MIDTERM EXAMINATION

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Useful identities

\[
\text{sinc}(t) \longleftrightarrow \text{rect}(f)\\
\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}\\
\text{rect}(f) = \begin{cases} 
1 & |f| \leq \frac{1}{2}, \\
0 & |f| > \frac{1}{2}.
\end{cases}
\]
1. (20 points) A voiceband telephone line channel has a bandpass frequency response spanning the frequency range 300-3300 Hz. Consider a modem for transmitting data with carrier frequency $f_c = 1800$ Hz.

(a) Sketch the approximate bandpass frequency response of the channel, indicating the location of the passband and carrier frequency.

(b) Translate this spectrum into its lowpass equivalent.

(c) What is the largest symbol rate that can be employed with this channel, assuming zero roll-off ($\alpha = 0$)?

(d) What is the largest symbol rate that can be employed with this channel, assuming 100% roll-off ($\alpha = 1$)?

(e) Assuming 64-QAM is employed, what is the largest bit rate that can be achieved with this channel employing 100% roll-off?

(f) Assume that the roll-off frequency response $X(f)$ is divided between the transmitter and the receiver as $G_T(f)$ and $G_R(f)$, respectively. Sketch the block diagram of the transmitter and describe the functional operation of each block.
Consider the channel frequency response $X(f)$ in the figure above.

(a) Does this channel satisfy the Nyquist criterion? Why?

(b) Calculate the derivative $X'(f)$ of $X(f)$ and sketch $X'(f)$ versus $f$.

(c) Express $X'(f)$ analytically.

(d) Noting $-j2\pi t x(t) \longleftrightarrow X'(f)$, calculate the impulse response $x(t)$.

(e) What are the zero crossings of $x(t)$?

(f) Compare this $x(t)$ with that of raised cosine. Which one would you prefer? Why?
3. (20 points) Let

\[ X_n = W_n + aW_{n-1} \]

where \(0 < a < 1\) and \(W_n\) is a white random process, \(E[W_n] = 0, E[W_n^2] = 1\). The desired response is \(d_n = X_{n+1}\).

(a) Calculate the optimum predictor filter of order \(M = 1\).

(b) Calculate the optimum predictor filter of order \(M = 2\).

(c) Calculate the mean square error for the optimum predictor filter of order \(M = 1\), \(J_{\text{min}}(1)\).

(d) Calculate the mean square error for the optimum predictor filter of order \(M = 2\), \(J_{\text{min}}(2)\).

(e) What can you say about \(J_{\text{min}}(1)\) and \(J_{\text{min}}(2)\)? Why?
4. (20 points) In this problem the cost function is given as

\[ J = w^2 + 2w + 1 \]

where \( w \) is a design parameter. The goal is to minimize \( J \). We would like to adaptively calculate \( w \) by using a gradient method as

\[ w_{k+1} = w_k - \frac{1}{2}\Delta g_k \quad k = 0, 1, 2, \ldots \]

where \( g_k \) is the gradient of \( J \) with respect to \( w \), evaluated at \( w_k \).

(a) Calculate the adaptive algorithm for given \( J \).
(b) Considering this adaptive algorithm as a difference equation, calculate the range of \( \Delta \) for which the algorithm converges.
(c) What is the optimum value \( w_{\text{opt}} \) of \( w \) to minimize \( J \)?
(d) Let \( \Delta = 0.5 \), \( w_0 = 4 \). Calculate the values of \( w_k \) for \( k = 1, 2, 3, 4 \).
(e) Plot \( J \) against \( w \) and place the values you found above on this plot.
(f) Assume now that \( J = w^3 + 3w^2 + 3w + 1 \). Can you use a gradient method to adaptively calculate \( w_{\text{opt}} \) so that \( J \) will be minimized? Why?
5. (10 points) Recall that in the true gradient algorithm

\[ c_{k+1} = c_k - \Delta G_k \]

where

\[ G_k = -p + \Gamma c_k. \]

Let

\[ q_k = c_k - c_{opt} \quad k = 0, 1, 2, \ldots \]

and show that

\[ q_k = \prod_{i=1}^{k} (I - \Delta \Gamma) q_0 \quad k = 1, 2, 3, \ldots . \]
6. (10 points) It is desired to build a zero-forcing decision feedback equalizer for a channel whose equivalent discrete-time bandlimited model is given as

\[ X(z) = \frac{20}{3} \frac{z^2 - \frac{3}{7}z + 1}{z^2 - \frac{20}{12}z + 1}. \]

The channel has additive white Gaussian noise with PSD \( N_0 / 2 \) W/Hz.

(a) Calculate the feedback filter \( D(z) \).
(b) Calculate the feedforward filter \( C(z) \).
(c) Calculate \( J_{\text{min}} \).