Optimal Layered Transmission Over Quasi-Static Fading Channels

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Outline

- Motivation
- Problem Formulation
- Proposed Solution
- Numerical Results
- Conclusion
Motivation

- Lossy transmission over a block fading channel (image/video transmission)
- Design metric: Expected end-to-end distortion
- **Goal**: Minimize the expected distortion under given constraints (bandwidth, power, delay, ...)

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Joint Source-Channel Coding

- Delay-limited, single-block coding
- Shannon’s separation theorem does not hold:
  - Non-ergodic channel
  - Channel has memory
  - Bounded delay/complexity
- Source and channel coders have to be optimized jointly
Problem Formulation

- $M_t$ transmit, $M_r$ receive antennas
- $H_{M_t \times M_r}$: Channel matrix, complex, random
- $H$ known at the receiver only
- Block fading: $H$ constant over $M$ channel uses
- Instantaneous channel capacity

\[ C = \log_2 |I_R + \frac{SNR}{M_t} H^* H| \]

- Zero Shannon capacity
- Outage capacity: $P_{out}(R, SNR) = \text{Prob}\{C < R\}$
Problem Formulation

• Source model:
  • Ergodic, complex, unit variance, Gaussian
  • Successively refinable
  • MSE distortion
  • $D(R) = 2^{-R}$, for $R$ bits/source symbol

• Problem: Minimize the expected distortion
  • Information-theoretical bound on the achievable performance
  • Optimal solution
Layered Transmission

- $M$: Size of the fading block
- Transmit $K$ source symbols per $M$ channel use
- $b = M/K$: Bandwidth expansion factor
- Partition $i$:
  - Transmission rate: $R_i$ bits/channel use
  - Size: $\alpha_i$ where $\sum \alpha_i = 1$
Layered Transmission

• Unequal protection assumption: $R_n \leq R_{n+1}$
  • Layers $n$ and $n+1$ experience the same capacity
  • Layer $n$ has to be decoded before layer $n+1$ can be used (successive refinement)
• Not necessarily the best strategy
Cost Function

- \( P_R = \text{Prob}\{C < R\} \) given SNR
- \( \text{Prob}\{\text{First n layers decoded}\} = \)
  \[ \text{Prob}\{R_n \leq C < R_{n+1}\} = P_{R_{n+1}} - P_{R_n} \]
- Expected distortion
  \[ E_D(R, \alpha) = \sum_{n=0}^{N} (P_{R_{n+1}} - P_{R_n}) 2^{-b(\sum_{k=1}^{n} \alpha_k R_k)} \]
Previous Work

• Distortion exponent: [Laneman et. al, 2005]

$$\Delta = - \lim_{SNR \to \infty} \frac{\log \mathcal{E}_D}{\log SNR}$$

• SIMO/MISO case: [Gunduz & Erkip, 2006]

$$\Delta = m \left[ 1 - \frac{1}{(1 + \frac{b}{Nm})^N} \right] \quad m = \max(M_t, M_r)$$

• Distortion exponent:
  • Partial characterization of the solution
  • Asymptotic measure ($SNR \to \infty$)
Optimization

- Rate allocation + partitioning

\[
\begin{align*}
\min_{R, \alpha} & \quad \mathcal{E}_D \quad (R, \alpha) \\
0 & \leq R_i \leq R_{i+1} \\
0 & \leq \alpha_i \leq 1
\end{align*}
\]

\[R = (R_1, \cdots, R_N)\]
\[\alpha = (\alpha_1, \cdots, \alpha_{N-1})\]
\[\alpha_N = 1 - \sum_{i=1}^{N-1} \alpha_i\]

- Brute-force search \(\mathcal{O}(|\mathcal{R}|^N)\) for known \(\alpha\)

- \(2N - 1\) variables, non-convex, constrained NLP, difficult to solve
**Iterative Optimization**

- **Initialize:** \( k = 0 \), Arbitrary partition \( \alpha^0 \)
- **Optimize:**
  
  \[
  R^{k+1} = \operatorname{arg\ min}_R \mathcal{E}_D(R, \alpha = \alpha^k) \tag{1}
  \]
  
  \[
  \alpha^{k+1} = \operatorname{arg\ min}_\alpha \mathcal{E}_D(R = R^{k+1}, \alpha) \tag{2}
  \]
- **Iterate:** \( k = k + 1 \), Go to (1)
- Guaranteed convergence to a local minima
Rate Optimization

Lemma 1: $N$-layer expected distortion is given by

$$
\mathcal{E}_D^1(R, \alpha) = D_1(1, R_1)
$$

$$
\mathcal{E}_D^2(R, \alpha) = D_2(\alpha_1, R_1, D_1(\alpha_2, R_2))
$$

$$
\cdots
$$

$$
\mathcal{E}_D^N(R, \alpha) = D_2(\alpha_1, R_1, D_2(\alpha_2, R_2, \cdots, D_2(\alpha_{N-1}, R_{N-1}, D_1(\alpha_N, R_N)) \cdots))
$$

where

$$
D_1(\beta, r) \triangleq P_r + (1 - P_r)2^{-b\beta r}
$$

$$
D_2(\beta, r, t) \triangleq (1 - 2^{-b\beta r})P_r + 2^{-b\beta r} t
$$
Lemma 2: For $N \geq 2$ and a given $\alpha$, the optimal value $(R_2^*, \ldots, R_N^*)$ is given by

$$(R_2^*, \ldots, R_N^*) = \arg \min_{R_2, \ldots, R_N} C$$

where

$$C = D_2(\alpha_2, R_2, D_2(\alpha_3, R_3, \ldots, D_2(\alpha_{N-1}, R_{N-1}, D_1(\alpha_N, R_N)) \ldots))$$

- Layers $2, \ldots, N$ can be optimized independently of layer 1.
Rate Optimization Algorithm

Optimal rate assignment for a given $\alpha$

$$R_N^* = \arg \min_{R_N} D_1(\alpha_N, R_N)$$

$$R_n^* = \arg \min_{R_n} D_2(\alpha_n, R_n, C_{n+1}^*), \ 1 \leq n \leq N - 1$$

where

$$C_n^* = D_2(\alpha_n, R_n^*, D_2(\alpha_{n+1}, R_{n+1}^*, \cdots, D_2(\alpha_{N-1}, R_{N-1}^*, D_1(\alpha_N, R_N^*) \cdots))$$

- Linear complexity $\mathcal{O}(N|R|)$
- Globally optimal
Optimal Partitioning

• \( \mathcal{E}_D(R, \alpha) = \sum_{n=0}^{N} (P_{R_{n+1}} - P_{R_n}) \cdot 2^{-b(\sum_{k=1}^{n} \alpha_k R_k)} \)

• Constrained convex program

\[
\min_{\alpha} \quad \mathcal{E}_D(R, \alpha) \\
0 \leq \alpha_i \leq 1
\]

• **Lemma 3**: \( \mathcal{E}_D^* \) is a non-increasing function of \( N \)

• Use lemma 3 to get rid of the constraints

• Solve \( \frac{\partial \mathcal{E}_D(R, \alpha)}{\partial \alpha_i} = 0 \)
Optimal Partitioning

Optimal partition for a given $R$

$$\alpha_k^* = \frac{1}{b R_k} \log_2 \left( \frac{S_1 - S_2}{(P_{R_k} - P_{R_{k-1}}) R_{k-1}} \right), \quad 2 \leq k \leq N$$

$$\alpha_1^* = 1 - \sum_{k=2}^{N} \alpha_k^*$$

where

$$S_1 = (R_N - R_{k-1})(1 - P_{R_N})2^{-b \sum_{j=k+1}^{N} \alpha_j^* R_j}$$

$$S_2 = R_{k-1} \sum_{i=k}^{N-1} (P_{R_{i+1}} - P_{R_{i}})2^{-b \sum_{p=k+1}^{i} \alpha_p^* R_p}$$

- Globally optimal
- $O(N)$ complexity
Numerical Results

Rayleigh Fading Channel, 1x1, b=1

Expected Distortion (dB) vs SNR (dB) for different values of N:
- N=1
- N=2
- N=5
- N=500

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Numerical Results

Rayleigh Fading Channel, 2x2, b=1

- SNR (dB)
- Expected Distortion (dB)

- N=1
- N=2
- N=5
- N=500

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Numerical Results

Rayleigh Fading Channel, N=5, b=1

Expected Distortion (dB) vs. SNR (dB)
Numerical Results

![Graphs showing numerical results for different layers and signal-to-noise ratios.](image)
Concluding Remarks

• Contribution:
  • Explicit rate/partition optimization
  • Optimal, linear complexity solution

• Extensions:
  • Non-Gaussian sources (image/video)
  • Other transmission strategies (Broadcast, hybrid digital-analog, ...