REDUCED FDTD FORMULATION (R-FDTD) FOR THE ANALYSIS
OF 30 GHz DIELECTRIC RESONATOR COUPLED TO A MICROSTRIP LINE

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Abstract-In this paper a reduced formulation for the standard finite difference time domain method (R-FDTD), based on the divergence free nature of the electric and magnetic field displacement is presented. This new approach for the solution of Maxwell equations allows a memory reduction in the storage of the field components with almost no computational cost. As validation of the new technique, the resonance frequencies of a dielectric resonator coupled to a microstrip line printed on alumina substrate is studied. Results of measured and calculated resonant frequencies are provided, confirming the validity of the novel numerical technique.

I. INTRODUCTION
With the increase in speed and memory storage in modern computer systems, the Finite Difference Time Domain technique (FDTD) [1] for the solution of electromagnetic problems is becoming rapidly an attractive choice due to its programming simplicity and flexibility to analyze wide range of structures. The simplicity and flexibility of this technique has the drawback of high memory requirement and computational power necessary to analyze large geometries. In this work, a modified version of FDTD with increased memory efficiency is presented and applied to the study of a dielectric resonator coupled to a microstrip line printed on alumina substrate. The memory reduction is achieved taking advantage of the spatial dependence of the field components. A net reduction of 33% in the storage requirement is obtained at almost no computational cost. This new technique uses the spatial dependence which exists among the electric and magnetic scalar field components, whenever a charge free region exists. This means that for each set of two field components, respectively for E and H, the third component can be uniquely calculated from (2). We demonstrate how this fact can be incorporated in the FDTD formulation for the 2-D case. The extension to 3-D is straightforward. Assume that we based on the existence of regions where \( \nabla \cdot \mathbf{D} = 0 \), the treatment of dielectric discontinuities is taken into account automatically.

At the metal interfaces the assumption of charge free region obviously fails. A novel approach to solve this problem is proposed. In this formulation metals are treated as dielectrics having infinite dielectric constant (10^8 is sufficient for numerical purpose), avoiding the necessity of local treatments.

II. THEORY
The standard Yee algorithm [1] for the solution of Maxwell equations is based on their discretization in space and time. Starting from system (1), the time marching solution is obtained using a leap-frog scheme [2] to propagate from each component of \( \mathbf{E} \) to \( \mathbf{H} \) and vice versa.

\[
\begin{align*}
\nabla \times \mathbf{E} &= -\frac{\partial (\mu \mathbf{H})}{\partial t} \\
\nabla \times \mathbf{H} &= \frac{\partial (\varepsilon \mathbf{E})}{\partial t}
\end{align*}
\] (1)

Because this formulation involves single derivatives in time and space for both fields, the storage of six scalar quantities for each cell location in the computational domain is required. Consider for the moment the case of a charge free region (\( \rho = 0 \)). The well known divergence equations for \( \mathbf{E} \) and \( \mathbf{H} \) then apply [3].

\[
\begin{align*}
\nabla \cdot (\varepsilon \mathbf{E}) &= 0 \\
\nabla \cdot (\mu \mathbf{H}) &= 0
\end{align*}
\] (2)

These equations demonstrate a spatial dependence between the electric and magnetic field components. This means that for each set of two field components, respectively for \( \mathbf{E} \) and \( \mathbf{H} \), the third component can be uniquely calculated from (2). We demonstrate how this fact can be incorporated in the FDTD formulation for the 2-D case. The extension to 3-D is straightforward. Assume that we...
have a 2-D problem involving $E_x, E_y$ and $H_z$ and we want to eliminate the explicit use of $E_y$. The second equation of system (2) is then trivially satisfied while we can discretize the first equation and rewrite it as:

$$E_{x(i,j+1)}^n = \frac{\varepsilon_{i,j+1}}{\varepsilon_{i,j+1}} E_{x(i,j)}^n + \frac{\Delta t}{\Delta y \mu_0} [E_{x(i,j+1)}^n - E_{x(i,j)}^n] + \frac{\Delta t}{\Delta x \mu_0} [\nu(i+1) - \nu(i)]$$

assuming for simplicity a square grid. Equation (3) shows us that, having $E_x(i,j)$ in every location, we can update spatially $E_x(i,j)$ along the y direction. All we need to store is a vector with the values of $E_x(i,j-1)$ along the x direction for every $i$. Equation (3) also shows us that we need $E_x(i,j)$, in order to begin this spatial update. To this end, we can apply the normal FDTD code for $j=1$, and then proceed based on (3). Therefore, the new FDTD formulation for each time step could be outlined as in the flowchart below. Note that this approach can treat any dielectric discontinuity or inhomogeneity naturally, without the need to subdivide the computational domain. Note also that at every time step we actually have all the field components available, we just don’t store one of them.

1. **E_X field update**
   Use normal FDTD to obtain $E_x(i,j)$

2. **H_z field update:**
   (a) Get $E_x(i,j)$ from FDTD and store in vector $v(i)$
   (b) for $j=1$ to $n_y-1$ for $i=1$ to $n_x$
   $$H_{z(i,j)}^{n+1} = H_{z(i,j)}^n + \frac{\Delta t}{\Delta y \mu_0} [E_{x(i,j+1)}^n - E_{x(i,j)}^n] + \frac{\Delta t}{\Delta x \mu_0} [\nu(i+1) - \nu(i)]$$
   end $i$
   Update $E_x(i,j+1)$ using Eq. (3) and store in $v$
   for $i=1$ to $n_x$
   $$v(i) = \frac{\varepsilon_{i,j+1}}{\varepsilon_{i,j+1}} v(i) + \frac{E_x(i,j+1)}{E_x(i,j)}$$
   end $i$
   end $j$

As we mentioned above, (2) holds for charge free regions, and this assumption obviously fails for conductors and source regions. Two ways are possible to overcome this limitation: one approach is to use, for these specific cells, the normal FDTD code to update $E_x(i,j)$. The increase in memory and complexity is not significant. A more efficient way is to numerically anneal the tangential field components, imposing a very large dielectric constant at the metal interface [3]. Doing so, allows us to treat the metal as regular dielectric, preserving the original formulation for charge free regions. This last approach has no computational cost whenever average dielectric constant is used for improved precision on dielectric interfaces [4]. Another instance where the displacement is not divergence free is at the source points. This is only an apparent problem, since the total charge can be calculated ahead from the excitation pulse imposed, and its value can be locally used (where the source exists) in $\nabla \cdot D=\rho$, to find the relationship among the field components. Finally, we want to mention that in [5] also a new technique has been proposed where, with the use of the vector potential, the authors where able to use two field components instead of three, to solve two dimensional scattering problems.

### III. MICROSTRIP COUPLED DIELECTRIC RESONATOR

In order to validate our new formulation (R-FDTD), a measurement was performed on a cylindrical resonator of diameter of 2.26 mm and height of 0.91 mm, made of a perovskite based on Ba,Zn, Ta-oxide. The dielectric resonator (manufactured by Trans-Tech model D8733-0089-036) has a dielectric permittivity of 30.15, and a Q factor at 10 GHz of 12,200. The resonator is placed on top of a dielectric substrate, the substrate is made of 124μm thick alumina (relative dielectric constant $\varepsilon_r=9.8$) with a ground metal (gold plated) on the other side as shown in Figure 1. A 9μm long offset conductor of 124 μm width is used to couple the field to the resonator. The whole substrate was enclosed on a rectangular waveguide whose dimensions...
are 2.5 x 4.6 x 7.5 mm to reduce radiation losses (see Figure 1).
The resonator was placed directly on the substrate (no spacer were used) with one edge exactly at the edge of the microstrip line (no overlapping). The measurement of the resonance frequency was done using a network analyzer (HP 8510C) with coaxial probes (Picoprobe Model 40 A).

Wrap around type ground transitions for the probes were done with silver epoxy. By using TRL [6] calibration technique 25 dB return loss sensitivity was achieved over the band going from 20-40 GHz. The total length of the microstrip from the two reference planes was 7.5 mm. The measured transmission coefficient for the proposed structure is reported in Figure 2. The desired resonance frequency is obtained at 29.862 GHz, also a box resonance is observed at 26.075 GHz and two higher resonance respectively at 36.875 GHz and 37.963 GHz. To simulate the described structure the whole domain was discretized with uniform grid having dimensions of 100x60x166 respectively along x,y, and z directions.

The corresponding grid size was Δx=45.2μm, Δy=41.33μm and Δz=45.2μm, this choice allows to fit at best with the grid the resonator height and the substrate dimensions. The excitation is obtained through a Gaussian pulse under the strip conductor having width T=10 ps, corresponding to a maximum frequency of 50 GHz. Reflection conditions are used on the metal side walls of the waveguide, and perfectly matched absorbing boundary conditions [7] are used at the two waveguide ends. Standard procedure is used to extract the transmission parameters [8]. The transmission coefficient obtained is shown in Figure 2 for normal FDTD, our formulation (R-FDTD) and is compared with measured data. The two calculated curves overlap completely, while some difference is observed with the measured data for the highest resonance frequencies.

IV. CONCLUSIONS

A modification of the original FDTD formulation, based on the spatial dependencies of the field components, aimed to reduce the memory requirement has been presented and tested. The new technique has been formulated to maintain the simplicity and generality of the original FDTD while reducing the stored field components to 2/3 at almost no computational cost. Perfect agreement between our formulation and standard FDTD is obtained, and validation with experimental results is given.

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REFERENCES


