex physical channel model in (7) when the number of BS antennas is sufficiently large, we will thus exclusively use the simplified SFCCM model in (11) in the rest of the paper.

Motivated by (11), we decompose the SFCRM as follows

$$\mathbf{G}_{k,\ell} = \mathbf{V}_M \mathbf{H}_{k,\ell} \mathbf{F}_{N_c \times N_g}^T \quad (12)$$

where

$$\mathbf{H}_{k,\ell} = \mathbf{V}_M^H \mathbf{G}_{k,\ell} \mathbf{F}_{N_c \times N_g}^* \in \mathbb{C}^{M \times N_g} \quad (13)$$

is referred to as the angle-delay (AD) domain channel response matrix (ADCRM) of the k th UT at OFDM symbol ℓ . A statistical property of the ADCRM is presented as follows.

Proposition 2: For massive MIMO-OFDM channels, when $M \rightarrow \infty$, elements of the ADCRM $\mathbf{H}_{k,\ell}$ satisfy

$$\begin{aligned} & \mathbb{E}\{[\mathbf{H}_{k,\ell+\Delta_\ell}]_{i,j} [\mathbf{H}_{k,\ell}]_{i',j'}^*\} \\ &= \varrho_k(\Delta_\ell) \delta(i - i') \delta(j - j') \cdot [\mathbf{\Omega}_k]_{i,j} \end{aligned} \quad (14)$$

where $\mathbf{\Omega}_k$ is given in (9).

Proposition 2 shows that, for massive MIMO-OFDM channels, different elements of $\mathbf{H}_{k,\ell}$ are approximately mutually statistically uncorrelated, which lends the ADCRM in (13) its physical interpretation. Note that $[\mathbf{\Omega}_k]_{i,j}$ corresponds to the average power of $[\mathbf{H}_{k,\ell}]_{i,j}$, and can describe the sparsity of the wireless channels in the AD domain. Hereafter we

will refer to $\mathbf{\Omega}_k$ as the AD domain channel power matrix (ADCPM) of UT k . The dimension of the ADCPM $\mathbf{\Omega}_k$ is much smaller than that of the SFCCM $\mathbf{R}_k$, and most elements in $\mathbf{\Omega}_k$ are approximately zero due to the channel sparsity. In addition, elements of $\mathbf{\Omega}_k$ can be estimated in an element-wise manner due to Proposition 2. Hereafter we will assume that the ADCPMs of all the UTs are known by the BS.

Before we conclude this section, we define the extended ADCRM $\bar{\mathbf{H}}_{k,\ell,(N_c)} \in \mathbb{C}^{M \times N_c}$ as follows

$$\bar{\mathbf{H}}_{k,\ell,(N_c)} \triangleq [\mathbf{H}_{k,\ell} \quad \mathbf{0}_{M \times (N_c - N_g)}]. \quad (15)$$

Similarly, the extended ADCPM, which corresponds to the power distribution of the extended ADCRM $\bar{\mathbf{H}}_{k,\ell,(N_c)}$, is defined as

$$\bar{\mathbf{\Omega}}_{k,(N_c)} \triangleq [\mathbf{\Omega}_k \quad \mathbf{0}_{M \times (N_c - N_g)}]. \quad (16)$$

Such definitions will be employed to simplify the analyses in the following sections.

III. APSPs FOR CE

With the sparse massive MIMO-OFDM channel model presented in the previous section, we propose APSPs for CE in massive MIMO-OFDM in this section. Due to the unitary equivalence between the ADCRM and the SFCCRM given in (12), we focus on estimation of the AD domain channels.

We assume that all the UTs simultaneously send pilot signals over one OFDM symbol, namely, the ℓ th OFDM symbol, in each frame. From (12), the SF domain signal received at the BS can be represented as

$$\begin{aligned} \mathbf{Y}_\ell &= \sum_{k'=0}^{K-1} \mathbf{G}_{k',\ell} \mathbf{X}_{k'} + \mathbf{Z}_\ell \\ &= \sum_{k'=0}^{K-1} \mathbf{V}_M \mathbf{H}_{k',\ell} \mathbf{F}_{N_c \times N_g}^T \mathbf{X}_{k'} + \mathbf{Z}_\ell \in \mathbb{C}^{M \times N_c} \end{aligned} \quad (17)$$

where $\mathbf{X}_k = \text{diag}\{\mathbf{x}_k\} \in \mathbb{C}^{N_c \times N_c}$ denotes the frequency domain pilot signal sent from the k th UT, $\mathbf{Z}_\ell$ is the noise matrix with elements identically and independently distributed (i.i.d.) as $\mathcal{CN}(0, \sigma_{\text{ztr}})$, and σ_{ztr} is the noise power.

The proposed APSP for a given UT k is given by

$$\mathbf{X}_k \triangleq \sqrt{\sigma_{\text{xtr}}} \mathbf{D}_{\phi_k} \mathbf{X}, \quad \phi_k = 0, 1, \dots, N_c - 1 \quad (18)$$

where $\mathbf{D}_{\phi_k} \triangleq \text{diag}\{\mathbf{f}_{N_c, \phi_k}\}$, $\mathbf{X}$ is an arbitrary diagonal matrix satisfying $\mathbf{X}\mathbf{X}^H = \mathbf{I}_{N_c}$, and σ_{xtr} is the pilot signal power. The APSP signal in (18) can be seen as a phase shifted version of $\sqrt{\sigma_{\text{xtr}}} \mathbf{X}$ with phase shift ϕ_k in the frequency domain.

From (18), it can be readily obtained that, for $\forall k, k'$,

$$\mathbf{X}_{k'} \mathbf{X}_k^H = \sigma_{\text{xtr}} \mathbf{D}_{\phi_{k'} - \phi_k} \quad (19)$$

which indicates that cross correlations of the APSPs for different UTs depends on the associated phase shift difference. Note that, for conventional PSOPs, phase shift differences for different pilots are set to satisfy the orthogonality condition $|\phi_{k'} - \phi_k| \geq N_g \quad \forall k' \neq k$. However, for our APSPs, the phase

shifts for different pilots are adjustable, which leads to more available pilots, and thus pilot overhead can be reduced.

After decorrelation and power normalization of (17), the BS can obtain the observation of the UL channel $\mathbf{H}_{k,\ell}$ given by (20), shown at the top of the next page, where (a) follows from (19), (b) follows from the property of unitary transformations, $\rho_{\text{tr}} \triangleq \sigma_{\text{xtr}}/\sigma_{\text{ztr}}$ is the signal-to-noise ratio (SNR) during the pilot segment, and $\mathbf{Z}_{\text{iid}}$ is the normalized noise matrix with i.i.d. elements distributed as $\mathcal{CN}(0, 1)$. Note that the pilot interference term $\mathbf{H}_{k',\ell}^{\phi_{k'} - \phi_k}$ in (20) satisfies

$$\begin{aligned} \mathbf{H}_{k',\ell}^{\phi_{k'} - \phi_k} &\stackrel{(a)}{=} \bar{\mathbf{H}}_{k',\ell,(N_c)} \mathbf{F}_{N_c}^T \mathbf{D}_{\phi_{k'} - \phi_k} \mathbf{F}_{N_c}^* \mathbf{I}_{N_c \times N_g} \\ &\stackrel{(b)}{=} \bar{\mathbf{H}}_{k',\ell,(N_c)} \mathbf{\Pi}_{N_c}^{\phi_{k'} - \phi_k} \mathbf{I}_{N_c \times N_g} \end{aligned} \quad (21)$$

where (a) follows from (15), $\mathbf{I}_{N_c \times N_g}$ denotes the matrix composed of the first N_g columns of $\mathbf{I}_{N_c}$, (b) follows from the permutation matrix definition $\mathbf{\Pi}_N^n \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{I}_{N-n} \\ \mathbf{I}_{(n)_N} & \mathbf{0} \end{bmatrix}$, and $\langle \cdot \rangle_N$ denotes the modulo- N operation. This indicates that the pilot interference term $\mathbf{H}_{k',\ell}^{\phi_{k'} - \phi_k}$ in (20) can be seen as a column truncated version of the extended ADCRM $\bar{\mathbf{H}}_{k',\ell,(N_c)}$ with column cyclic shift $\phi_{k'} - \phi_k$.

Recalling Proposition 2, elements of the ADCRM $\mathbf{H}_{k,\ell}$ are statistically uncorrelated. Then, elements of $\mathbf{H}_{k',\ell}^{\phi_{k'} - \phi_k}$ are also statistically uncorrelated, and the corresponding power matrix of $\mathbf{H}_{k',\ell}^{\phi_{k'} - \phi_k}$ can be defined as

$$\mathbf{\Omega}_{k',\ell}^{\phi_{k'} - \phi_k} \triangleq \bar{\mathbf{\Omega}}_{k',\ell,(N_c)} \mathbf{\Pi}_{N_c}^{\phi_{k'} - \phi_k} \mathbf{I}_{N_c \times N_g}. \quad (22)$$

With the channel observation $\mathbf{Y}_{k,\ell}$ in (20), and the property of statistical uncorrelation of the AD domain channel elements in Proposition 2, the minimum mean square error (MMSE) channel estimate $\hat{\mathbf{H}}_{k,\ell+\Delta_\ell}$ can be obtained in an element-wise manner as follows

$$[\hat{\mathbf{H}}_{k,\ell+\Delta_\ell}]_{i,j} = \frac{\varrho_k(\Delta_\ell) \cdot [\mathbf{\Omega}_k]_{i,j}}{\sum_{k'=0}^{K-1} [\mathbf{\Omega}_{k',\ell}^{\phi_{k'} - \phi_k}]_{i,j} + \frac{1}{\rho_{\text{tr}}}} [\mathbf{Y}_{k,\ell}]_{i,j} \quad (23)$$

and the corresponding MSE of CE (MSE-CE) is given by

$$\begin{aligned} \epsilon_k(\Delta_\ell) &\triangleq \sum_{i=0}^{M-1} \sum_{j=0}^{N_g-1} \mathbb{E} \left\{ \left| [\mathbf{H}_{k,\ell+\Delta_\ell} - \hat{\mathbf{H}}_{k,\ell+\Delta_\ell}]_{i,j} \right|^2 \right\} \\ &= \sum_{i=0}^{M-1} \sum_{j=0}^{N_g-1} \left\{ [\mathbf{\Omega}_k]_{i,j} - \frac{\varrho_k^2(\Delta_\ell) \cdot [\mathbf{\Omega}_k]_{i,j}^2}{\sum_{k'=0}^{K-1} [\mathbf{\Omega}_{k',\ell}^{\phi_{k'} - \phi_k}]_{i,j} + \frac{1}{\rho_{\text{tr}}}} \right\} \end{aligned} \quad (24)$$

where the orthogonality principle of MMSE estimation is used.

Note that there exist two typical frame structures for TDD massive MIMO transmission. One type of frame structure (which will be referred to as type-A) begins with the UL pilot segment, followed by the UL and downlink (DL) data segments. In the second type (which will be referred to as type-B), the UL pilot segment is placed between the UL data segment and DL data segment. For the proposed APSP approach, the delay between the tail-end symbols of the data segment and

$$\begin{aligned}
\mathbf{Y}_{k,\ell} &= \frac{1}{\sigma_{\text{xtr}}} \mathbf{V}_M^H \mathbf{Y}_\ell \mathbf{X}_k^H \mathbf{F}_{N_c \times N_g}^* \stackrel{(a)}{=} \mathbf{H}_{k,\ell} + \underbrace{\sum_{k' \neq k} \mathbf{H}_{k',\ell} \mathbf{F}_{N_c \times N_g}^T \mathbf{D}_{\phi_{k'} - \phi_k} \mathbf{F}_{N_c \times N_g}^*}_{\text{pilot interference} \triangleq \sum_{k' \neq k} \mathbf{H}_{k',\ell}^{\phi_{k'} - \phi_k}} + \underbrace{\frac{1}{\sigma_{\text{xtr}}} \mathbf{V}_M^H \mathbf{Z}_\ell \mathbf{X}_k^H \mathbf{F}_{N_c \times N_g}^*}_{\text{pilot noise}} \\
&\stackrel{(b)}{=} \mathbf{H}_{k,\ell} + \sum_{k' \neq k} \mathbf{H}_{k',\ell}^{\phi_{k'} - \phi_k} + \frac{1}{\sqrt{\rho_{\text{tr}}}} \mathbf{Z}_{\text{iid}}
\end{aligned} \tag{20}$$

the pilot segment will be longer than the PSOP approach due to the reduced pilot segment length. In addition, the APSP approach focuses on high mobility scenarios where channels vary relatively quickly. Thus the type-B frame structure is well-suited for the proposed APSP approach.

Before proceeding, we define the sum MSE-CE of all the UTs for a given delay Δ_ℓ as

$$\epsilon(\Delta_\ell) \triangleq \sum_{k=0}^{K-1} \epsilon_k(\Delta_\ell). \tag{25}$$

Due to the incurred pilot interference, the performance of the APSP-based CE, e.g., $\epsilon(\Delta_\ell)$ in (25), might deteriorate. However, we will show in the following that such effects can be eliminated with proper phase shift scheduling for different pilots.

Proposition 3: The sum MSE $\epsilon(\Delta_\ell)$ is lower bounded by

$$\begin{aligned}
\epsilon(\Delta_\ell) &\geq \varepsilon(\Delta_\ell) \\
&= \sum_{k=0}^{K-1} \sum_{i=0}^{M-1} \sum_{j=0}^{N_g-1} \left\{ [\boldsymbol{\Omega}_k]_{i,j} - \varrho_k^2(\Delta_\ell) \frac{[\boldsymbol{\Omega}_k]_{i,j}^2}{[\boldsymbol{\Omega}_k]_{i,j} + \frac{1}{\rho_{\text{tr}}}} \right\}
\end{aligned} \tag{26}$$

and the lower bound can be achieved under the condition that, for $\forall k, k'$ and $k \neq k'$,

$$\left(\bar{\boldsymbol{\Omega}}_{k,(N_c)} \boldsymbol{\Pi}_{N_c}^{\phi_k} \right) \odot \left(\bar{\boldsymbol{\Omega}}_{k',(N_c)} \boldsymbol{\Pi}_{N_c}^{\phi_{k'}} \right) = \mathbf{0} \tag{27}$$

where $\odot$ denotes the Hadamard product.

Proposition 3 shows that, with the proposed APSPs, the sum MSE-CE can be minimized when phase shifts for different pilots are properly scheduled according to the condition given in (27). The interpretation is very intuitive. With frequency domain phase shifted pilots, equivalent channels will exhibit corresponding cyclic shifts in the delay domain, as seen from (21). If the equivalent channel power distributions in the AD domain for different UTs are non-overlapping after proper pilot phase shift scheduling, the pilot interference effect can be eliminated, and the sum MSE-CE can be minimized.

Wireless channels are sparse in the AD domain in many practical propagation scenarios [14]–[16], and typically only a few elements of the ADCPM $\boldsymbol{\Omega}_k$ are dominant in massive MIMO-OFDM. In addition, channel sparsity patterns for different UTs, i.e., locations of dominant elements in $\boldsymbol{\Omega}_k$ for different k , are usually different. When such channel sparsity is properly taken into account, the equivalent AD domain channels for different UTs are almost non-overlapping with high probability, assuming proper pilot phase shifts. This

clarifies the feasibility of the proposed APSPs for massive MIMO-OFDM.

The condition in (27) cannot always be met in practice, and pilot phase shift scheduling, i.e., scheduling phase shifts for different pilots according to the ADCPMs, is of importance. Several scheduling criteria can be adopted. For example, if we schedule the pilot phase shifts based on the MMSE-CE criterion, the problem can be formulated as

$$\arg \min_{\{\phi_k\}} \epsilon(0). \tag{28}$$

Such a scheduling problem is combinatorial, and optimal solutions must be found through an exhaustive search. Note that optimal conditions in (27) are the same for $\forall \Delta_\ell$, thus the solution of (28) can also be expected to perform well when the objective function is changed to $\epsilon(\Delta_\ell)$ for $\forall \Delta_\ell \neq 0$. In addition, a simplified pilot scheduling algorithm can be developed motivated by the optimal condition in (27). In particular, one could schedule the pilot phase shifts to make the equivalent ADCPMs for different UTs as non-overlapping as possible. A detailed description of the algorithm can be found in [17].

IV. NUMERICAL RESULTS

In this section, we present numerical simulations to evaluate the performance of the proposed APSP approach in massive MIMO-OFDM. The system considered is equipped with a 128-antenna ULA at the BS with half wavelength antenna spacing. The number of UTs is set to $K = 42$ as in [1]. The major system parameters based on 3GPP LTE [18] are set as follows. The system bandwidth is 20 MHz, and the sampling duration equals $T_s = 32.6$ ns. The number of subcarriers is $N_c = 2048$, and the guard interval length is $N_g = 144$. We assume that transmissions from all the UTs are synchronized [19].

We consider channels with 20 taps in the delay domain, which exhibit an exponential power delay profile [19]

$$\mathbf{S}_k^{\text{del}}(\tau) \propto \exp(-\tau/\varsigma_k) \tag{29}$$

where ς_k denotes the channel delay spread of UT k . The q th channel tap of UT k is assumed to exhibit a Laplacian power angular spectrum [8], [19]

$$\mathbf{S}_{k,q}^{\text{ang}}(\theta) \propto \exp\left(-\sqrt{2}|\theta - \theta_{k,q}|/\varphi_{k,q}\right) \tag{30}$$

where $\theta_{k,q}$ and $\varphi_{k,q}$ represent the corresponding mean angle of arrival and angular spread for the given channel tap, respectively. We assume that the UTs are uniformly distributed

Fig. 1. Comparison of the achievable spectral efficiency for the APSP and PSOP approaches.

in a 120° sector, and the mean angle of arrival $\theta_{k,q}$ is uniformly distributed in the angle interval $[-\pi/3, \pi/3]$ in radians. We do not consider large scale fading in the simulations, and channels are normalized as $\sum_{i,j} [\Omega_k]_{i,j} = MN_c$. We consider high mobility suburban scenarios, and all the UTs exhibit the same Doppler, delay, and angular spread. Major channel statistical parameters based on the WINNER II channel model [19] are set as follows. The velocity is 250 km/h, and the corresponding normalized Doppler frequency is $\nu = 0.031$. The delay spread is $\varsigma = 0.77 \mu\text{s}$, and the angular spread is $\varphi = 2^\circ$.

We compare the achievable spectral efficiency of the proposed APSP and the conventional PSOP approaches. We assume that the frame length equals 7 OFDM symbols as in [1], and both UL and DL data transmissions occupy half of the data segment length. With the above settings, for the conventional PSOP approach, the required pilot segment length is $\lceil K/(N_c/N_g) \rceil = 3$ [1], and the type-A frame structure is adopted. For the proposed APSP approach, the pilot segment length is set to be 1, both the type-A and type-B frame structures are considered. A MMSE receiver and precoder are employed for both UL and DL data transmissions, and the SNR is assumed to be equal to the pilot SNR.

In Fig. 1, the cumulative distributions of the achieved spectral efficiency are depicted. It can be observed that the proposed APSP approach shows substantial performance gain in terms of achievable spectral efficiency over the conventional PSOP approach. Specifically, in the high mobility suburban scenarios with an SNR of 10 dB, the proposed APSPs can provide about 69% in average spectral efficiency gains over the conventional PSOPs. In addition, the type-B frame structure can provide a gain of about 64% over the type-A frame structure when APSPs are adopted.

V. CONCLUSION

In this paper, we proposed a CE approach with adjustable phase shift pilots (APSPs) for massive MIMO-OFDM to

reduce the pilot overhead, where the phase shifts for different pilots are adjustable. We first investigated the channel sparsity in massive MIMO-OFDM based on a physically motivated channel model. With this channel model, we investigated CE for massive MIMO-OFDM with APSPs, and provided an optimal pilot phase shift scheduling condition. The simulation results demonstrated that the proposed APSP approach can achieve significant gains in spectral efficiency compared with the conventional PSOP approach in high mobility scenarios.

REFERENCES

- [1] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.
- [2] C.-X. Wang, F. Haider, X. Q. Gao, X.-H. You, Y. Yang, D. Yuan, H. M. Aggoune, H. Haas, S. Fletcher, and E. Hepsaydir, "Cellular architecture and key technologies for 5G wireless communication networks," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 122–130, Feb. 2014.
- [3] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.
- [4] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, and R. Zhang, "An overview of massive MIMO: Benefits and challenges," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 742–758, Oct. 2014.
- [5] Y. G. Li, "Simplified channel estimation for OFDM systems with multiple transmit antennas," *IEEE Trans. Wireless Commun.*, vol. 1, no. 1, pp. 67–75, Jan. 2002.
- [6] I. Barhumi, G. Leus, and M. Moonen, "Optimal training design for MIMO OFDM systems in mobile wireless channels," *IEEE Trans. Signal Process.*, vol. 51, no. 6, pp. 1615–1624, Jun. 2003.
- [7] Y. Chi, A. Goma, N. Al-Dhahir, and A. R. Calderbank, "Training signal design and tradeoffs for spectrally-efficient multi-user MIMO-OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 10, no. 7, pp. 2234–2245, Jul. 2011.
- [8] L. You, X. Q. Gao, X.-G. Xia, N. Ma, and Y. Peng, "Pilot reuse for massive MIMO transmission over spatially correlated Rayleigh fading channels," *IEEE Trans. Wireless Commun.*, 2015, DOI: 10.1109/TWC.2015.2404839.
- [9] B. Clerckx and C. Oestges, *MIMO Wireless Networks: Channels, Techniques and Standards for Multi-Antenna, Multi-User and Multi-Cell Systems*, 2nd ed. Oxford, U.K.: Academic Press, 2013.
- [10] Y. G. Li, L. J. Cimini, Jr., and N. R. Sollenberger, "Robust channel estimation for OFDM systems with rapid dispersive fading channels," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 902–915, Jul. 1998.
- [11] G. Auer, "3D MIMO-OFDM channel estimation," *IEEE Trans. Commun.*, vol. 60, no. 4, pp. 972–985, Apr. 2012.
- [12] H. Yin, D. Gesbert, M. Filippou, and Y. Liu, "A coordinated approach to channel estimation in large-scale multiple-antenna systems," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 264–273, Feb. 2013.
- [13] A. Adhikary, J. Nam, J.-Y. Ahn, and G. Caire, "Joint spatial division and multiplexing—The large-scale array regime," *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6441–6463, Oct. 2013.
- [14] W. U. Bajwa, J. Haupt, A. M. Sayeed, and R. Nowak, "Compressed channel sensing: A new approach to estimating sparse multipath channels," *Proc. IEEE*, vol. 98, no. 6, pp. 1058–1076, Jun. 2010.
- [15] S. Payami and F. Tufvesson, "Channel measurements and analysis for very large array systems at 2.6 GHz," in *Proc. EuCAP*, Prague, Czech Republic, 2012, pp. 433–437.
- [16] X. Gao, F. Tufvesson, and O. Edfors, "Massive MIMO channels – Measurements and models," in *Proc. Annu. ASIOMAR*, Pacific Grove, CA, USA, 2013, pp. 280–284.
- [17] L. You, X. Q. Gao, A. L. Swindlehurst, and W. Zhong, "Channel acquisition for massive MIMO-OFDM with adjustable phase shift pilots," *IEEE Trans. Signal Process.*, 2015, submitted.
- [18] 3GPP TS 36.211 V12.4.0, "3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation (Release 12)," Tech. Rep., Dec. 2014.
- [19] IST-4-027756 WINNER II D1.1.2 V1.2, "WINNER II Channel Models," Tech. Rep., Feb. 2008.