Statistical Guarantee of QoS in Communication Networks with Temporally Correlated Loss

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Communication Loss Models

- Independent Loss: The probability of loss for consecutive units of information (UOIs) is independent.
- Correlated Loss: The probability of loss for consecutive units of information (UOIs) is correlated.

- UOIs: Bits, Symbols, Packets

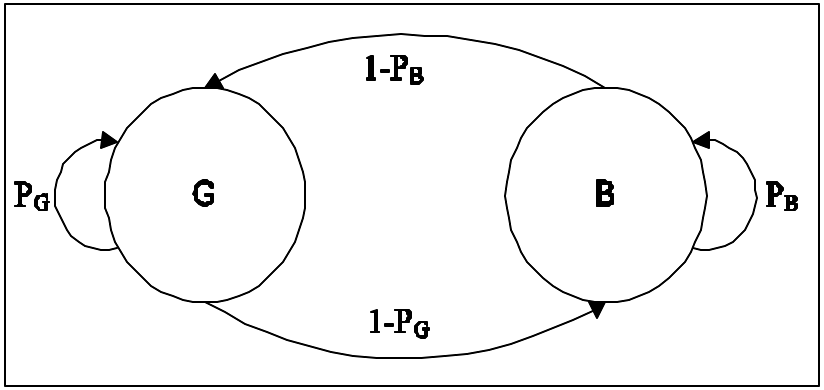
Independent Loss

- Loss is described by the Bernoulli model.
- Probability of loss for each UOI is identified by a value e, independent of other UOIs.
- Probability of transmitting n UOIs & receiving k UOIs is

$$P(n,k) = \binom{n}{k} e^{k} (1-e)^{n-k}, \quad n \geq k$$

Correlated Loss

- Loss is described by a finite-state Markov Chain [8]
- Special case of interest: 2-State Gilbert MC
- A UOI is received if the systems is in State G and lost in State B
  - Ext: Gilbert-Elliott Model w/ Non-Trivial Loss Prob's



- Probability of loss for each UOI is identified by the transition probabilities of the MC

$$P_G = g\,,\quad P_B = b$$

Motivation

- Correlated Loss
  - Wired line packet loss due to implementation of drop tail routing
  - Wireless fading channel bit and symbol loss
- Applications
  - Channel Coding [4]
  - Image/Audio Communication [2,10,11]
  - Network Loss Analysis [1]
  - Analog Channel Modeling [12]
  - Reliable/Real-Time Multicast [5,6,7,9]

Problem Specification

- What is the prob. of receiving exactly k UOIs from n transmitted UOIs?

  $$P(n,k) = P(n,k,G) + P(n,k,B), \quad n \geq k$$

- Recursive formula exists
  - May lead to high complexity solutions
  - Eliminates the possibility of analytically solving any related optimization problem

Recursive Solution

$$P(n,k) = P(n,k,G) + P(n,k,B), \quad n \geq k$$

$$P(n,k,G) = g\, P(n-1,k-1,G) + (1-b)\, P(n-1,k-1,B)$$

$$P(n,k,B) = (1-g)\, P(n-1,k,G) + b\, P(n-1,k,B)$$

Closed-Form Solution (CFS)

- For $n = v + z, \quad k = v,$ we have

$$P(v+z,v) = P(v+z,v,G) + P(v+z,v,B)$$

$$P(v+z,v,G) = g^{v-z}(1-b)(1-g)\{\sum_{i=0}^{z-1}\binom{z-1}{i}\binom{v}{i+1}(bg)^{z-1-i}[(1-b)(1-g)]^{i}\}g_{ss}$$

$$+g^{v-z-1}(1-b)\{\sum_{i=0}^{z}\binom{z}{i}\binom{v-1}{i}(bg)^{z-i}[(1-b)(1-g)]^{i}\}b_{ss}$$

$$P(v+z,v,B) = g^{v-z+1}(1-g)\{\sum_{i=0}^{z-1}\binom{z-1}{i}\binom{v}{i}(bg)^{z-1-i}[(1-b)(1-g)]^{i}\}g_{ss}$$

$$+g^{v-z}(1-b)(1-g)\{\sum_{i=0}^{z-1}\binom{z}{i+1}\binom{v-1}{i}(bg)^{z-1-i}[(1-b)(1-g)]^{i}\}b_{ss}$$

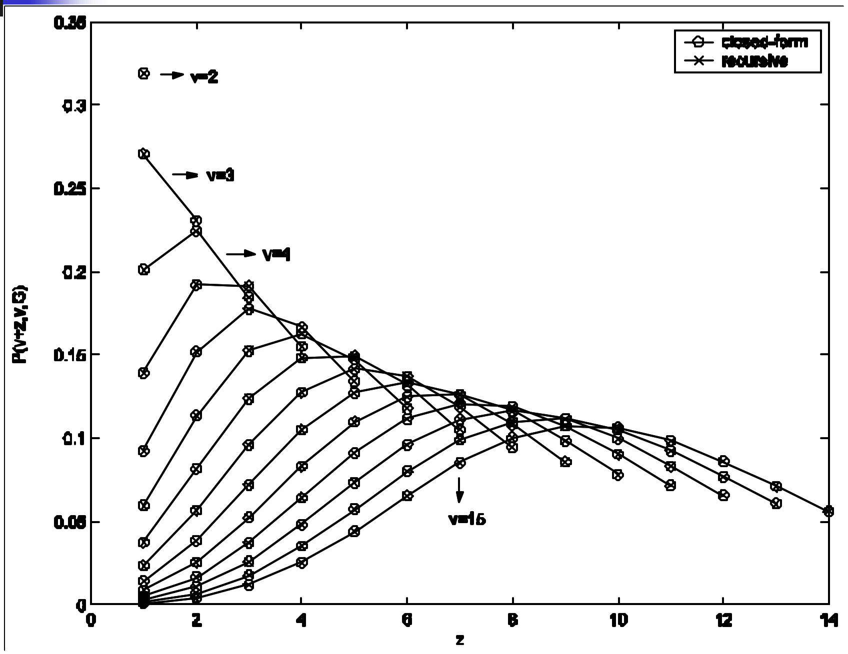
where $v \geq z + 2 \geq 3$ with

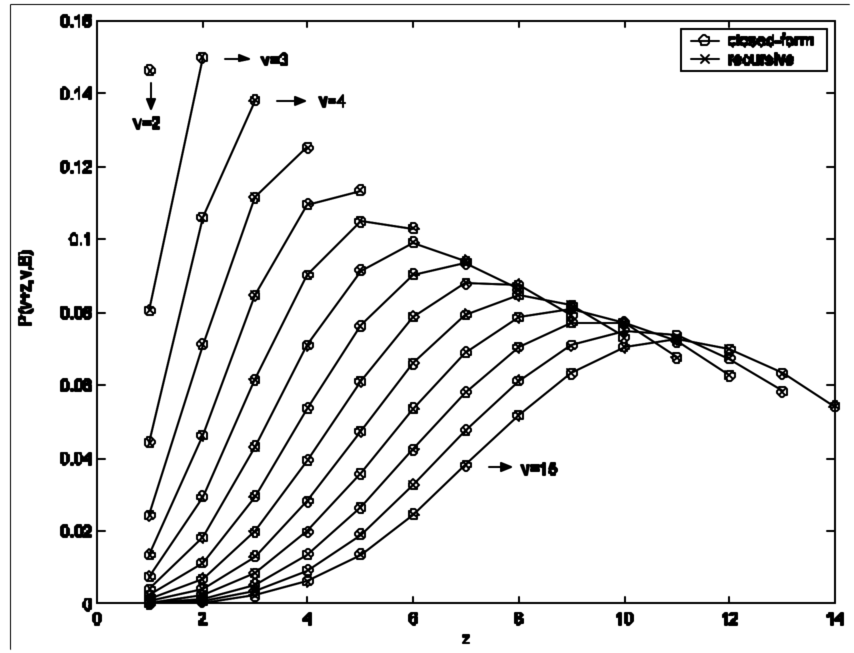
$$g_{ss} = \frac{1-b}{2-g-b}, \quad b_{ss} = \frac{1-g}{2-g-b}$$

Proof

- Use $\binom{j}{i}+\binom{j}{i+1}=\binom{j+1}{i+1}$ and $\binom{j}{0}=\binom{j}{j}=1$
  to prove if CFS holds for v and z=1,
  then it holds for v+1 and z=1
- Similarly, prove if CFS holds for v and z=2,
  then it holds for v+1 and z=2
- Utilize 2-D Mathematical Induction
  - To prove if CFS holds for v and a fixed z
    then it holds for v+1 and the same fixed z
  - To prove if CFS holds for z and a fixed v
    then it holds for z+1 and the same fixed v

Numerical Validation





A comparison plot of $P(v+z, v, G/B)$ vs z between the packet arrival results of closed-form and iterative solutions for the choice of parameters $v \in [2,15]$ and $z \in [1, v-1]$.

Statistical Guarantee Algorithm

Statistical Guarantee for Arrival of Minimum $k = v$ UOIs from $n = v + z$ transmitted UOIs with probability Π or better

Complexity $O(zv),\ z \ll v$ vs. recursive complexity $O(v^2)$

- $for \quad (z = 1 \quad to \quad v) \quad \{$
  - **Calculate** $P(v+z, v) = P(v+z, v, G) + P(v+z, v, B)$
  - **If** $\Psi(v+z, v) \geq \Pi$ **Break**

    where $\Psi(u, v) = \sum_{i=v}^{u} P(u, i)$

$\} \quad /*\ for \quad (z = 1 \quad to \quad v) * /$

- **Report the number of required packets,** $u = v + z$

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