Layered Media Multicast Control (LMMC): Error Control

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Abstract

In this paper, we present an optimal solution to the problem of error control in layered and replicated media systems satisfying real-time constraints. In doing so, we rely on an apriori estimate of loss along with a hybrid proactive FEC-ARQ scheme to statistically guarantee the quality of service for receivers. Our optimal Layered Media Multicast Control (LMMC) solution to a formulation of the error control problem analytically determines the redundancy assignment of individual groups associated with layered media or replicated media minimizing a cost metric defined over wasted bandwidth of redundancy. We also relate rate allocation and receiver partitioning aspects of LMMC to its error control aspect within the context of this research work.

Index Terms

Multicast IP Networks, Layered Media, Replicated Media, Error Control, Apriori Estimate of Loss, Statistical Guarantee of QoS.

I. Introduction

Transmitting real-time compressed digital media over multicast IP networks has been the subject of heavy research in the recent years as surveyed by Li et al. in [15] and the references cited therein. Replicated media streams approach first presented by Cheung et al. [8] within the context of DSG protocol and layered media streams approach first proposed by Deering et al. [9] in the context of multicast routing and by McCanne et al. [21] in the context of RLM protocol are convincingly the two most important methods in this area.

Realtime video and audio have limited tolerance for random loss within the compressed digital stream. The quality of decoded media at the receiver is subject to significant degradation as the result of excessive loss from network congestion or latency. In order to overcome the loss effects, error control techniques can be used. There have been three general error control approaches in the context of media multicasting.

The first approach is retransmission-based automatic repeat request (ARQ) approach. Different ARQ approaches and protocols have resulted from the interest in providing resilient multicast service for real-time data, where retransmissions occur only if data can be delivered before a real-time deadline. Two of such approaches are the error control aspect of LVMR presented by Li et al. [17] and STORM presented by Xu et al. [34]. Both approaches form virtual trees with the source as the root and receivers as internal and leaf nodes. Recovery is implemented by sending all repair requests and retransmissions using unicasting along this tree. Repairs can be performed with low latency provided that they do not need to traverse numerous links within the receiver-based tree. However, substantial delays can occur with loss close to the source. In such cases, the transmission path to a receiver can be significantly longer than the multicast route direct from the source since repairs occur as a series of unicast transmissions between receivers.

The second approach is pure Forward Error Correction (FEC) approach. Error correcting codes were initially applied in domains where bits could be erroneous or missing and later applied to repairing packet loss at the network layer. In the latter case, FEC is used to reduce the bandwidth overhead of repairing errors or loss in packet streams. The sender forms blocks of packets where each block consists of a subset of the data packets it wishes to deliver reliably. The number of data packets that are used to form a block is commonly referred to as the block size, B. The sender inputs B packets into its encoder then generating repair packets for that block. A receiver uses its decoder to recover B data packets from any combination of B distinct packets of the block and/or repairs generated for the block. Many researchers used different variations of packet-level FEC in the context of reliable and/or real-time multicasting applications. Rubenstein et al. [27] introduced the idea of using real-time reliable multicast using proactive FEC, Brockners [6] used FEC to accommodate congestion in reliable multicast bulk data transfer. Rhee et al. [25] presented a multicast layering scheme in which the FEC repair packets were proactively distributed among multiple multicast groups. Kasera et al. [12] used multiple multicast grouping to solve the repair locality problem where receivers who lost the same packets join the same multicast group for retransmission of the packets. Zhang et al. [38] investigated the techniques for optimal assignment of proactivity factor in case of deploying proactive FEC techniques.

The third approach is hybrid FEC-ARQ approach. Hybrid approaches that combine FEC and ARQ have been proposed and classified for repairing loss and noise at the bit and packet-level. Type I hybrid approach suggests sending data and proactively sending FEC repairs but retransmitting data directly if these repairs are insufficient. Type II hybrid approach uses FEC to send repairs based on retransmission requests, and sends nothing proactively. Towsley et al. [30] showed that a hybrid approach combining FEC with ARQ could significantly reduce bandwidth requirements of a large reliable multicast session over the one using stand-alone ARQ. Nonnenmacher et al. [23] presented a preliminary analysis that compares the benefits of combining local recovery with an FEC-ARQ hybrid technique. Maxemchuk et al. [20] proposed a distributed repair architecture known as client-server recovery. Kermode [13] introduced SHARQFEC a hierarchical reliable multicast error recovery protocol. Carle et al. [7] provided an extension to SHARQFEC protocol as an adaptive hybrid protocol for supporting real-time continuous media. Yoon et al. [35] introduced ARM yet another hybrid error control protocol.

The main objective of the current research work is to provide an analytical framework for the error control of layered and replicated media systems over multicast IP networks. The scope of our work addresses the issues of media systems error control and Quality of Service (QoS) within the context of Layered Media Multicast Control (LMMC) protocol. We also relate the error control scope of LMMC to its rate allocation and partitioning scope discussed in [36].

In this study, we assume the existence of congestion and flow control mechanisms capable of dynamically addressing inter-session fairness issue, i.e., a fair distribution of available bandwidth among multiple media and other sessions such as TCP sessions. Typical examples of such mechanisms are given in [31], [18], [33], [24], and [32]. Rather, the error control aspect of LMMC manifests in dynamic distribution of an available bandwidth among data and redundant traffic portions. For each individual multicast group related to a layered or a replicated media system, LMMC specifies the assignment of data and redundancy bandwidths such that the resulting redundancy bandwidth wastage is minimized.

The main contribution of this paper is in four areas. First, the paper studies temporally independent and correlated loss models utilized in a type I hybrid FEC-ARQ error control scheme considering real-time requirements of media multicast systems. The paper does so by extending the results of the studies proposed by us [37], Rubenstein et al. [27], and Towsley et al. [30]. In this area, the analysis results allow individual receivers of each multicast group to provide the source with an apriori estimate of their redundancy requirement in order to statistically guarantee quality of service. Second, the paper formulates an optimal control problem aiming at minimization of a cost function related to the redundancy wasted bandwidth. In its general form, the formulation of LMMC optimal error control problem resembles the formulation of the LMR protocol proposed by Rhee et al. [25] within the context of reliable multicast. However, it differs from LMR considering specific constraints of real-time latency. The paper also proposes an analytical solution to the optimal control problem. Third, the paper analyzes the impact of feedback implosion when dealing with large multicast groups and changes the general formulation of the problem such that feedback implosion impact can be effectively eliminated. LMMC analytical approach in this area also proposes a solution with much lower complexity than the dynamic programming solution of [25]. Finally, the paper relates the results of the current study as the error control aspect of LMMC to its rate allocation and receiver partitioning aspects proposed in [36] as an integrated solution for media multicast systems. By solving rate allocation, receiver partitioning, and error control problems within the scope of a hybrid media systems protocol, LMMC also resolves the so-called dynamic adjustment of proactivity factor problem for its hybrid FEC-ARQ approach.

In summary given the overall available bandwidth to different groups of a media session, LMMC solution to a formulation of the error control problem identifies available bandwidth to data and redundancy portions statistically guaranteeing quality of service for individual receivers of each group. In doing so, LMMC error control solution minimizes the overall bandwidth wastage of the session due to deployment of error control schemes while not compromising specific constraints of quality of service and session latency. It is important to note that the technique proposed in this paper can be independently applied to both replicated media as well as layered media systems.

An outline of the paper follows. In Section II, we adopt the notion of round-based delivery of real-time reliable multicast information for LMMC error control scheme while considering temporally independent and correlated loss models for a type I hybrid FEC-ARQ approach utilized in our study. In this section, we provide an analysis of statistically guaranteeing quality of service for different size multicast groups in media systems. In Section III, we analytically solve the formulation of the optimal control problem considering wasted bandwidth of individual multicast groups. We also consider the impact of feedback implosion when dealing with large multicast groups and propose an effective tuning in LMMC formulation that is capable of eliminating implosion impacts. In Section IV, we relate the error control aspect of LMMC to its rate allocation and partitioning to offer an integrated solution for media multicast systems. In Section V, we integrate the results of our analytical work into LMMC error control protocol. In Section VI, we focus on performance evaluation and provide simulation results along with practical considerations. Finally, Section VII concludes the paper providing a discussion of future work and concluding remarks.

II. LMMC Analysis of Redundancy

We begin our analysis by providing a brief overview of a media session according to LMMC rate allocation and partitioning work in [36]. Consider a multicast media session with a partitioning of receivers into K data groups. For a media session with N receivers and K data groups, each group $k \in \{1, \cdots, K\}$ consists of N_k receivers such that $N = \sum_{k=1}^{K} N_k$. For such a media session a set $P = \{G_1 | \cdots | G_K\}$ is called a partitioning of the receiver set $\{1, \cdots, N\}$ if P is a decomposition of the set of receivers into a family of disjoint sets. The term group rate is used to denote aggregated receiving data rate of a receiver in the group while the term layer rate is used to denote transmission data rate to a specific layer. For an ordered partitioning of receivers into K data groups with ordered group data rates of $g_1, g_2, \cdots, g_K$ such that $g_1 \leq g_2 \leq \cdots \leq g_K$, the layer data rates of a layered media session are calculated in the form of

$$g_1, g_2 - g_1, g_3 - g_2, \cdots, g_K - g_{K-1} \tag{1}$$

A receiver in data group k subscribes to data layers 1 through k receiving an aggregated data rate of g_k. Interpretation of our formulation in case of replicated media streams is also straight forward. For an ordered partitioning of the receivers into K data groups $G_1, G_2, \cdots, G_K$ with ordered group data rates of $g_1, g_2, \cdots, g_K$ such that $g_1 \leq g_2 \leq \cdots \leq g_K$, the layer data rates are the same as the group data rates. A receiver in group k only subscribes to layer k receiving a data rate of g_k.

We now turn our focus on the analysis of redundancy for a layered media session. We start by adopting the general notion of round-based delivery of real-time multicast information for LMMC error control scheme as proposed in [27] and continue by making necessary changes to make the original protocol fit into the framework of LMMC. We begin our discussion by providing the definition of a statistical guarantee for quality of service in a custom tailored type I hybrid FEC-ARQ scheme utilized in our study and continue by investigating how our definition is applied to temporally independent and correlated loss models relying on the so-called Bernoulli and Gilbert models respectively. We also introduce two alternatives appropriate for moderate and large size multicast groups in media systems with negligible NAK traffic in the latter case.

A round-based hybrid FEC-ARQ error recovery scheme for delivering multicast information appropriately applies to real-time scenarios in which a hard deadline has to be met. This deadline typically has to do with the availability of data at the playback time in a multimedia application. For each receiver, a hard deadline can be expressed in terms of the available number of rounds. Assuming that a hard deadline is given by τ_k time units for a data group G_k and a receiver i in data group G_k measures the average round trip time of a packet from the session source to be RTT_i time unit, the number of available rounds for receiver i is calculated as

$$RD_i = \lfloor \frac{\tau_k}{RTT_i} \rfloor \tag{2}$$

Applying the round-based concept to individual data groups $\{G_1, \cdots, G_K\}$ of a media session, the available number of rounds for data group G_k is defined as

$$\Gamma_k = \min_i RD_i \quad \forall i \in G_k \tag{3}$$

In the original round-based protocol of [27], the authors introduce two statistical methods relying on which a receiver can recover a block of data with a given probability, Π. In the first method Last Round Guarantee (LRG), a receiver guarantees that if a last round is necessary enough repairs are delivered in that round to assure the conditional probability of receiving all packets in the block is greater than the given probability Π. In the second method Block Good Put (BGP), a receiver achieves an overall block good put rate such that the data block is recovered on or before going to the last round with the given probability Π. Considering that the receiver has to specify the number of packets going to the last round, neither one of these methods are appropriate for error recovery techniques relying on an apriori knowledge or estimate of loss.

In what follows we propose a novel method appropriate for error recovery techniques relying on an apriori estimate of loss. In the first step of our method, we provide an analysis of calculating the number of required redundant packets in order to guarantee recovering a data block with a probability greater than a given probability Π. Considering the fact that the analysis of the first step calculates the number of redundant packets independent of the round-based recovery scheme, we fit the results of the first step into a round-based scheme in the second step.

Prior to proposing new techniques that can be effectively employed for error recovery techniques relying on an apriori estimate of loss, we briefly explain the single-state Bernoulli and the two-state Gilbert models. The simplest loss model describing packet loss in the Internet is the single state Bernoulli model assuming the probabilities of loss among different packets are temporally independent. In the Bernoulli model, the probability of receiving at least v packets from u transmitted packets is given by

$$D(u,v) = \sum_{i=v}^{u} \binom{u}{i} (1-p)^i p^{(u-i)} \tag{4}$$

for a packet loss probability of p. Bernoulli loss model is a suitable tool for capturing the loss pattern of slowly varying network conditions such as dedicated ISDN lines and/or controlled processes by means such as packet interleaving insertion methods. However as pointed out in [22], [27], and many other articles, Internet packet loss typically undergoes burst loss representing temporally correlated loss. This is related to the fact that many of the routers utilized in the Internet have deployed drop-tail routing. The two-state Gilbert loss model provides an elegant mathematical model to capture the loss behavior of ever-changing network conditions. In the Gilbert model, packet loss is described by a two-state Markov chain as described in Figure 1. The first state G known as the GOOD state represents the receipt of a packet while the other state B known as the BAD state represents the loss of a packet. The GOOD state introduces a probability $P_G = \gamma$ of staying in the GOOD state and a probability $1 - P_G$ of transitioning to the BAD state while the BAD state introduces a probability $P_B = \beta$ of staying in the BAD state and a probability $1 - P_B$ of transitioning to the GOOD state. The parameters γ and β can be typically measured from the observed loss rate and burst length.

In [37] we show that assuming an arbitrarily far start instant of time for the Markov chain of Figure 1, the closed-form expression for receiving exactly v packets from $v + z$ transmitted packets under the Gilbert loss model is given by

$$P(v+z, v) \ = \ P(v+z, v, G) + P(v+z, v, B) \tag{5}$$

where $P(v+z, v, G)$ the probability of receiving exactly v packets from $v+z$ transmitted packets and

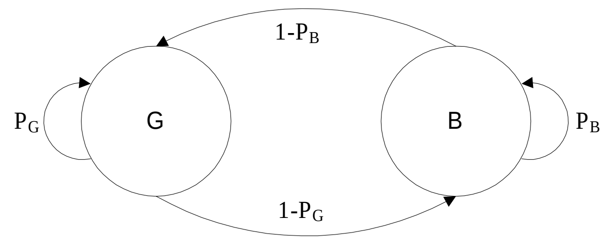

Fig. 1. The two-state Gilbert loss model with the state transition probabilities $1 - P_G$ and $1 - P_B$ for $P_G = \gamma$ and $P_B = \beta$.

winding up in the GOOD state is given by

$$\begin{aligned} P(v+z,v,G) \ = \\ \gamma^{v-z}\,(1-\beta)\,(1-\gamma)\,\{ \\ \sum_{i=0}^{z-1} \tbinom{z-1}{i} \tbinom{v}{i+1} (\beta\,\gamma)^{z-1-i}\,[(1-\beta)\,(1-\gamma)]^i \\ \}\,g_{ss} \quad + \quad \gamma^{v-z-1}\,(1-\beta)\,\{ \\ \sum_{i=0}^{z} \tbinom{z}{i} \tbinom{v-1}{i} (\beta\,\gamma)^{z-i}\,[(1-\beta)\,(1-\gamma)]^i \\ \}\,b_{ss} \end{aligned} \tag{6}$$

$P(v+z,v,B)$ the probability of receiving exactly v packets from $v+z$ transmitted packets and winding up in the BAD state is given by

$$\begin{aligned} P(v+z,v,B) \ = \\ \gamma^{v-z+1}\,(1-\gamma)\,\{ \\ \sum_{i=0}^{z-1} \tbinom{z-1}{i} \tbinom{v}{i} (\beta\,\gamma)^{z-1-i}\,[(1-\beta)\,(1-\gamma)]^i \\ \}\,g_{ss} \quad + \quad \gamma^{v-z}\,(1-\beta)\,(1-\gamma)\,\{ \\ \sum_{i=0}^{z-1} \tbinom{z}{i+1} \tbinom{v-1}{i} (\beta\,\gamma)^{z-1-i}\,[(1-\beta)\,(1-\gamma)]^i \\ \}\,b_{ss} \end{aligned} \tag{7}$$

for $v, z \in \{1, 2, 3, \cdots\}$, steady state probability of the GOOD state $g_{ss} = \frac{1-\beta}{2-\gamma-\beta}$, steady state probability of the BAD state $b_{ss} = \frac{1-\gamma}{2-\gamma-\beta}$, and the following initial conditions.

$$\begin{aligned} P(v,0,G) &= 0 \\ P(v,v,B) &= 0 \\ P(v,v,G) &= \gamma^v \, g_{ss} + (1-\beta)\gamma^{(v-1)} \, b_{ss} \\ P(v,0,B) &= (1-\gamma)\beta^{(v-1)} \, g_{ss} \; + \; \beta^v \, b_{ss} \end{aligned} \tag{8}$$

Utilizing equality

$$D(v+z,v) \; = \; \sum_{i=v}^{v+z} P(v+i,i) \tag{9}$$

and imposing a practical upper bound of v on the value of z, [37] also introduces the following algebraic placement algorithm with a time complexity of $\mathcal{O}(z\,v)$ to calculate the smallest number of required transmitted packets $u = v + z$ in order to guarantee the receipt of at least v packets with a probability Π or better for a system governed by the Gilbert loss model.

Statistical Guarantee for Packet Arrival Algorithm

- $for \quad (z = 1 \quad to \quad v) \, \{$
  - Calculate $P(v+z,v) = P(v+z,v,G) + P(v+z,v,B)$ from Equation (6) and Equation (7).
  - If $D(v+z,v) \geq \Pi$ Break.

  $\}$ /* $for \quad (z = 1 \quad to \quad u)$ */
- Report the number of required packets, $u = v + z$.

Taking into consideration specific design issues of LMMC pertaining to combining its rate allocation/partitioning aspects with its error control aspect and considering the above algorithm, we now propose two new alternatives in providing a statistical guarantee within the context of our current discussion. In the first alternative which we refer to as the Dynamic Mode (DM) of requesting redundant packets, we propose that an individual receiver i of a media session data group G_k waits until the last round in order to report its required redundancy by finding u_i either from the Bernoulli model or from the Gilbert model assuming the receiver is in need of v_i packets going to the last round. An individual receiver i then reports $r_i = \min(u_i, B_k)$ as its redundancy requirement where B_k indicates the block size for data group G_k [1]. We note that DM method is essentially an enhanced version of LRG adopted for layered media systems. The major differences between DM and LRG methods are utilization of closed-form solutions rather than recursive solutions in case of utilizing the Gilbert loss model and considering an upper bound on the number of redundant packets.

In the second alternative which we categorize under the Static Mode (SM) of requesting redundant packets, we propose that an individual receiver i of a media session data group G_k carries out an apriori estimate of loss. In our analysis pertaining to SM alternative, we consider a block recovery probability of Π_k with equal per round probabilities of π_k for the available number of rounds in data group G_k of a media session. Assuming the source of a media session only initiates a new transmission round for the receivers of data group G_k as the result of receiving at least one NAK from the receivers of the group, we relate the two quantities as

$$\Pi_k = 1 - (1 - \pi_k)^{\Gamma_k} \tag{10}$$

yielding

$$\pi_k = 1 - \sqrt[\Gamma_k]{1 - \Pi_k} \tag{11}$$

Hence, given the overall probability of block recovery Π_k for data group G_k, the per round probability of block recovery is calculated from Equation (11).

In the second alternative a receiver obtains an estimate of required redundant packets by assuming that it receives an expected number of packets according to its probability distribution $D(u_i, v_i)$ going from

[1] For practical reasons, we place an upper bound equal to the block size on the redundancy requirement of data group G_k.

one round to another. Inserting an assurance coefficient ψ in the range of $1 \leq \psi < 2$ and starting from an initial value of $v_i = (\psi - 1)\ B_k$ for the first round, the number of requested packets u_i requested by receiver i in each round is hence calculated by deducting the expected number of arrived packets in the previous round from the current value of v_i. Consequently, receiver i of data group G_k calculates the number of packets for round j indicated by u_i^j based on the expected number of required packets for round j indicated by v_i^j as

$$D(u_i^j, v_i^j) \geq \pi_k \tag{12}$$

utilizing either the Bernoulli or the Gilbert loss model. We note that Equation (12) holds assuming $v_i^j = v_i^{j-1} - \bar{u}_i^{j-1}$ for $v_i^1 = (\psi - 1)\ B_k$ and realizing the fact that the term $\bar{u}_i^{j-1}$ indicates the expected number of arrived packets in round $j - 1$. We also note that $\bar{u}_i^{j-1} = (1 - p).u_i^{j-1}$ in case of utilizing the Bernoulli model and $\bar{u}_i^{j-1} = g_{ss}.u_i^{j-1}$ in case of utilizing the Gilbert model. Overall required redundancy of receiver i is, then, calculated as

$$r_i = \min(\sum_{j=1}^{\Gamma_k} u_i^j, B_k) \tag{13}$$

The receiver then announces its overall redundancy and per round required redundancy sequence r_i and $\{u_i^1, \cdots, u_i^{\Gamma_k}\}$ to the source.

From a complexity stand point, our approach introduces a time complexity of $\mathcal{O}(B_k)$ in case of utilizing the Bernoulli loss model. The complexity of our approach is $\mathcal{O}(z\ B_k)$ in case of utilizing the Gilbert loss model where z is the smallest number chosen in order to statistically guarantee the receipt of at least v packets from $v + z$ transmitted packets. We note that in the latter case, the complexity of our approach matches that of a dynamic programming approach $\mathcal{O}(B_k^2)$ only in its worst case scenario.

We note that the main objective in the second alternative is to provide the receivers with an opportunity to recover a block with equal probabilities π_k in each round. The latter is of special interest from the design stand point of LMMC in which an apriori estimate of receivers loss is required in order to combine rate allocation and receiver partitioning aspects of a media system with its error control aspect. At the end of this section, we mention that the overhead of combining rate allocation, receiver partitioning, and error control aspects of a media system makes DM and SM methods suitable for moderate and large size multicast groups respectively. We leave the discussion of rate allocation/partitioning aspects of LMMC with its error control aspect to Section IV.

III. LMMC Optimal Solution to The Error Control Problem

A. General Formulation

Having calculated the required redundancy for individual receivers of a multicast group in a media multicast group, we now focus on the formulation of the optimal error control problem and LMMC's analytical solution to the problem. We formulate our layered real-time error control problem in a way similar to Layered Multicast Recovery (LMR) protocol proposed in [25]. However, we make note of the differences in the formulation as well as the solution. We note that unlike the formulation of [25] that is intended for reliable multicast, the formulation of our problem is within the context of layered or replicated media systems and is hence subject to real-time constraints applied to media systems. In addition, because of targeting at providing a set of integrated protocols for media systems in conjunction with what was discussed in [36], we rely on an apriori estimate of redundancy. Finally, rather than relying on dynamic programming, we propose a lower complexity analytical solution to the problem within the context of LMMC error control protocol. In our error control model for media systems, we associate ς_k multicast *redundancy* groups with every individual data group G_k. Although we apply a fixed value to parameter ς_k in our formulation, we mention that the choice of ς_k is a design parameter with the objective of providing a balance between bandwidth wastage and overhead of managing multicast groups.

The sequence of events is as follows. First, the source polls individual receivers about their redundancy requirement with the details of polling mechanism discussed in Section V. Receivers then respond based on one of DM or SM schemes of Section II indicating the number of redundant packets required to statistically guarantee the recovery of data blocks. We note that the process of collecting redundancy information is subject to feedback implosion and address the implosion problem in the next subsection.

Assuming a block size of B_k for data group G_k, the source transmits B_k data packets to data group G_k followed by ρ_j redundant packets for $j = 1, \cdots, \varsigma_k$ to ς_k independent redundancy groups. From a layering stand point, the formulation of the error control problem is similar to the two-phase rate allocation and partitioning problem of our earlier work in [36]. This means that a receiver can subscribe to a redundancy group only if it has already subscribed to all of the previous redundancy groups. However, we note that in this case the collection of redundancy groups $\{1, \cdots, \varsigma_k\}$ combined together are considered to be the error control groups associated with data group G_k in the rate allocation and partitioning problem.

In this analysis, we consider a partitioning of the receivers of data group G_k into ς_k groups according to their redundancy requirement. For data group G_k with N_k receivers we associate ς_k redundancy groups, each redundancy group ρ_j with $j \in \{1, \cdots, \varsigma_k\}$ carrying a portion of redundant traffic. For a data group G_k, a set $\Omega_k = \{R_1 | \cdots | R_{\varsigma_k}\}$ is called a partitioning of the receiver set of data group $G_k = \{1, \cdots, N_k\}$ if Ω_k is a decomposition of the set of receivers into a family of disjoint sets. The term group rate is used to denote the aggregated receiving redundancy rate of a receiver in the group while the term layer rate is used to denote the transmission redundancy rate to a specific layer. For an ordered partitioning of receivers into ς_k redundancy groups with ordered group redundancy rates of $\rho_1, \rho_2, \cdots, \rho_{\varsigma_k}$ such that $\rho_1 \leq \rho_2 \leq \cdots \leq \rho_{\varsigma_k}$, the layer redundancy rates of a layered error control scheme are calculated in the form of

$$\rho_1, \rho_2 - \rho_1, \rho_3 - \rho_2, \cdots, \rho_{\varsigma_k} - \rho_{\varsigma_k - 1} \tag{14}$$

A receiver in redundancy group j subscribes to layers 1 through j receiving an aggregated redundancy rate of ρ_j. A receiver, hence, subscribes to a subset of redundancy groups associated with its calculated loss rate. As loss rate varies over the time, a receiver can join extra or leave already subscribed multicast redundancy groups to dynamically adapt to its redundancy requirements with the cost of joining and leaving multicast groups being less than retransmission cost. As it will be explained in the following sections, LMMC error control protocol employs built-in mechanisms in order to control the overhead of receivers joining and leaving multicast groups considering real-time constraints of media systems. We note that the implications of utilizing such built-in mechanisms manifests in controlling the timings in which receivers are allowed to subscribe to extra redundancy groups.

In order to formulate a per group error control problem for the individual data groups G_k with $k \in \{1, \cdots, K\}$ of a media session, we first define the function EC_k as the bandwidth wastage of data group G_k over all of its redundancy groups R_j with $j \in \{1, \cdots, \varsigma_k\}$ as

$$EC_k \stackrel{\text{def}}{=} \sum_{j=1}^{\varsigma_k} \sum_{i \in R_j} (\rho_j - r_i) \tag{15}$$

Assuming there exists a per data group upper bound on the maximum number of redundant packets in the form of $\max_i r_i = U_k$ where $U_k \leq B_k$, we formulate the optimal error control problem of the individual data groups G_k of a media session as

$$\min_{\rho_1, \cdots, \rho_{\varsigma_k}} EC_k \quad = \quad \min_{\rho_1, \cdots, \rho_{\varsigma_k}} \sum_{j=1}^{\varsigma_k} \sum_{i \in R_j} (\rho_j - r_i) \tag{16}$$

$$\text{Subject To:} \qquad r_i \leq \rho_j \quad \forall i \in R_j \tag{17}$$

$$\rho_{\varsigma_k} \leq B_k \quad j \in \{1, \cdots, \varsigma_k\}$$

where ς_k with $k \in \{1, \cdots, K\}$ is the number of redundant groups associated with data group G_k. We note that taking into consideration the efficiency issue as well as the relationship between the group rates and the layer rates, we have limited the redundancy bandwidth of each group in the form of $\rho_{\varsigma_k} \leq B_k$ for $k \in \{1, \cdots, K\}$.

We are now in a position to provide the LMMC solution to Equation (16) with Constraint (17). Rather than relying on a dynamic programming approach as suggested in [25], we utilize an analytical approach in solving the problem. Considering the general objective of minimizing the cost function of Equation (15), it is imperative to assign a receiver i with required redundancy r_i to the redundancy group R_j with the group redundancy rate ρ_j for a set of given group redundancy rates $\{\rho_1, \cdots, \rho_{\varsigma_k}\}$, if the receiver bandwidth wastage $(\rho_j - r_i) \geq 0$ is minimized for the choice of ρ_j. As the result, we make the observation that the optimal receiver partitioning strategy has to assign receiver i with the isolated rate r_i to the redundancy group R_j with the group redundancy rate ρ_j such that

$$0 \leq (\rho_j - r_i) \leq (\rho_l - r_i) \quad l \in \{1, \cdots, \varsigma_k\} \tag{18}$$

It is proven in **Lemma (II.1)** of [26] that for such partitioning of the receivers utilized in LMMC formulation, the optimal redundancy rate of each partition is equal to the largest redundancy requirement of the receivers of that specific partition, i.e.,

$$\rho_j^* = \max_{i \in R_j} r_i \quad j \in \{1, \cdots, \varsigma_k\} \tag{19}$$

Let us now pay attention to the implication of the latter result in case of applying an optimal partitioning strategy to a simple partitioning of the receivers into two redundancy groups. For an ordered partitioning $\Omega_k = \{R_1, R_2\}$ of the receivers $G_k = \{1, \cdots, M_1, M_1 + 1, \cdots, M_2\}$ with M_1 indicating the last receiver of partition R_1 and M_2 indicating the last receiver of partition R_2, we note that an ordered partitioning $\Omega_k = \{R_1, R_2\}$ has to be changed moving a receiver s with redundancy requirement r_s and all of the receivers with greater redundancy requirements from partition R_1 to partition R_2 if the resulting cost function (EC^n) is less than the current cost function (EC^c) , i.e.,

$$\begin{aligned}
&EC^n \; < \; EC^c \Rightarrow \\
&\sum_{j=1}^{s-1}(r_{s-1} - r_j) + \sum_{j=s}^{M_1}(r_{M_2} - r_j) \; < \; \sum_{j=1}^{M_1}(r_{M_1} - r_j) \Rightarrow \\
&\sum_{j=1}^{s-1} r_{s-1} + \sum_{j=s}^{M_1} r_{M_2} \; < \; \sum_{j=1}^{M_1} r_{M_1} \Rightarrow
\end{aligned}$$

$$(s-1).r_{s-1} \;+\; (M_1 - s + 1).r_{M_2} \;<\; M_1.r_{M_1} \tag{20}$$

In other words, a receiver s with redundancy requirement r_s and all of the receivers with greater redundancy requirements in partition R_1 have to move to partition R_2 if

$$M_1\,(r_{M_2} - r_{M_1}) \;<\; (s-1)\,(r_{M_2} - r_{s-1}) \tag{21}$$

Likewise, a receiver t with redundancy requirement r_t and all of the receivers with lower redundancy requirements in partition R_2 have to move to partition R_1 if

$$M_1\,(r_{M_2} - r_{M_1}) \;<\; t\,(r_{M_2} - r_t) \tag{22}$$

Generalizing this approach for an ordered partitioning $\{R_1, \cdots, R_{\varsigma_k}\}$ of the receivers, we propose the following iterative algorithm to solve the optimal error control problem of Equation (16) with Constraint (17).

LMMC Error Control Algorithm: An Iterative Layered Partitioning Approach

- Step 1: Start from an initial ordered partitioning of the receivers by uniformly distributing the receivers among the redundancy groups. In addition, set the initial iteration number $it = 0$ and the maximum number of iterations it_{max}.
- Step 2: Calculate the optimal redundancy rates of each partition R_j with $j \in \{1, \cdots, \varsigma_k\}$ from Equation (19) and the resulting error control cost function EC_k from Equation (15). Save the previously calculated EC_k in variable q_1 and the currently calculated EC_k in variable q_2.
- Step 3: If $\frac{|q_1 - q_2|}{q_1} < \delta$ or $it > it_{max}$ STOP.
- Step 4: $for \quad (j = \varsigma_k \quad downto \quad 2)\ \{$
  - Repartition groups $j-1$ and j according to Equation (21) and Equation (22).

  $\}$ /* $for \quad (j = \varsigma_k \quad downto \quad 2)$ */

- Step 5: Go back to Step 2.

For the sake of clarity we note again that as a part of an integrated media session solution, the above algorithm has to be independently applied to redundancy multicast groups R_j with $j \in \{1, \cdots, \varsigma_k\}$ of individual data groups G_k where $k \in \{1, \cdots, K\}$ in a media session.

In LMMC error control algorithm, the initial conditions are chosen in the first step. While the second step selects the optimal redundancy rates, the third step merely checks to terminate the algorithm according to the specified conditions. Finally, the fourth step indicates the re-partitioning strategy. We also note that LMMC error control algorithm moves multiple receivers with the same redundancy requirements from one redundancy group to another together.

Theorem 3.1: "LMMC Error Control Algorithm" given in this section converges to a local minimum.

A formal proof is given in Appendix I. Intuitively, LMMC algorithm is employing steepest descent optimal control strategy. It is important to note that considering the convergence speed of the proposed LMMC algorithm as proven by steepest descent approach and supported by our simulation results of Section VI, the use of LMMC error control algorithm yields fast converging results.

B. Suppressing the Impact of Feedback Implosion

As mentioned in [25], the original formulation of the error control problem for reliable multicast systems and consequently the one in Section III.A are subject to feedback implosion. In this section, we make the necessary changes in LMMC error control protocol to address the feedback implosion issue. Our modified protocol can be applied to both reliable multicast systems as well as real-time media systems over multicast IP networks.

We start by pointing out that in our specific case, feedback implosion is the result of individual receivers reporting their required redundancy to the source. However it can be easily observed that for a block size of B_k in group G_k with $k \in \{1, \cdots, K\}$ satisfying Constraint (17), all of the receivers' reported redundancy numbers are in the range of $[1, B_k]$ [2]. The source can, hence, rely on a hierarchical tree-based feedback aggregation protocol similar to the one proposed in [19] or [14] to poll the receivers for their required redundancy. To be more specific, the source first sends a message to data group G_k specifying the block size B_k for the group. The source then continues by multicasting individual polling packets for the number of redundant packets i in the range $[1, B_k]$ to find out about the number of receivers associated to each number i. The method can effectively eliminate the feedback implosion effect and depending on the desired granularity, the source can even announce a subset of the numbers in the range of $[1, B_k]$ as the acceptable values of redundancy to the receivers. In the latter case, each receiver can report the closest number to its calculated redundancy to its upstream and eventually the source.

The use of the above-mentioned method changes the formulation of the error control problem of group G_k from Equation (16) with Constraint (17) to

$$\min_{\rho_1, \cdots, \rho_{\varsigma_k}} ECW_k = \min_{\rho_1, \cdots, \rho_{\varsigma_k}} \sum_{j=1}^{\varsigma_k} \sum_{i=1}^{B_k} w_{ji} \left(\rho_j - i\right) \tag{23}$$

$$\text{Subject To:} \quad \rho_{\varsigma_k} \leq B_k \tag{24}$$

where w_{ji} indicates the weighting function associated with the receivers in group j requesting redundancy i and $\sum_{j=1}^{\varsigma_k} \sum_{i=1}^{B_k} w_{ji} = N_k$. It is also important to note that $w_{ji} = 0$ if $\rho_j < i$ or if a receiver does not belong to redundancy group j. Also note that in Equation (23), the redundancy requirements of the set of receivers accepts values from the set of natural numbers in the range of $[1, B_k]$ and the inner sum upper limit has been changed to B_k from the range of the redundancy group receivers in Equation (16).

[2]Make note of the fact that receivers with zero redundancy requirement will have no impact on the cost function of Equation (16).

Equation (23) with Constraint (24) can be solved using the same method utilized in Section III.A making note of the fact that Theorem (3.1) still holds. The proof is similar to that of Theorem (3.1) and is skipped. The time complexity of solving for this equation is $\mathcal{O}(IB_k)$ where I indicates the number of iterations.

IV. LMMC Integration of Error Control with Rate Allocation and Partitioning

In [36], we formulated rate allocation and receiver partitioning problems of layered media systems within the context of LMMC protocol in the form of

$$\max_{g_1,\cdots,g_K} IRFA_{Tot} \equiv \max_{g_1,\cdots,g_K} \sum_{k=1}^{K} IRFA_k \tag{25}$$

$$= \max_{g_1,\cdots,g_K} \sum_{k=1}^{K} \sum_{i \in G_k} \frac{(2+a) r_i g_k}{g_k^2 + a r_i g_k + r_i^2}$$

$$\text{Subject To:} \quad g_k \leq \text{BWL}_k \quad k = 1, \cdots, K \tag{26}$$

$$g_k \leq \text{BWF}_k \quad k = 1, \cdots, K \tag{27}$$

where BWL_k in the constraint of Equation (26) was defined as $\text{BWL}_k \equiv \min_{i \in G_k} \frac{r_i}{1-L_i}$, and the constraint of Equation (27) indicated the available group bandwidth as the result of enforcing a per group inter-session fairness algorithm. Further, the group fairness utility function $IRFA_k$ was defined as

$$IRFA_k \equiv \sum_{i \in G_k} E(r_i, g_k) = \sum_{i \in G_k} \frac{(2+a) r_i g_k}{g_k^2 + a r_i g_k + r_i^2} \tag{28}$$

By defining $BWA_k \equiv \min(\text{BWL}_k, \text{BWF}_k)$, we converted the rate allocation problem to

$$\max_{g_1,\cdots,g_K} IRFA_{Tot} = \max_{g_1,\cdots,g_K} \sum_{k=1}^{K} IRFA_k \tag{29}$$

$$= \max_{g_1,\cdots,g_K} \sum_{k=1}^{K} \sum_{i \in G_k} \frac{(2+a) r_i g_k}{g_k^2 + a r_i g_k + r_i^2}$$

$$\text{Subject To:} \quad g_k \leq \text{BWA}_k \quad k = 1, \cdots, K \tag{30}$$

We noted while Constraint (26) reflected the receivers bandwidth processing capabilities, Constraint (27) was the result of employing a flow control mechanism with the objective of enforcing inter-session fairness among different flows. Further, we converted the general problem of Equation (29) and Constraint (30) to an optimization problem without constraints by defining a Lagrangian function in the form of

$$\begin{aligned} LG_{IRF} &= IRFA_{Tot} + \sum_{k=1}^{K} \mu_k \left(g_k - \mathrm{BWA}_k \right) \\ &= \sum_{k=1}^{K} IRFA_k + \sum_{k=1}^{K} \mu_k \left(g_k - \mathrm{BWA}_k \right) \end{aligned} \tag{31}$$

where the parameters μ_k for $k \in \{1, \cdots, K\}$ were the Lagrange multipliers in the Lagrangian Equation (31). Considering the specific form of the function $IRFA_{Tot}$ and the constraint set of (30), we obtained the optimal solution by decomposing the system of $2K$ equations and $2K$ unknowns from $\nabla IRFA_{Tot}(g^*) = 0$ and the constraints (30) into K pairs of independent equations. Equation (32) shows the simplified formulation applied to the set of $k \in \{1, \cdots, K\}$ independent problems.

$$\begin{aligned} \max_{g_k} IRFA_k &= \max_{g_k} \sum_{i \in G_k} \frac{(2+a) r_i g_k}{g_k^2 + a r_i g_k + r_i^2} \\ \text{Subject To:} \quad & g_k \leq \mathrm{BWA}_k \end{aligned} \tag{32}$$

Relating the error control aspect of LMMC as described in the previous sections with its rate allocation and partitioning aspect as described in [36] is now straight forward. We relate the two aspects by applying the results of error control problem in adjusting the constraints of rate allocation and partitioning problem in each iteration of the rate allocation and partitioning iterative algorithm. More precisely, let us assume that solving the error control problem of the sample group G_k with the block size B_k yields to calculations of the redundancy rates ρ_j with $j \in \{1, \cdots, \varsigma_k\}$. We first define *Coefficient of Redundancy* for group G_k as

$$CoR_k \equiv \frac{B_k}{B_k + \rho_{\varsigma_k}} \tag{33}$$

Multiplying the constraint BWA_k by CoR_k, we can consider the impact of LMMC error control aspect in solving LMMC rate allocation and partitioning problem. In essence, our proposed technique divides the available bandwidth BWA_k between the data and redundancy components with weighting functions of CoR_k and $(1 - \text{CoR}_k)$ respectively. It is obvious that the proposed change only impacts the numerical value of the constraint BWA_k in each iteration of the rate allocation and partitioning algorithm without changing the optimal control problem (32).

Consequently, the integration process of rate allocation and partitioning aspect of LMMC with its error control aspect expands the formulation of two-phase optimal control rate allocation and partitioning problem into a three-phase problem. While the first two phases deal with the rate allocation and partitioning problems for individual layers of the media session, the third phase copes with the end-to-end error control problem. Considering the fact that the solution to LMMC three-phase optimal problem is sub-optimal due to the impact of its phasing approach, we expand the scope of LMMC to an iterative approach that is capable of reaching a near-optimal solution. We argue that a near-optimal solution can be reached by iteratively calculating the overall bandwidth available to layers redundancy as obtained in the third phase and applying the resulting coefficient of redundancy CoR_k to the constraint BWA_k. In what follows we propose the integrated iterative LMMC algorithm to simultaneously solve rate allocation, receiver partitioning, and error control problems.

LMMC Iterative RAP-EC [3] Algorithm:

- Step 1: Given the redundancy requirements of the receivers, start from an initial ordered partitioning of the receivers by uniformly distributing receivers among the existing groups. In addition, set the initial iteration number $j = 0$ and the maximum number of iterations j_{max}.
- Step 2: For each group G_k with $k \in \{1, \cdots, K\}$ obtain the adjusted Constraint (30) resulting from multiplying BWA_k by the coefficient of redundancy CoR_k. Calculate $\text{CoR}_k = \frac{B_k}{B_k + U_k}$ with $U_k = \max_i u_i$ and u_i indicating the redundancy requirement of receiver i for $\forall i \in G_k$.
- Step 3: Calculate the optimal group rates $g^* = \{g_1^*, \cdots, g_K^*\}$ and the resulting session utility

[3] Rate Allocation-Partitioning-Error Control

$IRFA_{Tot}$ by numerically solving the set of optimal control problems of Equation (32). Save the previously calculated $IRFA_{Tot}$ in variable x_1 and the currently calculated $IRFA_{Tot}$ in variable x_2.

- Step 4: If $\frac{|x_1 - x_2|}{x_1} < \delta$ or $j > j_{max}$ STOP.
- Step 5: $for \quad (k = 2 \quad to \quad K) \{$
  - Calculate the partitioning threshold $\sqrt{g_{k-1} g_k}$.
  - Repartition groups $k-1$ and k. For every receiver belonging to groups $k-1$ or k and isolated rate r_i, assign the receiver to group k if $r_i > \sqrt{g_{k-1} g_k}$ and one of the following conditions

$$\frac{r_i}{1 - L_i} \geq g_k^*$$

$$C_2\, g_k^* < \frac{r_i}{1-L_i} < g_k^*$$

    holds. Otherwise, assign the receiver to group G_{k-1}.
  - Calculate redundancy rates and redundancy partitioning of group G_k according to LMMC Error Control Algorithm of Section III.
  - Calculate coefficient of redundancy CoR$_k$ for group G_k from Equation (33) and obtain the adjusted Constraint (30) resulting from multiplying BWA$_k$ by CoR$_k$.
  - Calculate the new optimal sending rate of group G_k according to the new partitioning.

  $\}$ /* $for \quad (k = 2 \quad to \quad K)$ */
- Step 6: Go back to Step 3.

Theorem 4.1: Convergence of “LMMC Iterative RAP-EC Algorithm” given in this section is guaranteed.

The formal proof is identical to the proof of Theorem (5.1) given in [36] considering the fact that the impact of adjusting the constraint BWA$_k$ in each iteration does not impact the bounded non-decreasing sequence of utility function values at each step of the algorithm guaranteeing convergence to a fixed point.

Again, our algorithm is employing steepest descent optimal control strategy In practice, the use of the iterative method is a factor of time complexity and the speed of convergence. The iterative method can be effectively deployed in environments with relatively stable assignment of the bandwidth such as the scenarios encountered in admission control problems. In environments with rapidly varying available bandwidth such as the scenarios encountered in congestion control problems, the sub-optimal solution with few or one iteration may be deployed. Considering the lower time complexity of the error control problem, the time complexity of LMMC RAP-EC algorithm is $\mathcal{O}(IKN \log N)$ the same as LMMC RAP algorithm reported in [36].

A discussion of practicality is in order here. In many cases, different solutions aiming at solving different pieces of a high level problem cannot be integrated together because of conflicting underlying assumptions. Consequently, successful integration of research results plays a crucial role in their real-world acceptance. We argue that LMMC has achieved an accepted level of practicality by proposing a method of jointly solving two different sub-problems of error control and rate allocation within the context of the high level problem of layered media multicast. We also argue that the only requirement for simultaneously solving the error control and the rate allocation problems within the context of LMMC protocol is to solve the two problems simultaneously together.

V. LMMC Error Control Protocol

This section focuses on describing LMMC error control protocol relying on the analytical study of the previous sections. Generally speaking, LMMC error control protocol relies on the source of a media system to solve the error control problem based on the information collected from the receivers of a media system. The information includes the number of available rounds and the redundancy requirement of individual receivers. The source repeats the calculations pertaining to the solution of the combined problem as the result of a significant potential change in the system. A significant potential change in the system can be flagged based on one of the following two events. First, when the source polling period timer goes off and second, when a significant change is reported by a designated receiver in the middle of a polling period. The latter may have been caused for example by occurrence of congestion in a segment of the network impacting the receivers of a specific zone. LMMC relies on designated per zone receivers to collect such information and notify the source about the existence of such conditions. At the beginning of every new polling period caused by expiration of the source timer or a significant redundancy change of a group of receivers in a local zone, the source probes the receivers for the number of available rounds as well as their redundancy requirement. Individual receivers then rely on the methods of Section II to calculate the number of rounds as well as their redundancy requirement. The source then proceeds with collecting and calculating the bandwidth assignment of data and redundancy traffic following the algorithm of Section III.B.

The source discovers the available number of rounds for each group by polling the group of receivers through multicasting pilot packets. The receivers then set their own timers with a random value reporting back the result after having an expired timer only if not having seen a value less than or equal to the calculated number of rounds from Equation (2). The number of available rounds Γ_k for group G_k is eventually announced to the group utilizing Equation (3). The source relies on a hierarchical tree-based feedback aggregation protocol similar to RMTP proposed in [19] to poll the receivers for their required redundancy. It first sends a message to data group G_k specifying the block size B_k for the group and continues by multicasting individual polling packets for the number of redundant packets i associated with the natural numbers in the range $[1, B_k]$. As the default rule of practice and considering the fact that the source requires an apriori estimate of the receivers' redundancy over the total number of rounds, receivers rely on the SM method of section II to estimate their redundancy requirement and report it back to the source. In this section, it makes sense to review the issues involved with the implementation of the SM method described in Section II. Noting that per round redundancy requirement of every receiver changes over a polling period according to Equation (12), the source needs to discover the number of available rounds Γ_k and the sequence of per round redundancy of the receivers $\{u_i^1, \cdots, u_i^{\Gamma_k}\}$ for each data group G_k. Having collected the information, the source needs to calculate per round distribution of redundancy groups and rates for every individual round of the polling period separately.

The source then continues by announcing data and redundancy rates of individual multicast groups. Each receiver then has the opportunity of subscribing to the appropriate number of multicast data groups as well as multicast redundancy groups satisfying its redundancy requirement. We note that although LMMC error control protocol allows the receivers to drop any number of layers that they are already subscribed to at any time, it only allows the receivers to subscribe to extra redundancy groups at the beginning of a polling period and after the new redundancy rates have been announced. This is necessary to control the overhead of multicast groups joins and leaves considering real-time constraints of media systems.

During a polling period, the source sends the data packets of a block for individual groups G_k with $k \in \{1, \cdots, K\}$ followed by the first round of redundant packets. At the end of each round, the first receiver not capable of recovering the data block with size B_k multicasts a NAK message to the group notifying the source about the need for initiating the next round. A receiver only sends a NAK message if it has neither been able to recover the block nor seen another NAK message with a sequence number matching the current round. The proposed mechanism effectively eliminates the NAK traffic as the overall number of transmitted NAKs is in the order of number of rounds Γ_k. Going from one round to another, the source only initiates another round if it has received a NAK request from one of the receivers of the group within a certain number of time units from the end of current round.

We also argue that as an alternative to the polling mechanism and only for moderate size group of receivers in which the overhead of dynamically calculating the number of redundant packets is acceptable, the source can rely on DM method of Section II to dynamically adjust the data and redundancy rates without relying on an apriori estimate of overall redundancy. In this scenario, each receiver calculates the number of required packets only going to the last round and reports the result to the source. Having started from an initial proactivity factor, the source then solves the combined problem of rate allocation and partitioning after collecting the redundancy information of the receivers going to the last round of transmitting a block.

Before we conclude this section, it is in order to provide a discussion of LMMC error control protocol practicality for real-time media systems. Perhaps the most important concern pertains to explaining why the latency of joining and/or leaving multicast trees does not make the protocol overhead prohibitive. We argue that LMMC error control protocol is custom tailored for media systems according to the following reasons. First, we note that having a reduced loss rate resulting in dropping redundant groups is not a problem as the receiver is not concerned with the delay of multicast tree topology changes in this case. This is of special importance in case of the SM approach of Section II in which a receiver needs a lower number of redundant packets going from one round to another. Second, calculation of the bandwidth for individual redundant groups is done considering redundancy requirements of individual receivers at the beginning. Third, the built-in polling mechanism of LMMC counts for adjusting the number of redundant packets according to the current loss condition of individual receivers so that the receivers do not have to subscribe to extra redundancy groups often. Considering the above factors, we do not anticipate having frequent changes in multicast tree memberships and LMMC error control protocol can be hence effectively deployed in real-time media systems.

VI. Numerical Performance Analysis

In this section, we present the numerical results of applying LMMC error control algorithm to a number of layered media scenarios. First, we compare LMMC results with the results of optimal LMR (OLMR) utilizing dynamic programming and heuristic LMR (HLMR) algorithms of [25]. In our comparisons, we review the performance of the approaches from the stand point of tracking the minimum value of the bandwidth wastage, time complexity indicated by experiment runtime, and space complexity indicated by memory allocation. Additionally, we review the scalability of the techniques by covering a relatively broad range of multicast group sizes ranging from hundreds to thousands of receivers. We remind that per group time complexity of LMMC error control algorithm is $\mathcal{O}(IB_k)$ and that of OLMR algorithm is $\mathcal{O}(\varsigma_k B_k^2)$. In addition, per round space complexity of the LMMC error control algorithm in our implementation is $\mathcal{O}(B_k)$ where as the space complexity of OLMR is $\mathcal{O}(B_k^2)$ assuming block size B_k indicates an upper bound on the maximum required redundancy. In our simulations, we ran in excess of $20,000$ experiments with different number of groups K, different group sizes N_k with $k \in \{1, \cdots, K\}$, and different receiver redundancy requirements.

Figures 2 through 7 compare the sample results of LMMC algorithm with those of OLMR and HLMR algorithms for some individual data groups. In each experiment, we have relied on a normal random number generator simulating receiver loss rates within the range of $[1\%, 30\%]$ consistent with real network traces reported in [25]. Different figures have been obtained for different choices of different parameters of interest. Different parameters of interest include block size indicated by B and the number of redundancy groups associated with the individual data group ς. In our simulations B is set at 64, 128, and 256 packets; ς is set at 2 and 3. The x-axis of each curve is always in logarithmic scale indicating different values of the group size from the set $\{100, 300, 1000, 3000, 10000, 30000, 100000\}$. Each figure consists of two pairs of curves. The first set of curves compare the bandwidth wastage or redundancy cost of the three techniques. While LMMC and OLMR keep a close bandwidth wastage across the board, we observe that for group sizes of 1000 or more the bandwidth wastage of HLMR departs from the other two. Considering the results, we note that HLMR can be effectively used only if the distribution of the redundancy is not highly skewed and the group size is not very large. Additionally for about $20,000$ experiments made by us, we observed a maximum 6% cost advantage of OLMR over LMMC. Consider-

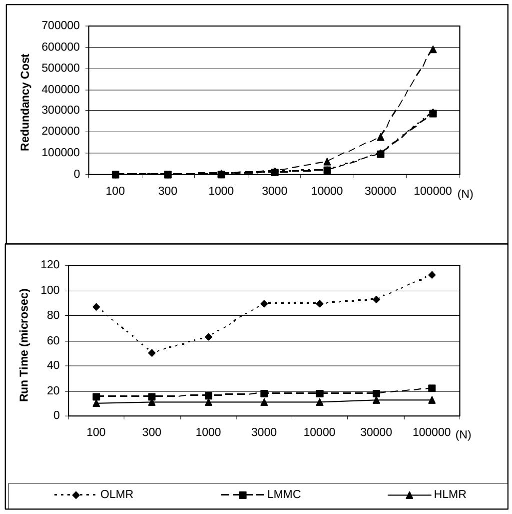

Fig. 2. Redundancy cost and runtime comparison of LMMC, OLMR, and HLMR methods vs number of receivers (N) for ς=2 and block size of $B = 64$.

ing the fact that a dynamic programming approach identifies a global optimum where as a gradient-based approach identifies a local optimum, our experiments indicate impressive convergent behavior of LMMC. The second pair of curves display the runtime of the experiments as an indicator of the time complexity of the three techniques. To our expectation, the complexity of HLMR for a small size group is the lowest among the three considering its negligible overhead of computation. In this area, a review of the results reveals closeness of LMMC results to those of HLMR. The review also reveals great performance advantage of LMMC over OLMR consistent with the time complexity analysis reporting a linear dependency and a quadratic dependency on the value of B in the runtime of LMMC and OLMR respectively.

Next, we provide sample simulation results of implementing integrated LMMC rate allocation, partitioning, and error control schemes. In our simulations of integrated media systems, we rely on generalizations of normal distribution namely bi- and tri-modal distributions to generate receiver isolated rates. We select the means of distributions from the set of $\{64Kbps, 1Mbps, 100Mbps\}$ to properly represent dialup, Cable/DSL, and high-speed LAN users. For each distribution, we also set the standard deviation of the distribution at 20% of the mean value. Considering the location of the means, the choice of standard

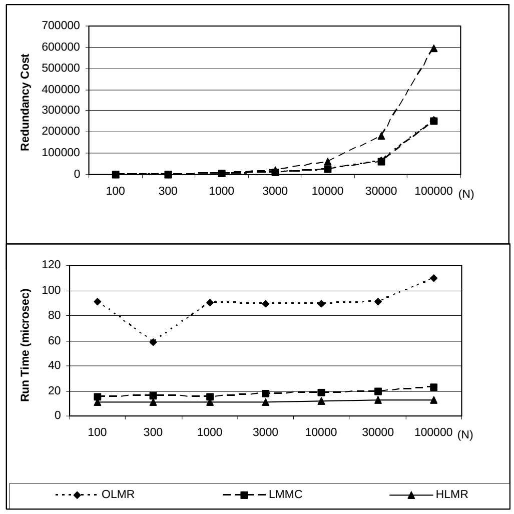

Fig. 3. Redundancy cost and runtime comparison of LMMC, OLMR, and HLMR methods vs number of receivers (N) for ς=2 and block size of $B = 64$.

deviations yields successive distributions remain disjoint with a certainty better than 99.7%. Tables I and II show rate allocation, partitioning, and redundancy calculation results of applying LMMC algorithm to couple scenarios of combined media systems.

In the rest of this section, we discuss some of the practical findings of our experimentations pertaining to the comparison and combination of LMMC error control technique applied as a reliable multicast technique [4] with FEC-based and ARQ-based techniques. In our experiments, we looked at the impact of utilizing LMMC in conjunction with ARQ-based SRM recovery [10], as well as hierarchical scoping techniques such as scoped SRM [29], and SHARQFEC [13]. The following summarizes our findings.

- First, we have observed that utilizing LMMC error control relying on proactive FEC-based recovery greatly reduces the overall amount of redundant traffic compare to reactive ARQ-based recovery utilized in single-scoped SRM.
- Second, we have seen that utilizing layered recovery in a hybrid technique resulting from the com-

[4] We have investigated such a scenario by relaxing real-time constraints and calculating receiver redundancies based on the probability of recovery.

Data Group	Receiver IDs	Data Range	Optimal Rate
$G1$	$\{1..361\}$	$[211..682]$	253.83
Redundancy	R. Group	R. Range	Optimal R.
	$R11$	$[1..5]$	5
	$R12$	$[6..18]$	18
	$R13$	$[19..40]$	40
Parameters	BWL	BWF	CoR
	263.75	800.00	0.9624
Data Group	Receiver IDs	Data Range	Optimal Rate
$G2$	$\{362..10000\}$	$[683..167726]$	818.58
Redundancy	R. Group	R. Range	Optimal R.
	$R21$	$[1..5]$	5
	$R22$	$[6..19]$	19
	$R23$	$[20..44]$	44
Parameters	BWL	BWF	CoR
	853.75	95000.00	0.9588

TABLE I

INTEGRATED RATE ALLOCATION PARTITIONING (RAP) AND ERROR CONTROL (EC) LMMC RESULTS WITH A RUNTIME OF $2992msec$. RAP PARAMETERS ARE $a = 1.5329$, RECEIVER LOSS TOLERANCE 20%, $K = 2$, $C_2 = 0.79$. UNIFORM EC PARAMETERS ARE $\varsigma_k = 3$ AND $B_k = 1024$.

Data Group	Receiver IDs	Data Range	Optimal Rate
$G1$	$\{1..5014\}$	$[25..423]$	27.16
Redundancy	R. Group	R. Range	Optimal R.
	$R11$	$[1..37]$	37
	$R12$	$[38..77]$	77
Parameters	BWL	BWF	CoR
	31.25	50.00	0.86927
Data Group	Receiver IDs	Data Range	Optimal Rate
$G2$	$\{5015..6642\}$	$[428..813]$	465.85
Redundancy	R. Group	R. Range	Optimal R.
	$R21$	$[1..38]$	38
	$R22$	$[39..76]$	76
Parameters	BWL	BWF	CoR
	535.00	850.00	0.87075
Data Group	Receiver IDs	Data Range	Optimal Rate
$G3$	$\{6643..20000\}$	$[814..167634]$	877.04
Redundancy	R. Group	R. Range	Optimal R.
	$R31$	$[1..38]$	38
	$R32$	$[39..82]$	82
Parameters	BWL	BWF	CoR
	1017.50	90000.00	0.86195

TABLE II

INTEGRATED RATE ALLOCATION PARTITIONING (RAP) AND ERROR CONTROL (EC) LMMC RESULTS WITH A RUNTIME OF $7787msec$. RAP PARAMETERS ARE $a = 1.5329$, RECEIVER LOSS TOLERANCE 20%, $K = 3$, $C_2 = 0.79$. UNIFORM EC PARAMETERS ARE $\varsigma_k = 2$ AND $B_k = 512$.

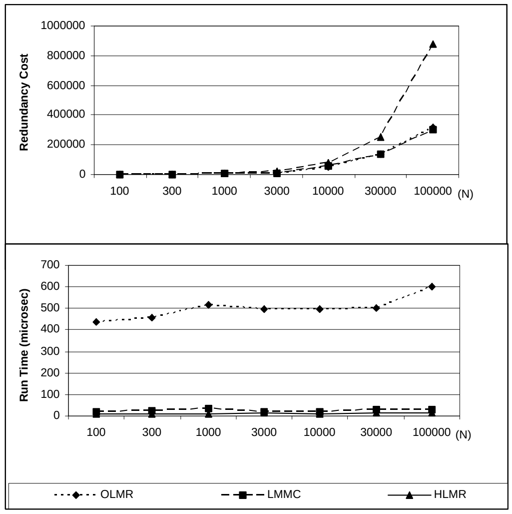

Fig. 4. Redundancy cost and runtime comparison of LMMC, OLMR, and HLMR methods vs number of receivers (N) for ς=3 and block size of $B = 128$.

bination of LMMC with SRM significantly reduces the overall amount of redundant traffic. In our experiments we have also observed that increasing the number of recovery layers has led to lower amount of redundant traffic at the expense of higher protocol overhead. Despite the fact that we did not see the threshold point in our experiments with up to 5 groups, we expect that increasing the number of redundant groups beyond a certain point is not justified considering the extra amount of overhead.

- Third, we have been able to achieve great repair locality by combining LMMC layering technique with a hierarchical technique such as scoped SRM or SHARQFEC.

We note that most of our findings are consistent with the results reported in [25]. Finally, we need to discuss the impact of increasing the number of layers in solving the global problem of layered or replicated media systems. Considering the layering architecture of LMMC, we argue that utilizing a relatively large number of redundant multicast groups associated with each individual data group may be overhead prohibitive from the stand point of joining and leaving multicast groups and as discussed in Section V. As a cautionary step, our implementation of LMMC error control relies on utilizing up to

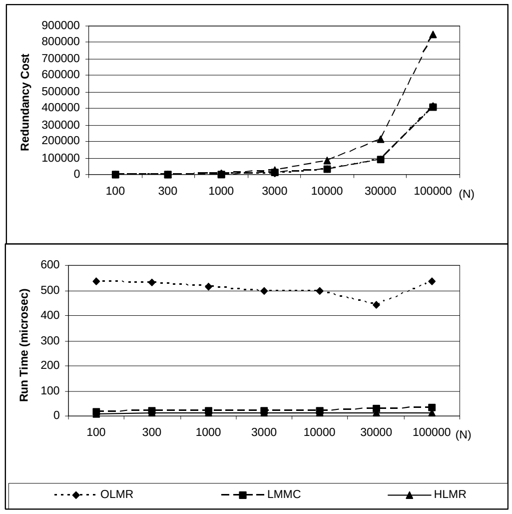

Fig. 5. Redundancy cost and runtime comparison of LMMC, OLMR, and HLMR methods vs number of receivers (N) for ς=3 and block size of $B = 128$.

three redundant groups associated with each individual data group. We also note that utilizing a limited number of redundant groups reduces the effectiveness of HLMR approach compare to OLMR approach and LMMC approach.

VII. Conclusion

In this paper, we studied Layered Media Multicast Control (LMMC) solution to a formulation of the optimal error control problem for layered and replicated media systems over multicast IP networks. We assumed the existence of congestion and flow control mechanisms specifying a fair bandwidth available to a media session. We aimed at providing an analytical solution to a formulation of the problem minimizing the bandwidth wastage of individual multicast groups while effectively eliminating the impact of feedback implosion and providing a statistical guarantee for the quality of service of each receiver. We also related the error control aspect of LMMC to its rate allocation and partitioning aspects as parts of an integrated solution for media multicast systems. Considering the scalability of LMMC error control approach, we showed that the approach could be effectively adopted in different size point-to-multipoint groups as well

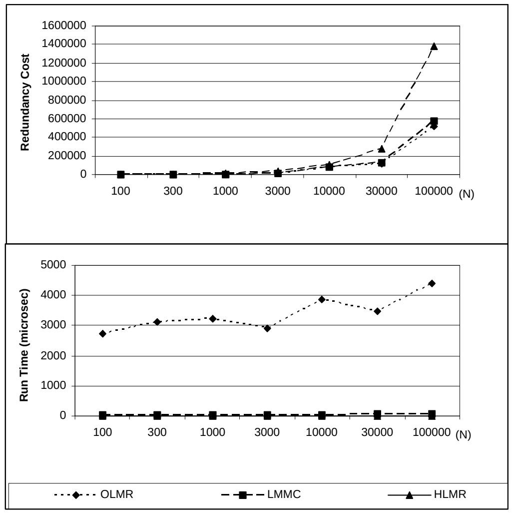

Fig. 6. Redundancy cost and runtime comparison of LMMC, OLMR, and HLMR methods vs number of receivers (N) for ς=4 and block size of $B = 256$.

as different speeds of load change in the network. Finally, we evaluated the performance of LMMC solution and illustrated its applicability in realistic network topologies through the use of simulations.

At the end, we would like to revisit our brief review of [36] about the issues involved in the distribution of media systems over multicast networks. In that article, we mentioned that the main issues involved with media multicast systems are rate allocation, receiver partitioning, error control, flow control (inter-session fairness), congestion control, and accommodation of hybrid wired and wireless networks. In addition, practical considerations such as scalability and feedback implosion have always played a key role in applicability of any media distribution algorithm. Envisioning LMMC as a complete framework for media systems, we are currently working on the flow and congestion control aspects of LMMC. Our objective is to integrate the existing aspects of LMMC rate allocation, receiver partitioning, and error control techniques with their counter parts addressing inter-session fairness and congestion control issues for media systems. In addition, we are working on fine tuning LMMC in order to be able to accommodate hybrid wired and wireless media systems.

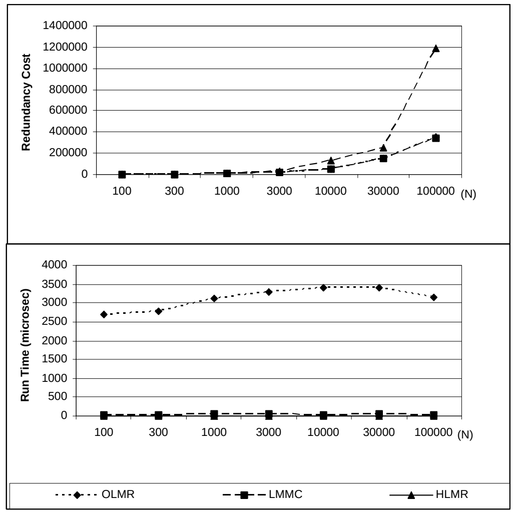

Fig. 7. Redundancy cost and runtime comparison of LMMC, OLMR, and HLMR methods vs number of receivers (N) for ς=4 and block size of $B = 256$.

APPENDIX I

PROOF OF THEOREM 3.1

Let us make note of the fact that the cost function of Equation (15) consists of a finite number of functions, one for each receiver. These functions are all positive, with a minimum value of zero. Consequently, the positive cost function of Equation (15) has a lower bound. Next, we observe that the cost function of Equation (15) can only decrease in each step considering the operating mechanism of the individual phases of LMMC error control algorithm. Therefore, the sequence of cost function values at each step of the algorithm is a non-increasing sequence with a lower bound. We also note that any non-increasing sequence with a lower bound would converge to a finite number also known as a fixed point. In case of our optimization problem, converging to a fixed point is equivalent to satisfying necessary condition for optimality defined below.

For a data group G_k of a media session with N_k receivers, ς_k redundancy groups, partitioning $\Omega_k = \{R_1|\cdots|R_{\varsigma_k}\}$ and redundancy group rate set $\rho = \{\rho_1, \cdots, \rho_{\varsigma_k}\}$, the necessary condition for optimality is defined over partitioning Ω^* and redundancy rate set ρ^* in two steps considering the impact of LMMC iterative error control approach. First, for a fixed partitioning Ω_k^{fixed} and the redundancy group rate set ρ^* such that

$$ECW_k(\Omega_k^{fixed}, \rho^*) \leq ECW_k(\Omega_k^{fixed}, \rho) \tag{34}$$

for every $\rho \neq \rho^*$. Second, for a fixed redundancy group rate set ρ^{fixed} and the partitioning Ω_k^* such that

$$ECW_k(\Omega_k^*, \rho^{fixed}) \leq ECW_k(\Omega_k, \rho^{fixed}) \tag{35}$$

for every $\Omega_k \neq \Omega_k^*$.

Since, the two-step necessary condition for optimality holds in each individual step of LMMC error control algorithm, we conclude that LMMC iterative algorithm converges to a local minimum. **QED**

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