

# Layered Media Multicast Control (LMMC): Rate Allocation and Partitioning

Homayoun Yousefi'zadeh, *Member, IEEE*, Hamid Jafarkhani, *Senior Member, IEEE*,  
and Amir Habibi

**Abstract**—The objective of layering techniques of distributing multimedia traffic over multicast IP networks is to effectively cope with the challenges in continuous media applications. The challenges include heterogeneity, fairness, real-time constraints, and quality of service. We study the problem of rate allocation and receiver partitioning in layered and replicated media systems. We formulate an optimization problem aimed at maximizing a close approximation of the so-called max-min fairness metric subject to loss and bandwidth constraints. Our optimal Layered Media Multicast Control (LMMC) solution to the problem analytically determines the layer rates and the corresponding partitioning of the receivers. Our simulation results show the effectiveness of our proposed solution in realistic scenarios.

**Index Terms**—Layered Media, Replicated Media, Multicast IP Networks, Heterogeneity, Optimality, Fairness Extrapolation, Rate Allocation, Receiver Partitioning.

## I. INTRODUCTION AND RELATED WORK

TRANSMITTING realtime compressed digital media over multicast IP networks has been the subject of heavy research in the recent years as surveyed by Li. et al. in [17] and the references cited therein. In a typical multicasting transmission scenario, a source generates realtime media traffic following a periodic pattern. The periodic pattern of realtime media traffic generated at a source consists of many frames in a unit of time at a variable bit rate, i.e., the number of bits per frame varies for individual frames. The receivers rely on a preserved frame periodicity at the time of play back. Data not available at the play back time is considered lost. In addition, the delay jitter or the difference in the delay of packets arrived at the receivers has to be small. In order to accommodate the latter need, buffering techniques at the receiver can be employed. A review of the literature reveals three different adaptive bit-rate media multicasting schemes for the transmission of digital media. The schemes are described below.

(1) Single stream adaptive approach was first presented by Bolot et al. [4] and Ammar [2] in which a single encoded video stream is transmitted by the source with feedback returned from the receivers to the source. The source uses the feedback information to adapt its data rate. One of the potential problems with this approach is the problem of feedback implosion for a large number of receivers attempting to return feedback to the source. Practical video multicast protocols targeting a large number of

receivers are required to address this issue. While it is straightforward to implement, the single stream adaptive approach is unable to properly address the problem of receiver heterogeneity.

(2) Replicated media streams approach was first presented by Cheung et al. [5] within the context of DSG protocol as an extension to the single stream approach that is capable of addressing the heterogeneity issue. In this approach, the source sends multiple streams carrying the same video with different qualities and bit rates. Each stream is obtained by encoding the video with different compression parameters and is sent to a different multicast group. Each individual receiver is able to join and change its group according to its capacity. While the simplicity of this scheme in addressing the heterogeneity issue is attractive, it has the drawback of requiring the network to carry redundant information of replicated media streams.

(3) Layered media streams approach was first proposed by Deering et al. [6] in the context of multicast routing and further enhanced by McCanne et al. [20] in the context of RLM protocol, Amir et al. [1] in the context of SCUBA protocol, and Li et al. [18] in the context of rate control aspect of LVMR protocol. The approach relies on the ability of many video compression schemes to divide their output bit stream into layers; a base layer and one or more enhancement layers. The base layer can be independently decoded providing a basic level of video quality. The enhancement layers can only be decoded together with the base layer providing improvements to video quality. This approach is also known as successive refinability approach in the context of source coding literature and was discussed by Jafarkhani et al. in [12] and references therein. Using this capability, a video multicast source could send each layer to a different multicast group. Receivers would then join at least the base layer group and join as many enhancement layer groups as their capacities allow. Layered media approach provides an elegant and efficient way to deal with the heterogeneity issue at the expense of protocol complexity.

As a real-world example of the subject material of this study, one can consider the transmission of a digital video stream to the members of a pay-per-view entertainment club. Club members are typically connected through dial-up, ISDN, Cable/DSL, 10baseT Ethernet, 100baseT Ethernet, and gigabit Ethernet lines and consequently belong to different bandwidth groups. The differences among processing power, topology, and protocol implementation at the receiving ends typically cause deviations from the nominal bandwidth capabilities of the members in each bandwidth group. Assuming either the replicated or the layered media system approach is used for the

Homayoun Yousefi'zadeh and Hamid Jafarkhani are with the Department of Electrical Engineering and Computer Science at the University of California, Irvine; E-mail: [hyousefi,hamidj]@uci.edu. Amir Habibi is with TSS Wireless, Los Angeles, CA; E-mail: amir@tsswireless.com.

transmission of the video stream, an important practical question is that what is the optimal number of groups for transmitting the stream? The answer is typically specified by considering the trade off between receiving ends' bandwidth heterogeneity and the incurring overhead in source encoding, receiver decoding, and multicast addressing. Without considering coding and multicasting overhead, the number of groups is directly mapped to the number of bandwidth categories. However, it is often required to select a smaller number of groups than the number of bandwidth categories in order to reduce the overhead. In such cases, a number of high bandwidth groups may be combined into one group in order to address the tradeoff between heterogeneity and overhead.

The material proposed in this paper is most closely related to the following articles. In [14] and the follow-on work of [15], Jiang et al. explore the issue of improving inter-receiver fairness in multicast ATM sessions with an Available Bit Rate (ABR). In order to determine the optimal partitioning and allocation of the group rates, the authors formulate a max-min fairness optimization problem subject to the maximum loss tolerance of a set of receivers. The authors apply their formulation to replicated media systems in the context of DSG protocol. They also provide a set of heuristic rules for solving the formulated problem. Their three proposed heuristic rules are consistent with our practical discussion of the previous paragraph and are intended for ensuring (1) dissimilar receivers are not grouped together, i.e., a set of receivers are increasingly ordered and partitioned in terms of their isolated rates, (2) receivers of similar performance levels are grouped together, i.e, the normalized standard deviations of the isolated rates of the receivers in each partition are relatively small, and (3) a group of receivers can only be split into two groups if the difference between the resulting group rates is larger than the smaller group rate. In [16], the same group of authors apply their work to Internet-driven applications with the considerations of TCP-friendliness. In [29], Yang et al. provide a dynamic programming algorithm to simultaneously solve the problems of optimal partitioning and rate allocation for layered media systems.

The main objective of the current research work is to provide an analytical framework for the partitioning strategy and rate allocation of both layered and replicated media systems over multicast IP networks in the context of Layered Media Multicast Control (LMMC) protocol. In this study, we assume the existence of congestion and flow control mechanisms capable of dynamically addressing inter-session fairness issue, i.e., a fair distribution of available bandwidth among multiple media and other sessions such as TCP sessions. Typical examples of such mechanisms are given in [26], [19], [28], [22], and [27]. In addition, our work of [30] proposes a framework of flow control for layered and replicated media streams. The main contributions of this paper are in three areas. First, the paper introduces an analytical approach in which a non-continuously differentiable max-min fairness function is extrapolated by a class of mathematically well-behaved continuously differentiable functions. The extrapolated functions satisfy the conditions required for applicability of traditional optimization techniques. Second, the paper provides an analytical solution to a formulation of the optimal rate allocation problem of the replicated and layered

media systems. Third, the paper offers a near optimal receiver partitioning strategy maximizing the enhanced fairness utility metric for any set of allocated layer rates.

Specifically, we formulate a two-phase optimization problem of partitioning and rate allocation after extrapolating the so-called max-min fairness metric with a mathematically well-behaved function. In the first phase, we analytically solve the optimal rate allocation problem for individual layers of the media session assuming the number of layers is given. The solution to this first problem considers receiver heterogeneity, i.e., the variation of the bandwidth among different receivers of the target session by means of maximizing the extrapolated inter-receiver fairness metric. In the second phase, we provide an optimal partitioning strategy for the layered media session based on the allocation rates of the first phase. The solution to the second problem maximizes the overall fairness utility function of the media session. Considering the phasing approach of our solution, we introduce an iterative approach that can reach a near-optimal solution by iteratively applying the partitioning result of the second phase to the first phase and solving the optimal rate allocation problem with the new partitioning strategy. This is equivalent to employing steepest descent optimization strategy and is guaranteed to reach an  $\epsilon$ -neighborhood of a local optimal point if such a point exists.

In summary given the overall available bandwidth to a media session, the LMMC solution to the formulation of the problem identifies the optimum rates for each individual layer and the corresponding receiver partitioning such that the fairness utility function of the session is maximized while satisfying the problem constraints. To the best of our knowledge, this is a unique approach providing an analytical solution to the rate allocation problem of layered media in multicast networks.

An outline of the paper follows. In Section II, we formulate the two-phase receiver partitioning and rate allocation problem considering individual receivers max-min fairness. In Section III, we analytically solve the optimal rate allocation problem of the first phase assuming a given partitioning. In Section IV, we use the allocated rates of Section III to obtain a near-optimal partitioning strategy. In Section V, we introduce an iterative approach relying on the solutions of Section III and IV to reach a near-optimal solution. Section VI focuses on performance evaluation and includes the simulation results along with practical considerations. Finally, Section VII contains a discussion of the future work and concluding remarks.

## II. FORMULATION OF THE PROBLEM BY MEANS OF FAIRNESS EXTRAPOLATION

In this section, we focus on the general rate allocation and partitioning problem of the layered and replicated media sessions. The problem aims at transmitting a stream of digital media to a set of receivers with different bandwidth capabilities such that each receiver can create a reconstruction of the stream with a quality proportional to its own bandwidth capability. We formulate the problem in a manner similar to that of [16], [15], and [29] with an extra constraint on the overall available bandwidth to the session. The previous problems without any constraints can hence be considered as a specific case of our problem.

Consider a multicast media session with a partitioning of the receivers into  $K$  groups. Recall that for a media session with  $N$  receivers and  $K$  groups, a set  $P = \{G_1 | \dots | G_K\}$  is called a partitioning of the receiver set  $R = \{1, \dots, N\}$  if  $P$  is a decomposition of the set  $R$  into a family of disjoint sets. Make note of the fact that we are formulating the problem for a given number of groups. The impact of the changes in the number of groups  $K$  is investigated in Section VI. The term group rate is used to denote the aggregate receiving rate of a receiver in the group while the term layer rate is used to denote the transmission rate to a specific layer. For an ordered partitioning of the receivers into  $K$  groups with ordered group rates of  $g_1, g_2, \dots, g_K$  such that  $g_1 \leq g_2 \leq \dots \leq g_K$ , the layer rates of a layered media session are calculated in the form of,

$$g_1, g_2 - g_1, g_3 - g_2, \dots, g_K - g_{K-1} \quad (1)$$

A receiver in group  $k$  subscribes to layers 1 through  $k$  receiving an aggregate rate of  $g_k$ .

Interpretation of our formulation in the case of replicated media streams is also straight forward. For an ordered partitioning of the receivers into  $K$  groups  $G_1, G_2, \dots, G_K$  with ordered group rates of  $g_1, g_2, \dots, g_K$  such that  $g_1 \leq g_2 \leq \dots \leq g_K$ , the layer rates are the same as the group rates. A receiver in group  $k$  only subscribes to layer  $k$  receiving a rate of  $g_k$ . The interpretation difference has a minor impact on the formulation and consequently the solution of the problem in some special cases which will be discussed in Section III.

The optimization problem is formulated by means of defining a per receiver max-min fairness utility with the objective of maximizing the session utility defined as the sum of receiver utilities over the layered media session. Each receiver is assumed to have an isolated multi-rate max-min fair rate of  $r_i$  as described in both [14] and [23]. This is the reception rate of the receiver and is typically determined by a network bottleneck link from the source to the receiver or the receiver itself. For the clarity of representation, we also assume that the receivers are numbered such that their isolated rates are in a non-decreasing order, i.e.,  $r_1 \leq r_2 \leq \dots \leq r_N$ . In addition, each receiver  $i$  is assumed to have a loss tolerance  $L_i$  identified as its largest acceptable loss rate. Therefore, the group rates  $g_k$  should satisfy the following inequality for individual receivers of groups  $G_1, \dots, G_K$ .

$$g_k \leq \frac{r_i}{1 - L_i} \quad \forall i \in G_k, k = 1, \dots, K \quad (2)$$

In [14], a class of fairness utilities  $\mathcal{F}(r_i, g_k)$  are defined for receiver  $i$  of group  $G_k$  by means of satisfying the following conditions:

- $\mathcal{F}(r_i, g_k) \in [0, 1]$ .
- $\mathcal{F}(r_i, r_i) = 1$ .
- $\mathcal{F}(r_i, g_k) < 1$  if  $r_i \neq g_k$ .
- $\mathcal{F}(r_i, g_k)$  is non-decreasing in the range  $[0, r_i]$ .
- $\mathcal{F}(r_i, g_k)$  is non-increasing in the range  $(r_i, \infty)$ .

In this paper, we work with the most widely accepted example of such utility functions, the so-called max-min fairness

utility function defined as

$$F(r_i, g_k) = \frac{\min(r_i, g_k)}{\max(r_i, g_k)} = \begin{cases} \frac{g_k}{r_i} & : g_k \leq r_i \\ \frac{r_i}{g_k} & : g_k \geq r_i \end{cases} \quad (3)$$

The group utility for the group  $G_k$  with a group rate  $g_k$  is defined as,

$$IRF_k = \sum_{i \in G_k} F(r_i, g_k) = \sum_{i \in G_k} \frac{\min(r_i, g_k)}{\max(r_i, g_k)} \quad (4)$$

In order to assign priorities to the different receivers of a group, the fairness utilities of the receivers can be multiplied by a parameter  $\alpha_i$  with the following characteristics,

$$\begin{aligned} \sum_{i=1}^N \alpha_i &= 1 \\ 0 \leq \alpha_i &\leq 1 \quad \text{for } i = 1, \dots, N \\ \alpha_i &= 0 \quad \text{for } i \notin G_k \end{aligned} \quad (5)$$

The choice of parameters  $\alpha_i$  is a design decision allowing for unequal contribution of the receivers to a group utility according to their importance. The parameters may be statically assigned or dynamically vary over time. Generally speaking, the choice of parameters  $\alpha_i$  does not have any significant impact in our study. The session utility of the partitioning  $P = \{G_1 | \dots | G_K\}$  is defined as

$$\begin{aligned} IRF_{Total} &= \sum_{k=1}^K \sum_{i \in G_k} F(r_i, g_k) \\ &= \sum_{k=1}^K \sum_{i \in G_k} \frac{\min(r_i, g_k)}{\max(r_i, g_k)} \end{aligned} \quad (6)$$

The objective of both heuristics given in [15] and the dynamic programming algorithm given in [29] is to determine the optimal partitioning and the optimal layer rate allocations such that the function defined in (6) is maximized considering receivers loss constraints. The rate allocation optimization problem is, then, formulated as

$$\max_{g_1, \dots, g_K} IRF_{Total} = \max_{g_1, \dots, g_K} \sum_{k=1}^K IRF_k \quad (7)$$

$$= \max_{g_1, \dots, g_K} \sum_{k=1}^K \sum_{i \in G_k} \frac{\min(r_i, g_k)}{\max(r_i, g_k)}$$

$$\begin{aligned} \text{Subject To: } g_k &\leq \frac{r_i}{1 - L_i} \\ i &\in G_k, k = 1, \dots, K \end{aligned} \quad (8)$$

for the optimal partitioning  $P^* = \{G_1^* | G_2^* | \dots | G_K^*\}$  leading to the calculation of the optimal rates  $g_1^*, g_2^*, \dots, g_K^*$ .

In **Theorem (1)** of [29] the existence of an ordered receiver partitioning that maximizes the function defined in (6) is proven assuming the receiver utility function  $\mathcal{F}(r, g)$  satisfies a Receiver Utility Property (RUP). The RUP holds for a receiver with an isolated rate  $r$  in a group  $G$  with a group rate  $g$  if,

- $\mathcal{F}(r, g)$  is non-decreasing in the interval  $[0, g]$  and non-increasing in the interval  $[g, \infty)$  for a fixed  $r$ ; and
- $\mathcal{F}(r, g)$  is non-decreasing in the interval  $[0, r]$  and non-increasing in the interval  $[r, \infty)$  for a fixed  $g$ .

We now introduce an extrapolation technique to replace the non-continuously differentiable max-min fairness utility for the receiver  $i$  of group  $G_k$  defined in (3) with a mathematically well-behaved function over the real numbers axis while satisfying RUP. Such an extrapolation technique provides us with the opportunity to introduce a more effective solution to the problem of rate allocation and partitioning in terms of time and space complexity. By mathematically well-behaved, we mean that our so-called extrapolated function  $E(r_i, g_k)$  is continuously differentiable and has no poles over the real numbers axis. We select a rational function  $E(r_i, g_k)$  in the form of

$$E(r_i, g_k) = \frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} \quad (9)$$

and note that not only  $E(r_i, g_k)$  is well behaved for parameter  $a$  satisfying the boundary condition  $-2 < a < 2$ , but it satisfies the boundary and maximum conditions of function  $F(r_i, g_k)$ . The matter is best explained by a graphical illustration. Fig. (1) shows generic sample plots of  $F(r, g)$  and  $E(r, g)$  versus  $g$  for a fixed  $r$ . It is important to note that since both  $F(r, g)$  and  $E(r, g)$  functions can transparently interchange the variables  $r$  and  $g$  we could consider the plots  $F(r, g)$  and  $E(r, g)$  versus  $r$  for a fixed  $g$ , instead. Next, we employ least square error

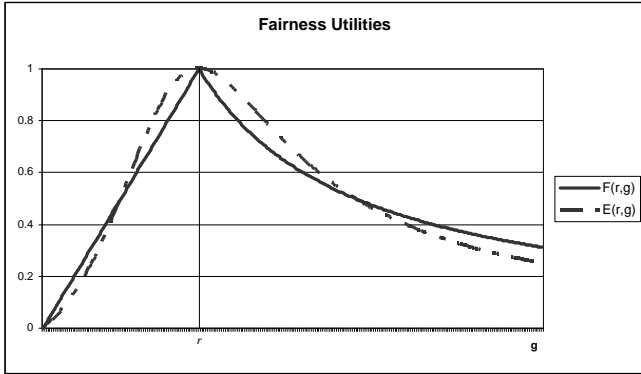


Fig. 1. Plots of  $F(r, g)$  and  $E(r, g)$  versus  $g$  for a fixed  $r$ .

estimation technique to find the optimum value of the parameter  $a$  within the interval of interest  $[0, \frac{r_i}{1-L_i}]$  considering the constraint function of (8) and as shown below.

$$\begin{aligned} \min_a [ \text{LSE}(a, r_i, L_i) ] &\equiv \quad (10) \\ \min_a [ &\int_0^{r_i} \left( \frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} - \frac{g_k}{r_i} \right)^2 dg_k \\ &+ \int_{r_i}^{\frac{r_i}{1-L_i}} \left( \frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} - \frac{r_i}{g_k} \right)^2 dg_k ] \end{aligned}$$

Solving Equation (10) for different values of  $r_i$  and  $L_i$  in the intervals of interest reveals the range  $[-1.6012, -1.5153]$  for the optimal value of parameter  $a$ . In our calculations, we perform a table look up operation to extract the optimal value of parameter  $a$ . Appendix I describes the details of the extrapolation technique.

We now formulate the new rate allocation problem with

an extra constraint on the available bandwidth to individual groups of the session as

$$\max_{g_1, \dots, g_K} IRFA_{Total} \equiv \max_{g_1, \dots, g_K} \sum_{k=1}^K IRFA_k \quad (11)$$

$$= \max_{g_1, \dots, g_K} \sum_{k=1}^K \sum_{i \in G_k} \frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2}$$

$$\text{Subject To: } g_k \leq \text{BWL}_k \quad k = 1, \dots, K \quad (12)$$

$$g_k \leq \text{BWF}_k \quad k = 1, \dots, K \quad (13)$$

where  $\text{BWL}_k$  in the constraint of Equation (12) is defined as  $\text{BWL}_k \equiv \min_{i \in G_k} \frac{r_i}{1-L_i}$ , the same as that of (8), and the constraint of Equation (13) indicates the available group bandwidth as the result of enforcing a per group inter-session fairness algorithm. Further, the function  $IRFA_k$  is the group fairness utility defined as

$$IRFA_k \equiv \sum_{i \in G_k} E(r_i, g_k) = \sum_{i \in G_k} \frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} \quad (14)$$

By defining  $\text{BWA}_k \equiv \min(\text{BWL}_k, \text{BWF}_k)$ , we convert the rate allocation problem to

$$\max_{g_1, \dots, g_K} IRFA_{Total} = \max_{g_1, \dots, g_K} \sum_{k=1}^K IRFA_k \quad (15)$$

$$= \max_{g_1, \dots, g_K} \sum_{k=1}^K \sum_{i \in G_k} \frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2}$$

$$\text{Subject To: } g_k \leq \text{BWA}_k \quad k = 1, \dots, K \quad (16)$$

We note the difference between the loss tolerance constraints  $\text{BWL}_k$  and the group bandwidth upper bounds  $\text{BWF}_k$ . While the former reflects the receivers bandwidth processing capabilities, the latter is the result of employing a flow control mechanism with the objective of enforcing inter-session fairness among different flows.

### III. PHASE 1: LMMC OPTIMAL SOLUTION TO THE RATE ALLOCATION PROBLEM

In this section, we provide an analytical solution to the optimal rate allocation problem formulated by Equation (15) and Constraint (16) that can be applied to both layered media and replicated media sessions. Appendix II includes the solution for another case in which an overall available bandwidth for the session is given instead of the available bandwidth to individual groups of the session. The general problem of Equation (15) and Constraint (16) can be converted to an optimization problem without constraints by defining a Lagrangian function in the form of

$$\begin{aligned} LG_{IRF} &= IRFA_{Total} + \sum_{k=1}^K \mu_k (g_k - \text{BWA}_k) \\ &= \sum_{k=1}^K IRFA_k + \sum_{k=1}^K \mu_k (g_k - \text{BWA}_k) \quad (17) \end{aligned}$$

where the parameters  $\mu_k$  for  $k = 1, \dots, K$  are the Lagrange multipliers in the Lagrangian Equation (17). The solution to the unconstrained problem can then be obtained by solving  $\nabla LG_{IRF}|_{g^*} = 0$ . However considering the specific form of the function  $IRFA_{Total}$  and the constraint set of (16), the most straight forward way of solving for the optimal solution is to decompose the system of  $2K$  equations and  $2K$  unknowns obtained from  $\nabla IRFA_{Total}(g^*) = 0$  and the constraints (16) into  $K$  pairs of independent equations. This is in essence equivalent to solving the set of  $K$  individual unconstrained problems of  $\nabla IRFA_k(g_k^*) = 0$  and then investigating the impact of applying the corresponding inequality constraint  $g_k \leq BWA_k$  on individual results. Equation (18) shows the simplified formulation applied to the set of  $K$  independent problems.

$$\begin{aligned} \max_{g_k} IRFA_k &= \max_{g_k} \sum_{i \in G_k} \frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} \quad (18) \\ \text{Subject To:} & \quad g_k \leq BWA_k \end{aligned}$$

where  $k = 1, \dots, K$ . The set of optimization problems of Equation (18) can be solved by finding the roots of the following equations,

$$\frac{\partial IRFA_k}{\partial g_k} = \sum_{i \in G_k} \frac{(2+a)r_i(r_i^2 - g_k^2)}{(g_k^2 + a r_i g_k + r_i^2)^2} = 0 \quad (19)$$

and extracting the global maximum from among the set of local optimal points satisfying Constraint (16) and

$$\frac{\partial^2 IRFA_k}{\partial g_k^2} = \sum_{i \in G_k} \frac{2(2+a)r_i(g_k^3 - 3r_i^2 g_k - a r_i^3)}{(g_k^2 + a r_i g_k + r_i^2)^3} \leq 0 \quad (20)$$

Prior to proceeding with the solution to individual optimization problems, we review the mathematical characteristics of the function  $IRFA_k$ . We first note that the function  $IRFA_k$  is non-decreasing in the interval  $[0, r_{k_{min}}]$  and non-increasing in the interval  $[r_{k_{max}}, \infty]$  where  $r_{k_{min}}$  indicates the minimum isolated rate and  $r_{k_{max}}$  indicates the maximum isolated rate of the receivers belonging to group  $G_k$ . This is true because the function  $IRFA_k$  consists of a sum of a number of the receiver utility functions  $E(r_i, g_k)$  which are all non-decreasing in the interval  $[0, r_{k_{min}}]$  and non-increasing in the interval  $[r_{k_{max}}, \infty]$ . Consequently, Equation (19) has no roots in the intervals  $[0, r_{k_{min}}]$  and  $[r_{k_{max}}, \infty]$ . We also remind that any acceptable optimal point has to satisfy Constraint (16). Combining the above conditions, we can argue that for  $r_{k_{min}} \geq BWA_k$  the optimal solution equals to  $BWA_k$  and for  $r_{k_{min}} < BWA_k < r_{k_{max}}$  any acceptable maximum point falls into the interval

$$[r_{k_{min}}, BWA_k] \quad (21)$$

For  $r_{k_{max}} < BWA_k$ , Constraint (16) has no impact on the optimal solution.

Generally speaking, the function  $IRFA_k$  can have up to  $N_k$  maximum points and  $N_{k-1}$  minimum points with  $N_k$  indicating the number of the receivers in group  $G_k$ . Finding the global maximum of the function  $IRFA_k$  is hence equivalent to applying a root finding algorithm on Equation (19) and extracting

the global maximum from the set of optimal points satisfying Inequality (20) and Constraint (16).

In our extensive set of simulations, we have consistently observed that the function  $IRFA_k$  includes a single global maximum point if the individual receiver utilities are distributed in such a way that every two consecutive isolated rates  $r_i$  and  $r_{i+1}$  satisfy the relationship  $r_{i+1} \leq 2r_i$ . The latter is a practical assumption for a set of receivers with similar bandwidth capabilities. Fig. (2) shows a typical  $IRFA_k$  function. Finding the global maximum of the function  $IRFA_k$  in such a case is hence equivalent to applying a single root finding algorithm such as bisection or Newton algorithms to Equation (19). These algorithms can identify the single root of Equation (19) with a time complexity of  $\mathcal{O}(N \log N)$ . We argue that if a media session

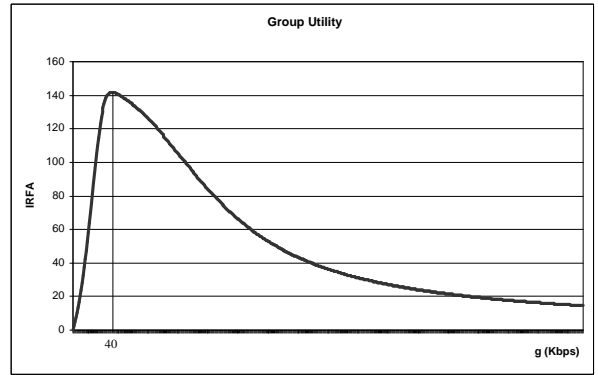


Fig. 2. A sample plot of the group utility  $IRFA_k$  versus  $g_k$  for a group including 200 receivers with isolated rates in the range of  $[32Kbps, 128Kbps]$  and every two consecutive isolated rates  $r_i$  and  $r_{i+1}$  satisfying  $r_{i+1} \leq 2r_i$ .

can choose the number of groups such that our heuristic rule of  $r_{i+1} \leq 2r_i$  is satisfied, all of the corresponding  $IRFA_k$  functions will only have one maximum point. We also argue that having a limited number of groups can only impact the number of optimum points for the function  $IRFA_K$  of the last group. To explain the latter claim, consider a scenario in which the receivers are distributed around  $S$  major categories of bandwidth while there are only  $K$  groups ( $K < S$ ) are available to accommodate the receivers. A real example of this situation is when you have receivers belonging to the bandwidth range of dial-up, cable,  $10Mbps$  LAN, and  $100Mbps$  LAN while there are only 3 groups available due to multicasting constraints. In such a scenario, the bandwidth and loss characteristics of the receivers in the lower bandwidth ranges maps the first  $K - 1$  bandwidth categories to the first  $K - 1$  groups while combining the rest of  $S - K + 1$  bandwidth categories in the last group. This creates a situation in which only the last group consists of a mix of receivers with significantly different bandwidth characteristics resulting in an  $IRFA_K$  function with multiple optimum points. Additionally, even in the case of observing multiple maximum points for the function  $IRFA_k$ , our numerical results have only shown one maximum for any subset of receivers with isolated rates satisfying  $r_{i+1} \leq 2r_i$ . Fig. (3) shows an example of such an  $IRFA_k$  function. In order to prevent a significant quality degradation at a receiver, we assume that the maximum acceptable loss tolerance of a receiver does not exceed 50%. This implicitly means that  $BWA_k$  defined as  $\min(BWL_k, BWF_k)$

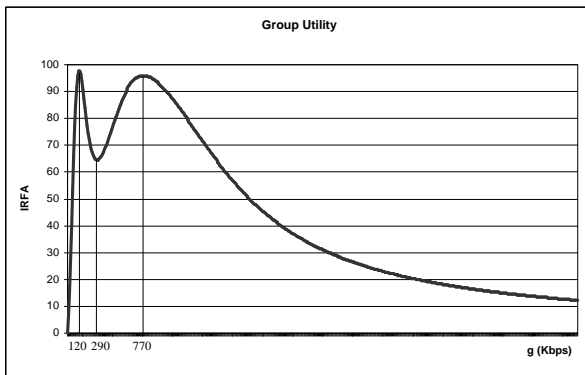


Fig. 3. A sample plot of the group utility  $IRFA_k$  versus  $g_k$  for a group including 200 receivers with isolated rates in the range of  $[64Kbps, 128Kbps]$  and  $[640Kbps, 1280Kbps]$ . In each interval, every two consecutive isolated rates  $r_i$  and  $r_{i+1}$  satisfy  $r_{i+1} \leq 2r_i$ .

with  $BWL_k$  defined as  $\min_{i \in G_k} \frac{r_i}{1-L_i}$  will typically not exceed  $2r_{kmin}$  where  $r_{kmin}$  indicates the minimum isolated rate of the receivers belonging to group  $G_k$ . Combining these observations, we come to the conclusion that in practical cases applying Constraint (16) limits the search to find the first optimum point of the function  $IRFA_k$ . Applying the interval of Equation (21), Newton, bisection or a similar numerical technique can be employed to find the first positive real maximum of  $IRFA_k$  function.

As an important special case and by substituting  $BWA_k$  with  $BWL_k$ , the general formulation of our problem reduces to the no flow constraint problem formulated in [29] and [15]. The problem can then be solved using the same technique as the one used to solve the general problem. It is now relevant to compare the time complexity of our algorithm with that of [29]. In practice, the time complexity of solving for optimum point of equation set (19) over all of the existing groups is  $\mathcal{O}(KN \log N)$ . The search for the root of Equation (19) determines the overall time complexity of the solution considering the fact that the rest of calculations are in the complexity order of  $\mathcal{O}(N)$ . The time complexity of the algorithm is by far better than  $\mathcal{O}(N^2)$  the complexity of the dynamic programming algorithm offered by [29]. This is aside from the fact that a dynamic programming approach in general does not provide an analytical solution to an optimization problem and the algorithm of [29] needs minor modifications to be able to solve the formulation of the general problem of Equation (15) considering the impact of enforcing a flow control algorithm.

Before we proceed to phase 2 of our solution, it is also relevant to investigate the impacts of facing some of the source and receiver limitation scenarios when solving LMMC optimization problem. First, we consider a source limitation scenario that appears in the form of discrete sending rates. Up until now, we have assumed that there is no limitation on the source sending rates, i.e., the source can control the group rates with fine granularity. In practice, layered encoding techniques may limit the source to some pre-determined quantized discrete group rates<sup>1</sup>.

<sup>1</sup>Examples of standard layered encoding techniques with pre-determined quantized discrete rates include MPEG-2 [8], H.263 [10], and new-generation MPEG-4 [9], AVC/H.264/ISO 14496-10 [11]. We note that the family of

There are two ways to cope with this issue in our rate allocation problem. The first approach is to change the formulation of our optimization problem from a NonLinear Programming (NLP) to a Mixed Integer NonLinear Programming (MINLP) in which the group rates can only take on discrete values. The solution to the new problem will then satisfy the discrete constraints. The second approach is to rely on the continuous optimal solution of the existing formulation and approximate it with the closest discrete rate. Although the approximated solution is sub-optimal in this case, it reduces the complexity of the problem to a great extent and yields acceptable results so long as the discrete achievable rates of the underlying encoder are not very far from each other. The latter is a reasonable assumption for many of the currently available encoders. Considering distribution of the discrete group rates, we choose the second approach as the practical way of coping with this issue in our optimization problem. This method is of special interest, considering the iterative nature of our two-phase solution as described in Section V.

Next, we consider a scenario in which the receivers introduce a zero loss tolerance. The only impact of facing a zero loss tolerance scenario with  $L_i = 0$  for  $i = 1, \dots, N$  in our optimization algorithm is to change the definition of  $BWL_k$  from  $BWL_k \equiv \min_{i \in G_k} \frac{r_i}{1-L_i}$  to  $BWL_k \equiv \min_{i \in G_k} r_i$  for  $k = 1, \dots, K$ . Since the previous constraint qualifications hold for  $0 \leq L_i < 1$  with  $i = 1, \dots, N$  we do not foresee any changes on the method of obtaining our optimal solution. However, we make note that this scenario greatly simplifies the results considering the fact that the function of Equation (15) would have no zero slope point satisfying Constraint (16) for  $N_k > 1$ . Intuitively, we anticipate that the optimal rate of each group is always less than or equal to the lowest isolated rate of the group.

#### IV. PHASE 2: LMMC NEAR-OPTIMAL PARTITIONING STRATEGY

In Section I, we briefly described the heuristic partitioning rules of [15]. We note that the heuristic rules are well categorized under probabilistic classification and clustering methods for non-convex optimization problems. In [31], we provide a formal classification method that is closely related to the partitioning heuristic rules. However, it is worth mentioning that the general short coming of probabilistic classification methods lies in the fact that they are typically appropriate for deduction techniques on the properties of mathematical concepts and closely related computational algorithms concepts rather than being useful for approximate or exact solutions to the optimization problems. Nevertheless, these techniques come handy in the case of solving optimization problems and in the absence of a formal solution.

In addition, the dynamic programming algorithm of [29] provides an optimal receiver partitioning strategy for a media session while computing the optimal layer rates. The main disadvantages of utilizing a dynamic programming approach to solve

MPEG standards originally supported successively refinable video in the range of several  $Mbps$  and eventually covered lower rates. The H.263 standard and the follow-on standards were originally designed to support successively refinable video at a wider range of rates starting at tens of  $Kbps$ .

an optimization problem are (1) the lack of providing an analytical answer, and (2) a relatively high degree of complexity. However, we make note of the fact that dynamic programming is one of the best tools and in many cases the only available tool for solving an optimization problem. Fortunately, this is not the case for a typical rate allocation problem.

Rather than relying on a dynamic programming approach, we introduce a near optimal partitioning strategy with time complexity of  $\mathcal{O}(\mathcal{N})$  for a layered media or a replicated media session and show that our partitioning strategy maximizes the session utility for a set of given group rates.

The fact that the extrapolated receiver fairness function  $E(r, g)$  satisfies RUP defined in Section II keeps the order of the resulting partitioning of this section.

Considering the general objective of maximizing the session utility of Equation (15) and for a set of given group rates  $\{g_1, \dots, g_K\}$ , it is imperative that a receiver with isolated rate  $r_i$  is assigned to the group with rate  $g_k$  if the receiver utility defined in (9) is maximized for the choice of  $g_k$ . As the result, we make the observation that the optimal receiver partitioning strategy has to assign the receiver with the isolated rate  $r_i$  to the group with rate  $g_k$  such that

$$E(r_i, g_k) \geq E(r_i, g_l) \quad l \in \{1, \dots, K\} \quad (22)$$

We now translate the latter observation to a simple group assignment mechanism. Let us first consider the fairness function of Equation (9) with parameter  $g_k$  and variable  $r_i$ . We note that in Sections II and III, the function of Equation (9) with parameter  $r_i$  and variable  $g_k$  was considered instead. Given the group rates  $\{g_1, \dots, g_K\}$ , we first plot the family of functions  $E(r_i, g_k)$  versus  $r_i$  for different parameter values of  $g_k$  where  $k = 1, \dots, K$ . Fig. (4) shows the sample plots for  $K = 3$ . Next, we find the intersection points of every two functions

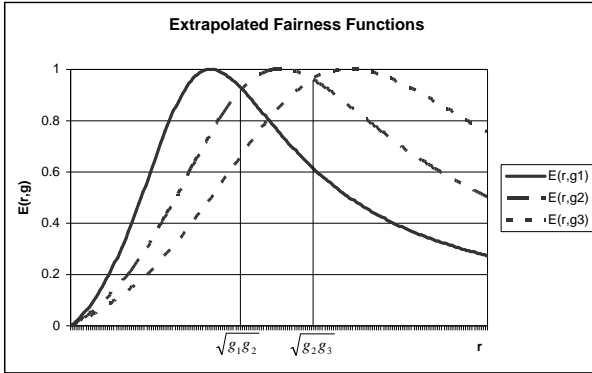


Fig. 4. Sample plots of  $E(r_i, g_k)$  versus  $r_i$  for three given values of  $g_k$ .

with consecutive group rates  $g_k$  and  $g_{k+1}$ . The values of  $r_i$  at the intersection points are obtained by finding the roots of the following set of equations for variables  $r_i$  and parameters  $g_k$  and  $g_{k+1}$  where  $k = 1, \dots, K$ .

$$E(r_i, g_k) = E(r_i, g_{k+1}) \quad (23)$$

Solving Equation (23) yields

$$\frac{(2 + a(r_i)) r_i g_k}{g_k^2 + a(r_i) r_i g_k + r_i^2} = \frac{(2 + a(r_i)) r_i g_{k+1}}{g_{k+1}^2 + a(r_i) r_i g_{k+1} + r_i^2} \quad (24)$$

Although in the general form of Equation (24) the parameter  $a$  is a function of the variable  $r_i$ , the solution to the equation can nevertheless be expressed in the following form after a bit of algebraic manipulation as

$$r_i = \sqrt{g_k g_{k+1}} \quad (25)$$

We now pay attention to the key characteristic of the intersection points of the curves that we refer to as partitioning thresholds.

**Theorem 4.1:** The value of the receiver utility as defined in Equation (9) is maximized for the choice of the group rate  $g_k$  for  $k > 1$  and  $k < K$  over the set of given group rates  $\{g_1, \dots, g_K\}$  if  $\sqrt{g_{k-1} g_k} < r_i \leq \sqrt{g_k g_{k+1}}$ . The receiver utility is maximized for the choice of the group rate  $g_1$  if  $r_i \leq \sqrt{g_1 g_2}$  and for the choice of the group rate  $g_K$  if  $r_i > \sqrt{g_{K-1} g_K}$ .

**Proof:** As graphically observed in Fig. (4), among the three functions  $E(r_i, g_1)$ ,  $E(r_i, g_2)$ ,  $E(r_i, g_3)$  the value of the function  $E(r_i, g_1)$  is the maximum if  $r_i \leq \sqrt{g_1 g_2}$ , the value of the function  $E(r_i, g_2)$  is the maximum if  $\sqrt{g_1 g_2} < r_i \leq \sqrt{g_2 g_3}$ , and finally the value of the function  $E(r_i, g_3)$  is the maximum if  $r_i > \sqrt{g_2 g_3}$ . The above observation graphically proves our claim for the partitioning of the receivers in the case of three groups. The graphical proof remains the same by expanding partitioning thresholds from  $\sqrt{g_1 g_2}$  to  $\sqrt{g_{K-1} g_K}$  for any number of given groups  $K$ . **QED**

We now realize that Theorem (4.1) provides the best overall repartitioning strategy for an unconstrained problem. There is also another issue that needs to be addressed in the case of solving the constrained problem of Equation (16). Considering the definitions of Equation (16) and Equation (12), the issue has to do with the fact that moving a receiver from group  $k - 1$  to group  $k$  can potentially introduce a new constraint for group  $k$ . If the new constraint is far from the existing optimal group rate  $g_k^*$ , it can cause a reduction in the utility sum of groups  $k - 1$  and  $k$  after repartitioning. There are two ways to resolve this issue. First, we can rely on statistical bounds to control the move of a receiver from group  $k - 1$  to group  $k$ . In this case a receiver is allowed to move from group  $k - 1$  to group  $k$  if **one** of the following conditions holds

$$\frac{r_i}{1 - L_i} \geq g_k^* \quad (26)$$

$$(\mu - C_1 \sigma < r_i) \quad \& \quad \left( \frac{r_i}{1 - L_i} < g_k^* \right)$$

where  $\mu$  and  $\sigma$  are the mean and standard deviation of the receivers in group  $k$ . In practice, we have observed that setting  $C_1 \in [0.9, 1]$  yields good results for different values of receivers' loss tolerance. Second, we can allow for moving a receiver from group  $k - 1$  to group  $k$  only if the newly introduced constraint is satisfying a deviation from the existing group  $k$  optimal rate. In the second case, a receiver is allowed to move from group  $k - 1$  to group  $k$  if **one** of the following conditions holds

$$\frac{r_i}{1 - L_i} \geq g_k^* \quad (27)$$

$$C_2 g_k^* < \frac{r_i}{1-L_i} < g_k^*$$

In practice, we have observed that setting  $C_2 \in [0.5, 0.9]$  yields good results for different values of receivers' loss tolerance. Note that, although it is unlikely for the same issue to reveal when moving a receiver from group  $k$  to  $k - 1$ , a similar approach can be used to avoid the problem.

LMMC near-optimal partitioning algorithm then reorders the receivers such that each receiver is moved to a group maximizing its individual utility according to Theorem (4.1) and one of the conditions (26) or (27). Such an algorithm introduces a time complexity order of  $\mathcal{O}(KN)$ . As an alternative and to achieve a more rapid convergence, we can also obtain the new optimal rate of the corresponding group of receivers while repartitioning. This is due to the fact that changing the partitioning thresholds yields a different optimal group rate for the group of receivers affected by the change in the sequence. Considering the added complexity for solving yet another optimization problem, this version of the algorithm introduces a time complexity order of  $\mathcal{O}(KN \log N)$ . The trade off between the two versions of the algorithm is the speed of convergence versus increased complexity. In practice, one selects the latter over the former if the higher speed of convergence justifies the increased complexity of the latter version. Otherwise, the former version is preferred. The second version of the optimal partitioning algorithm is summarized below. The first version is simply obtained by eliminating the last step of the loop.

#### LMMC Near-Optimal Partitioning Algorithm:

For every group of a media session and assuming the group rates  $\{g_1, \dots, g_K\}$  are given,

for  $(k = 2 \text{ to } K) \{$

- Calculate the partitioning threshold  $\sqrt{g_{k-1} g_k}$ .
- Repartition groups  $k - 1$  and  $k$ . For every receiver belonging to groups  $k - 1$  or  $k$  and isolated rate  $r_i$ , assign the receiver to group  $k$  if  $r_i > \sqrt{g_{k-1} g_k}$  and one of the conditions (26) or (27) hold. Otherwise, assign the receiver to group  $k - 1$ .
- Calculate the new optimal sending rate of group  $k$  according to the new partitioning.

$\} /* \text{ for } (k = 2 \text{ to } K) /*$

The other interesting characteristic of the intersection points of Equation (23) is that they remain the same for both the approximate and original fairness functions of Equation (9) and Equation (3). The latter is verified by observing that the partitioning thresholds of Equation (23) are also the intersection points of the fairness functions of Equation (3) for different values of  $g_k$  from the following equation

$$\frac{\min(r_i, g_k)}{\max(r_i, g_k)} = \frac{\min(r_i, g_{k+1})}{\max(r_i, g_{k+1})} \quad (28)$$

We conclude that the general algorithm of this section can be used in conjunction with any rate allocation algorithm by properly identifying partitioning thresholds. In specific, the algorithm of this section can also be used with a rate allocation algorithm relying on the fairness function of (4) in order to reach the optimal partitioning assuming a given set of group rates.

## V. LMMC NEAR-OPTIMAL ITERATIVE SOLUTION

In this section, we introduce an iterative approach that can reach a near-optimal solution considering the fact that the solution to our two-phase optimal problem is sub-optimal due to the impact of our phasing approach. A near-optimal solution can be achieved by iteratively applying the results of each phase as an existing condition to obtain the solution of the other phase. This is equivalent to applying the partitioning results of the second phase to the first phase and solving the optimal rate allocation problem again with the alternative partitioning strategy. The optimal layer rates of the first phase can then be applied to the near-optimal partitioning strategy of the second phase to partition the receivers according to the new set of rates. In what follows we propose the formal iterative algorithm of LMMC and prove that it yields a near-optimal solution considering the necessary condition for optimality defined below holds.

Recall that for a media session with  $N$  receivers,  $K$  groups and the group rate set  $g = \{g_1, \dots, g_K\}$ , a set  $P = \{G_1 | \dots | G_K\}$  is called a partitioning of the receiver set  $R = \{1, \dots, N\}$  if  $P$  is a decomposition of the set  $R$  into a family of disjoint sets. The necessary and sufficient condition for optimality is now defined over the partitioning  $P^*$  and the group rate set  $g^*$  such that

$$IRFA_{Total}(P^*, g^*) \geq IRFA_{Total}(P, g) \quad (29)$$

for every  $P \neq P^*$  and  $g \neq g^*$ . Considering the impact of LMMC phasing approach, the necessary condition for optimality is defined for the combination of two individual phases. In the first phase, we consider a fixed partitioning  $P_{fixed}$  and define the group rate set  $g^*$  such that

$$IRFA_{Total}(P_{fixed}, g^*) \geq IRFA_{Total}(P_{fixed}, g) \quad (30)$$

for every  $g \neq g^*$ . In the second phase, we consider a fixed group rate set  $g_{fixed}$  and define the partitioning  $P^*$  such that

$$IRFA_{Total}(P^*, g_{fixed}) \geq IRFA_{Total}(P, g_{fixed}) \quad (31)$$

for every  $P \neq P^*$ .

#### LMMC Iterative Rate Allocation-Partitioning Algorithm:

- Step 1: Start from an initial ordered partitioning of the receivers by uniformly distributing the receivers among the existing groups. In addition, set the initial iteration number  $j = 0$  and the maximum number of iterations  $j_{max}$ .
- Step 2: Calculate the optimal group rates  $g^* = \{g_1^*, \dots, g_K^*\}$  and the resulting session utility  $IRFA_{Total}$  by numerically solving the system of equations (19) while satisfying conditions (20) and (16). Save the previously calculated  $IRFA_{Total}$  in variable  $q_1$  and the currently calculated  $IRFA_{Total}$  in variable  $q_2$ .
- Step 3: If  $\frac{|q_1 - q_2|}{q_1} < \delta$  or  $j > j_{max}$  STOP.
- Step 4: for  $(k = 2 \text{ to } K) \{$ 
  - Calculate the partitioning threshold  $\sqrt{g_{k-1} g_k}$ .
  - Repartition groups  $k - 1$  and  $k$ . For every receiver belonging to groups  $k - 1$  or  $k$  and isolated rate  $r_i$ , assign the receiver to group  $k$  if  $r_i > \sqrt{g_{k-1} g_k}$  and

one of the conditions (26) or (27) hold. Otherwise, assign the receiver to group  $k - 1$ .

- Calculate the new optimal sending rate of group  $k$  according to the new partitioning.

} /\* for ( $k = 2$  to  $K$ ) \*/

- Step 5: Go back to Step 2.

In the algorithm above the initial conditions are chosen in the first step. While the second step solves the optimal rate allocation problem of the first phase in our two-phase approach, the third step merely checks to terminate the algorithm according to the specified conditions. The fourth step includes the solution to the second phase near-optimal partitioning approach while adjusting the optimal rate of the corresponding group according to the new partitioning. We note that the time complexity of our iterative algorithm is  $\mathcal{O}(IKN \log N)$  where  $I$  indicates the number of iterations. Comparing the overall complexity of LMMC algorithm with that of the dynamic programming algorithm of [29]  $\mathcal{O}(N^3)$ , LMMC algorithm achieves a much lower complexity.

**Theorem 5.1:** The convergence of “LMMC Iterative Rate Allocation-Partitioning Algorithm” mentioned in this section is guaranteed.

**Proof:** Let us make note of the fact that the session utility of Equation (15) consists of a finite number of fairness functions, one for each receiver. These functions are all positive, minimized at the value of zero, and maximized at the value of one. Consequently, the positive session utility function of Equation (15) has both a lower bound and an upper bound. Next, we observe that the session utility function of Equation (15) can only increase in each step considering the operating mechanism of the individual phases of our optimization algorithm. Therefore, the sequence of utility function values at each step of the algorithm is a non-decreasing sequence with an upper bound equal to the number of fairness functions. We also note that any non-decreasing sequence with an upper bound would converge to a finite number also known as a fixed point. We, hence, conclude that LMMC iterative approach converges to a fixed point. **QED**

Intuitively, LMMC algorithm is employing steepest descent optimization strategy and is guaranteed to reach a near-optimal point if such a point exists. It is important, however, to note the followings.

First, we note that the “LMMC Iterative Rate Allocation-Partitioning Algorithm” mentioned in this section converges to a local optimum in the case of solving the unconstrained problem. The claim is accurate considering the fact that the sequence of session utility functions of Equation (15) converges to a fixed point satisfying necessary conditions of (30) and (31) for optimality. We remind that we only claim reaching a near-optimal solution in the case of solving the constrained problem because of applying one of the conditions of (26) or (27). However, we conjecture that the proper choice of the parameters in (26) and/or (27) leads to reaching a local optimal solution as shown by our numerical results.

In practice, the use of the iterative method is a factor of time complexity and the speed of convergence. The iterative method can be effectively deployed in environments with moderate variations of the available per flow bandwidth. As an example, the scenarios encountered in admission control problems can be mentioned in which the assignment of per flow bandwidth is relatively stable. In environments with rapidly varying available bandwidth, the sub-optimal solution with few or no iteration may be deployed.

It is obvious that the initial choice of the partitioning strategy plays a crucial role in the convergence speed of the algorithm. As a practical alternative, the classification method of [31] may be deployed as the partitioning strategy. The use of our proposed algorithms yields fast converging results in most cases as shown by our simulation results.

## VI. NUMERICAL PERFORMANCE ANALYSIS

In this section, we present the numerical results of applying LMMC partitioning and rate allocation algorithms to a number of layered media scenarios and compare them with those of the dynamic programming algorithm of [29]<sup>2</sup>. We review the performance of both approaches from the stand point of tracking the maximum value of the utility function, time complexity indicated by experiment runtime, and space complexity indicated by memory allocation. Additionally, we review the scalability of the techniques by covering a relatively broad range of multicast group sizes ranging from hundreds to thousands of receivers. In our simulations, we rely on generalizations of normal distribution namely tri-, quad-, and pent-modal distributions to generate receiver isolated rates. We select the means of distributions from the set of  $\{128Kbps, 1Mbps, 10Mbps, 100Mbps, 1Gbps\}$ . We note that the choice of modal distributions with the above-mentioned means realistically represents the distribution of bandwidths associated with ISDN, Cable/DSL, low-speed LAN, high-speed LAN, and Gigabit LAN users. For each distribution, we also set the standard deviation of the distribution at 20% of the mean value. Considering the location of the means, the choice of standard deviations yields successive distributions remain disjoint with a certainty better than 99.7%. In our experiments, we make use of a host server with a 1.8GHz Pentium 4 CPU, 512MB of physical memory and 1GB of virtual memory. Further, we rely on the Gnu Scientific Language (GSL) optimization toolbox to provide a balance between the speed and robustness of program execution.

We recall that the time complexity of the iterative optimized LMMC algorithm is  $\mathcal{O}(IKN \log N)$  where  $I$  indicates the number of iterations and the time complexity of the dynamic programming (DP) algorithm of [29] is  $\mathcal{O}(N^3)$ . In addition, the space complexity of the LMMC algorithm in our implementation is  $\mathcal{O}(N)$  where as the space complexity of DP algorithm proposed in [29] is  $\mathcal{O}(N^2)$ . In our simulations, we ran in excess of 5000 experiments with different number of groups  $K$ , different group sizes  $N$ , and different receiver loss tolerance values.

<sup>2</sup>In our simulations, we relax the flow control constraint and assume  $BWA_k = BWL_k$ . The impact of applying the flow control constraint is to only change the value of the constraint  $BWA_k$ .

Fig. (5) through (10) compare the sample results of LMMC algorithm with those of DP algorithm of [29]. In each experiment, we have considered the same loss tolerance for all of the receivers of the session. Different figures have been obtained for different choices of loss tolerance set at 10%, 20%; and the number of groups set at 3, 4, and 5. The x-axis of each curve is always in logarithmic scale and includes values of  $N$  from the set  $\{100, 300, 1000, 3000, 10000, 30000, 100000\}$ . Each figure consists of two pairs of curves. The first pair of curves

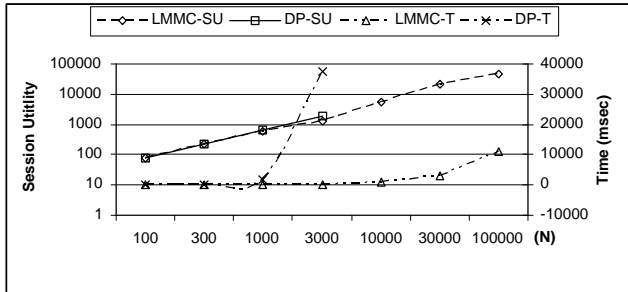


Fig. 5. Session Utility (SU) and Time (T) comparison of LMMC and DP versus number of receivers (N) for  $K=3$  and loss tolerance of 10%.

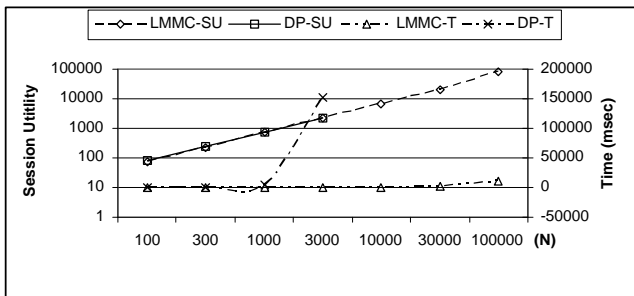


Fig. 6. Session Utility (SU) and Time (T) comparison of LMMC and DP versus number of receivers (N) for  $K=3$  and loss tolerance of 20%.

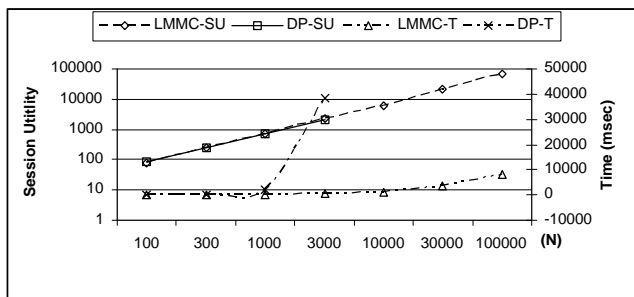


Fig. 7. Session Utility (SU) and Time (T) comparison of LMMC and DP versus number of receivers (N) for  $K=4$  and loss tolerance of 10%.

compare fairness results of the two techniques. In order to do a fair comparison, we have used the fairness function of Equation (9) for LMMC and the fairness function of Equation (3) for DP. Since the maximum of each individual receiver utility is the value 1, the number  $N$  indicates the corresponding upper bound on the fairness for both techniques. A review of the sample results of the figures shows a difference of less than 10%

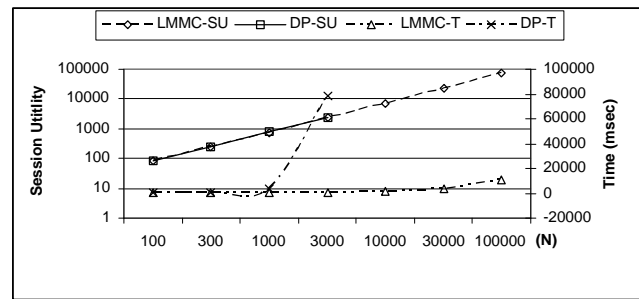


Fig. 8. Session Utility (SU) and Time (T) comparison of LMMC and DP versus number of receivers (N) for  $K=4$  and loss tolerance of 20%.

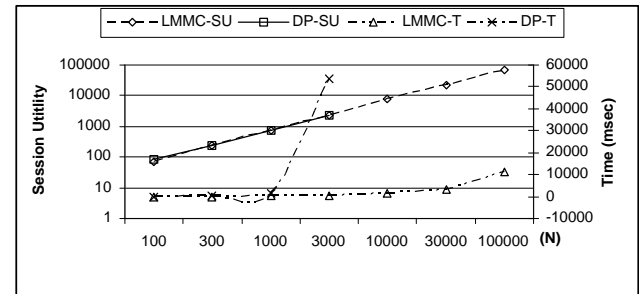


Fig. 9. Session Utility (SU) and Time (T) comparison of LMMC and DP versus number of receivers (N) for  $K=5$  and loss tolerance of 10%.

between the raw session utility values of the LMMC and the DP algorithms. Considering the fact that the fairness function of Equation (9) is an approximation of the fairness function of Equation (3) in the interval of interest, it is in order to mention that the session utility value is only a relative metric of performance comparison. Our overall conclusion is that both of the techniques are capable of tracking a maximum satisfying the existing constraints.

The second pair of curves display the runtime of the experiments as an indicator of the time complexity of the two techniques. In this area, a review of the results reveals the great performance advantage of LMMC over DP. We observe a nonlinear increase in the runtime of the DP algorithm where as LMMC algorithm curve indicates a linear increase. We also note that in each figure, the pair of the DP algorithm curves end at the value of 3000 receivers.

This is explained in terms of the time complexity and the space complexity of the DP algorithm. We argue that an in-

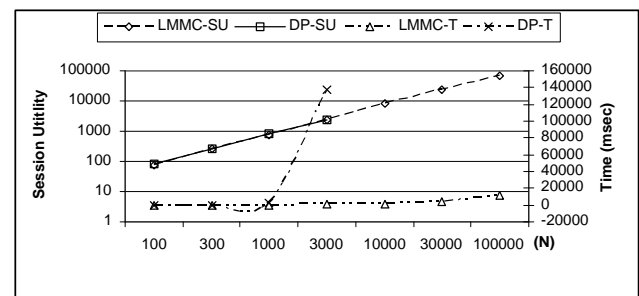


Fig. 10. Session Utility (SU) and Time (T) comparison of LMMC and DP versus number of receivers (N) for  $K=5$  and loss tolerance of 20%.

crease in the value of  $N$  increases the runtime of the algorithm proportional with the third power of  $N$  and consumes the memory proportional with the second power of  $N$ . In our experiments, the impacts of coping with higher time complexity and space complexity become significant for media sessions with more than 1000 receivers. The space complexity analysis also justifies the fact that we have not been able to run any experiment deploying DP algorithm for media sessions with 10000 or more receivers. We argue that although the specific numbers of our experiments are closely related to the capabilities of our host server, the same qualitative behavior is observed in general. It is obvious from our results that Bellman's **curse of dimensionality** defined in [3] shows its impact much more rapidly in the case of DP algorithm than the case of LMMC algorithm.

Finally, we would like to review the impacts of using criteria of criteria set (26) and criteria set (27) in controlling successive groups repartitioning. Generally speaking, we have observed that the proper choice of coefficients  $C_1$  in criteria set (26) and  $C_2$  in criteria set (27) mostly depends on the loss tolerance. The coefficients have to be chosen such that they enforce a narrower bound for smaller values of loss tolerance and a wider bound for larger values of loss tolerance. In our experiments using criteria of criteria set (27) has yielded better results than using criteria of criteria set (26). We have experimentally observed that for a loss tolerance of 10% a value of  $C_2 = 0.790$  best controls the repartitioning process while for a loss tolerance of 20% a value of  $C_2 = 0.885$  provides best repartitioning results. We have also observed that smaller values of loss tolerance increase the number of iterations required for the convergence of LMMC. This is explained considering the fact that smaller values of loss tolerance typically yield narrower bounds in criteria set (26) and criteria set (27) utilized to control the move of receivers from group  $k - 1$  to group  $k$  in each iteration. In general, utilizing narrower control bounds results in a higher number of iterations required for convergence. It is also worth mentioning that the distribution of receivers isolated rates plays an important role in the speed of convergence for both LMMC and DP algorithms.

In the rest of this section, we briefly discuss some of the practical issues. Although in this study we did not discuss many of the practical aspects of implementing LMMC technique, we have implicitly assumed the use of most of the known techniques in the course of implementation. First, we need to apply the comparison analysis of source centric and receiver centric methods to LMMC algorithms. Considering the coordination necessary to synchronize the operation between the sender and receivers in LMMC algorithm, it is classified under hybrid algorithms with the main focus on the sender. Next, we need to consider the issue of feedback implosion in the process of collecting the isolated rates and loss tolerance of the receivers of a large multicast group. We can address feedback implosion issue either as an end-to-end or as an intermediate issue. In the former case, we can deploy a selective feedback mechanism from the receivers to the source of the session. In the latter case, we can force the receivers to report their isolated rates and loss tolerance to their parent routers in the multicast tree. The routers can then

send aggregated feedback messages to the source in multiple intervals. As an example, the feedback suppression technique proposed in [7] can be used to suppress feedback implosion when practically implementing our algorithms.

Finally, we need to discuss the impact of increasing the number of layers in the extrapolated fairness utility of the overall session. In general, we find consistent results in our numerical analysis with what was reported in [29], i.e., in most cases one can achieve the best combination of receiver heterogeneity accommodation and protocol complexity by choosing 3 to 5 layers. We would also like to add that the best fairness results are typically obtained if the number of groups matches to the number of bandwidth ranges in which receiver isolated rates are distributed. In the latter scenario, each of the ranges can capture the bandwidth characteristics of a group of receivers. For example, receivers with isolated rates distributed in the range of  $64Kbps$  indicate dial-up users, receivers with isolated rates distributed in the range of  $1Mbps$  indicate Cable/DSL users, and receivers with isolated rates distributed in the range of  $100Mbps$  indicate fast LAN users. We make a practical observation that currently the number of these ranges does not exceed 5 considering the available bandwidths from dial-up, ISDN, Cable/DSL, Ethernet, and fast Ethernet. With the popularity of faster switched network interfaces such as Gigabit Ethernet and the obsolescence of slower switched network interfaces the number of the groups has to be proportionally adjusted in order for algorithms such as ours to provide best fairness results.

## VII. CONCLUSION

In this paper, we studied the Layered Media Multicast Control (LMMC) solution to the general problem of optimal partitioning and rate allocation for layered and replicated media systems over multicast IP networks. We assumed the existence of congestion and flow control mechanisms specifying the fair bandwidth available to the media session. We aimed at providing a two-phase iterative solution converging to a near-optimal solution. We solved the general problem of receiver partitioning and layering rate allocation by means of extrapolating max-min fairness utilities of individual receivers. We formulated LMMC rate allocation and partitioning problems as a two-phase optimization problem. In the first phase, we calculated the optimal rates allocated to the individual layers of a media session in an analytical form assuming a given initial partition. In the second phase, we introduced a formal approach that obtained the best partitioning of the receivers based on optimal allocated rates of the first phase while adjusting the optimal rates of the resulting groups. We showed that the partitioning strategy of the second phase is near-optimal for the specific allocated optimal layered rates. Considering the impact of LMMC phasing approach, we introduced an iterative method in which a near-optimal solution could be achieved by iteratively applying the results of each phase to another. We discussed that low complexity LMMC algorithms are ideal for dynamically solving the partitioning and rate allocation problems in media systems where the underlying network environments are subject to frequent changes in available fair bandwidth.

Considering the scalability of LMMC techniques, we showed that the techniques could be effectively adopted in different size point-to-multipoint groups as well as different speeds of load change in the network. We also demonstrated that LMMC techniques were able to achieve a very close approximation of max-min fairness considering the heterogeneity issue and the varying loss characteristics of different receivers. Finally, we evaluated the performance of LMMC solution and illustrated its applicability in realistic network topologies through the use of simulations.

At the end, we would like to briefly review the different issues involved in the distribution of media systems over multicast networks. Generally speaking, the main issues involved with media multicast systems are rate allocation, receiver partitioning, and end-to-end error control. In addition, practical considerations such as scalability and feedback implosion have always played a key role in applicability of any media distribution algorithm. We are currently working on a bundled solution integrating LMMC rate allocation and receiver partitioning technique with a counter part end-to-end error control technique for media systems. In addition, we are working on adding a real-time analysis of receivers isolated rates such that LMMC partitioning technique can dynamically select the best parameters when applying conditions of (26) and/or (27).

#### APPENDIX I

##### LEAST SQUARE ERROR EXTRAPOLATION OF THE MAX-MIN FAIRNESS FUNCTION

In this appendix, we introduce a least square error extrapolation technique for the max-min fairness function of Equation (3). The objective of our extrapolation technique is to provide an estimated function  $E(r_i, g_k)$  of function  $F(r_i, g_k)$  that minimizes the surface between the two curves shown in Fig. (1). We select a rational function of  $g_k$  and  $r_i$  in the form of

$$E(r_i, g_k) = \frac{N(r_i, g_k)}{D(r_i, g_k)} \quad (32)$$

where  $N(r_i, g_k)$  and  $D(r_i, g_k)$  are polynomials of  $r_i$  and  $g_k$ . Without loss of generality and to simplify the calculation, let us treat the variable  $r_i$  as a parameter and obtain the function  $E(g_k) = E(r_i, g_k)$  assuming  $\text{Deg}(N(r_i, g_k)) = \text{Deg}(N(g_k)) \leq M$  and  $\text{Deg}(D(r_i, g_k)) = \text{Deg}(D(g_k)) = M$  with respect to  $g_k$  and for the parameter  $r_i$ . The simplest rational function  $E(g_k)$  behaving close to  $F(r_i, g_k)$  is resulted by considering  $\text{Deg}(N(g_k)) + 1 = \text{Deg}(D(g_k)) = 2$  in the form of

$$E(g_k) = \frac{N(g_k)}{D(g_k)} = \frac{b g_k}{a_2 g_k^2 + a_1 g_k + a_0} \quad (33)$$

In the above equation, the parameters,  $b$ ,  $a_0$ ,  $a_1$ , and  $a_2$  are obviously functions of the parameter  $r_i$ . The following conditions assure that not only  $E(g_k) = E(r_i, g_k)$  is well behaved according to the description of Section II, but it satisfies the boundary and maximum conditions of function  $F(r_i, g_k)$ .

$$\begin{aligned} b &> 0, & a_0 &> 0, a_1 \geq 0, a_2 > 0 \\ E(0) &= 0 \Rightarrow a_0 \neq 0 \\ E(\infty) &= 0 \Rightarrow \text{Deg}(N(r)) < \text{Deg}(D(r)) \end{aligned}$$

$$\begin{aligned} E(r_i) = 1 &\Rightarrow a_2 r_i^2 + (a_1 - b) r_i + a_0 = 0 \\ E'(r_i) = 0 &\Rightarrow a_2 r_i^2 + a_1 r_i + a_0 - r_i(2a_2 r_i + a_1) \\ &= -a_2 r_i^2 + a_0 = 0 \\ &\Rightarrow a_0 = a_2 r_i^2 \\ \Delta D(r_i) < 0 &\Rightarrow a_1^2 - 4a_0 a_2 < 0 \\ &\Rightarrow |a_1| < 2\sqrt{a_2 a_0} \end{aligned} \quad (34)$$

Without loss of generality, we assume that  $a_2 = 1$  and  $a_1 = a r_i$ . Applying the conditions of Equation (34) to the general form of Equation (33) introduces the specific form of

$$E(r_i, g_k) = \frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} \quad (35)$$

with the boundary condition  $-2 < a < 2$  for the function  $E(r_i, g_k)$ . We note that the optimum choice of parameter  $a$  yields the best least square estimate for the original fairness function  $F(r_i, g_k)$  defined in (3). Applying least square estimation technique in the interval of interest  $[0, \frac{r_i}{1-L_i}]$  considering the constraint function of (8) yields the optimum value for parameter  $a$  in terms of parameters  $r_i$  and  $L_i$ .

$$\begin{aligned} \min_a [ \text{LSE}(a, r_i, L_i) ] &\equiv \quad (36) \\ \min_a [ \int_0^{r_i} (\frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} - \frac{g_k}{r_i})^2 dg_k \\ + \int_{r_i}^{\frac{r_i}{1-L_i}} (\frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} - \frac{r_i}{g_k})^2 dg_k ] \end{aligned}$$

Equation (37) provides a closed-form for the function  $\text{LSE}(a, r_i, L_i)$ . The solution to Equation (36) can be obtained by choosing the parameter  $a$  resulting in the least value for the function  $\text{LSE}(a, r_i, L_i)$  calculated from Equation (37) over a uniform partitioning of the interval  $(-2, 2)$ . The granularity of the partitioning depends on the desired precision in the numerical algorithm.

$$\begin{aligned} \text{LSE}(a, r_i, L_i) &= r_i(L_i + \frac{1}{3}) \\ &+ r_i \frac{a+2}{a-2} [(4-a) - \frac{(1-L_i)(a^2+a(1-L_i)-2)}{L_i^2-(a+2)L_i+(a+2)} \\ &\quad + a(a-2) \log(a+2)] \\ &- r_i \frac{a+2}{a-2} [\frac{2(a^3-2a^2-2a+6)}{\sqrt{(4-a^2)}} \arctan(\sqrt{\frac{2-a}{2+a}}) \\ &\quad - \frac{4(a-1)}{\sqrt{(4-a^2)}} \arctan(\frac{-L_i}{2-L_i} \sqrt{\frac{2-a}{2+a}})] \end{aligned} \quad (37)$$

Alternatively, a single non-parametric optimal value for parameter  $a$  is the one minimizing the integral of Equation (36) for a fixed value of loss tolerance  $L_i$  and calculated over a continuous range of isolated rates from 0 to  $r_{max}$  where  $r_{max}$  indicates the maximum feasible value of the receivers isolated rates. Considering the available bandwidth ranges, a feasible value for  $r_{max}$  is  $1Gbps$ .

$$\begin{aligned} \min_a [ \text{LSE}(a) ] &= \quad (38) \\ \min_a [ \int_0^{r_{max}} \int_0^{r_i} (\frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} - \frac{g_k}{r_i})^2 dg_k dr_i \\ + \int_0^{r_{max}} \int_{r_i}^{\frac{r_i}{1-L_i}} (\frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} - \frac{r_i}{g_k})^2 dg_k dr_i ] \end{aligned}$$

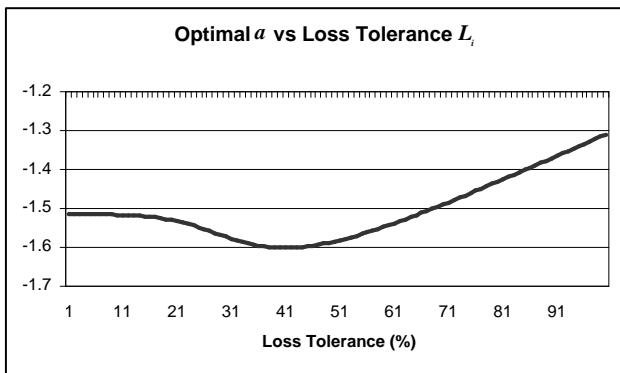


Fig. 11. The plot of optimal  $a$  versus loss tolerance  $L_i$ .

In solving the problem, we have observed that the optimal value of parameter  $a$  is only a function of parameter  $L_i$ . In other words, the optimal value of parameter  $a$  remains the same for a fixed value of parameter  $L_i$  and different choices of parameter  $r_i$  in the interval of interest. Fig. (11) plots the optimal value of parameter  $a$  versus the loss tolerance percentage  $L_i$ . Reviewing the figure in the interval of interest  $L_i \in [0\%, 50\%]$  reveals that the optimal value of parameter  $a$  is in the range of  $[-1.6012, -1.5153]$ . In our calculations, we extract the optimal  $a$  by performing a simple table look up operation.

We also note that with the proper choice of parameters in the general form of Equation (32) with  $M = 2$ , one can potentially model any function belonging to the class of fairness utilities satisfying the conditions defined in [14].

## APPENDIX II

### LMMC OPTIMAL SOLUTION TO THE RATE ALLOCATION PROBLEM WITH AN OVERALL AVAILABLE SESSION BANDWIDTH CONSTRAINT

In this appendix, we provide an analytical solution to the optimal rate allocation problem formulated by Equation (11), Constraint (12), and a new constraint replacing Constraint (13). We consider a scenario in which the overall available session bandwidth is given instead of the available bandwidths of the individual groups. We investigate the solution to this problem for both layered media and replicated media sessions. The interpretation of the problem for layered media sessions is fairly straight forward. First, we note that the constraint set of Equation (13) is reduced to a single constraint in the form of

$$g_K \leq \text{BWF}_K \quad (39)$$

considering the fact that the group rate  $g_K$  is the aggregate rate of layers  $1, \dots, K$  according to Equation (1). The problem of (15) and (16) can then be solved the same way as described in Section III by simply substituting  $\text{BWA}_k = \text{BWL}_k$  for  $k = 1, \dots, K - 1$ .

In the case of replicated media sessions, the constraint set of Equation (13) is reduced to a single constraint in the form of

$$\sum_{k=1}^K g_k \leq \text{BWF} \quad (40)$$

taking into consideration the fact that individual group rates do not include the aggregated sum of the previous layers. First, we convert the rate allocation optimization problem of (15) with inequality constraints to an optimization problem without constraints. We do so by defining the Lagrangian function of Equation (15) as

$$\begin{aligned} LG_{IRF} &= IRFA_{Total} \\ &+ \sum_{k=1}^K \mu_k (g_k - \text{BWL}_k) + \lambda (\sum_{k=1}^K g_k - \text{BWF}) \\ &= \sum_{k=1}^K IRFA_k \\ &+ \sum_{k=1}^K \mu_k (g_k - \text{BWL}_k) + \lambda (\sum_{k=1}^K g_k - \text{BWF}) \\ &= \sum_{k=1}^K \sum_{i \in G_k} \frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} \\ &+ \sum_{k=1}^K \mu_k (g_k - \text{BWL}_k) + \lambda (\sum_{k=1}^K g_k - \text{BWF}) \end{aligned} \quad (41)$$

where the parameters  $\lambda$  and  $\mu_k$  for  $k = 1, \dots, K$  are the Lagrange multipliers in the Lagrangian Equation (41). The unconstrained maximization problem is defined as

$$\begin{aligned} \max_{g_1, \dots, g_K} LG_{IRF} &= \max_{g_1, \dots, g_K} (\sum_{k=1}^K IRFA_k \\ &+ \sum_{k=1}^K \mu_k (g_k - \text{BWL}_k) + \lambda (\sum_{k=1}^K g_k - \text{BWF})) \\ &= \max_{g_1, \dots, g_K} (\sum_{k=1}^K \sum_{i \in G_k} \frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} \\ &+ \sum_{k=1}^K \mu_k (g_k - \text{BWL}_k) + \lambda (\sum_{k=1}^K g_k - \text{BWF})) \end{aligned} \quad (42)$$

### Conditions of Optimality: Constraint Qualifications

We now investigate the existence of necessary and sufficient optimality conditions also known as constraint qualifications. For our unconstrained maximization problem

$$\max_{g_1, \dots, g_K} LG_{IRF} \quad (43)$$

where  $g = \{g_1, \dots, g_K\}$  the constraint qualifications are expressed in terms of Lagrange multiplier theory revolving around conditions under which Lagrange multiplier vectors satisfying the following conditions are guaranteed to exist for a local maximum  $g^* = \{g_1, \dots, g_K\}$ .

$$\nabla LG_{IRF}(g^*) = \quad (44)$$

$$\begin{aligned} \nabla_{(g^*)} &\sum_{k=1}^K \sum_{i \in G_k} \frac{(2+a)r_i g_k}{g_k^2 + a r_i g_k + r_i^2} \\ &+ \nabla_{(g^*)} \sum_{k=1}^K \mu_k (g_k - \text{BWL}_k) \\ &+ \nabla_{(g^*)} \sum_{k=1}^K \lambda (g_k - \text{BWF}) = 0 \end{aligned}$$

$$\begin{aligned} \mu_k &\leq 0 & \forall k = 1, \dots, K \\ \mu_k &= 0 & \forall k \notin A(g^*) \end{aligned} \quad (45)$$

for  $A(g^*) = \{k \mid g_k^* - \text{BWL}_k = 0\}$  and

$$\begin{aligned} \lambda &\leq 0 \\ \lambda &= 0 & \forall k \notin B(g^*) \end{aligned} \quad (46)$$

for  $B(g^*) = \{k \mid g_k^* - \text{BWF} = 0\}$ . The constraint qualifications guarantee the existence of Lagrange multipliers for a given local maximum  $g^* = \{g_1^*, \dots, g_K^*\}$  if the inequality constraint

function of (40) and the inequality constraint functions of (12) are concave<sup>3</sup>.

Considering the fact that the Lagrangian function  $LG_{IRF}$  satisfies all of the conditions mentioned above, finding the optimal solution is equivalent to finding the solutions of Equation (44) in the appropriate group ranges. The solution to the nonlinear system of  $2K + 1$  equations and  $2K + 1$  unknowns provides the optimal rates  $g_k$  for  $k = 1, \dots, K$  as well as the optimal Lagrange multipliers. The system of  $2K + 1$  equations consists of the  $K$  gradient equations shown below plus  $(K + 1)$  constraint equations (12) and (40).

$$\frac{\partial LG_{IRF}}{\partial g_k} \Big|_{g_k^*} = \left( \sum_{i \in G_k} r_i (2 + a) \frac{r_i^2 - g_k^2}{(r_i^2 + a r_i g_k + g_k^2)^2} + \mu_k + \lambda \right) \Big|_{g_k^*} = 0 \quad (47)$$

where  $k = 1, \dots, K$ . The solution to the nonlinear system of  $2K + 1$  equations and  $2K + 1$  unknowns can be obtained by finding the positive real root of (47) such that  $r_{k_{min}} \leq g_k^* \leq r_{k_{max}}$  where  $r_{k_{min}}$  and  $r_{k_{max}}$  indicate the minimum and maximum isolated rates of the receivers belonging to group  $G_k$ . One can find the region in which the border line second condition of (47) holds. The time complexity of solving for the root of this equation over all of the existing groups is  $\mathcal{O}(KN \log N)$  and determines the overall complexity of the solution considering the fact that the rest of calculations are in the time complexity order of  $\mathcal{O}(N)$ . Note that the system of  $2K + 1$  equations and  $2K + 1$  unknowns in this case is more complicated than the case of layered media described in Section III, because of the coupling of the constraint (40) with individual gradient equations of (44).

## REFERENCES

- [1] E. Amir, S. McCanne, R. Katz, "Receiver-Driven Bandwidth Adaptation for Light-Weight Sessions," ACM Multimedia, November 1997.
- [2] M. H. Ammar, "Probabilistic Multicast: Generalizing the Multicast Paradigm to Improve Scalability," In Proc. IEEE INFOCOM, June 1994.
- [3] R. Bellman, "Adaptive Control Processes: A Guided Tour", Princeton University Press, 1961.
- [4] J. Bolot, T. Turetli, "A Rate Control Mechanism for Packet Video in the Internet", In Proc. IEEE INFOCOM, June 1994.
- [5] S. Cheung, M. H. Ammar, X. Li, "On the Use of Destination Set Grouping to Improve Fairness in Multicast Video Distribution," In Proc. IEEE INFOCOM, March 1996.
- [6] S.E. Deering, D.R. Cheriton, "Multicast Routing in Datagram Internetworks and Extended LANs," ACM Trans. Computer Systems, May 1990.
- [7] D. DeLucia, K. Obraczka "Multicast Feedback Suppression Using Representatives", In Proc. IEEE INFOCOM, April 1997.
- [8] ISO/IEC IS 13818-2, "Information Technology - Generic Coding of Moving Pictures and Associated Audio Information, Part 2: Video," 1994.
- [9] ISO/IEC 14496-2/FPDAM4, "Coding of Audio-Visual Objects, Part-2 Visual, Amendment 4: Streaming Video Profile," 2000.
- [10] ITU-T, "Video Coding for Narrow Telecommunication Channels at 64kbit/s," Recommendation H.263, 1996.
- [11] ISO/IEC JTC1/SC29/WG11MPEG02/N4920, "Text of Final Committee Draft of Joint Video Specification," (ITU-T Rec. H.264 — ISO/IEC 14496-10 AVC), 2002.
- [12] H. Jafarkhani, V. Tarokh, "Design of Successively Refinable Trellis Coded Quantizers," IEEE Trans. Information Theory, July 1999.
- [13] J. M. Jaffe, "Bottleneck Flow Control," IEEE Trans. Communications, July 1981.
- [14] T. Jiang, M. H. Ammar, E. W. Zegura, "Inter-Receiver Fairness: A Novel Performance Measure for Multicast ABR Sessions," In Proc. ACM SIGMETRICS, June 1998.

- [15] T. Jiang, M. H. Ammar, E. W. Zegura. "On the use of Destination Set Grouping to Improve Inter-Receiver Fairness for Multicast ABR Sessions," In Proc. IEEE INFOCOM, March 2000.
- [16] T. Jiang, E. W. Zegura, M. Ammar, "Inter-receiver Fair Multicast Communication over the Internet," In Proc. ACM NOSSDAV, June 1999.
- [17] X. Li, M. Ammar, S. Paul, "Video Multicast Over the Internet," IEEE Network Magazine, April 1999.
- [18] X. Li, S. Paul, M. H. Ammar, "Layered Video Multicast with Retransmissions (LVMR): Evaluation of Hierarchical Rate Control," In Proc. IEEE INFOCOM, March 1998.
- [19] X. Li, S. Paul, M. Ammar, "Multi-Session Rate Control for Layered Video Multicast," In Proc. SPIE Multimedia Computing and Networking, January 1999.
- [20] S. McCanne, V. Jacobson, M. Vetterli, "Receiver Driven Layered Multicast," In Proc. ACM SIGCOMM, September 1996.
- [21] P. Moghe, I. Rubin, "Reserving for Future Clients in Multipoint Application-Why and How?," IEEE JSAC, April 1997.
- [22] W. Ren, K. Siu, H. Suzuki, "On the Performance of Congestion Control Algorithm for Multicast ABR Service in ATM," In Proc. IEEE ATM Workshop, August 1996.
- [23] D. Rubenstein, J. Kurose, D. Towsley, "The impact of Multicast Layering on Network Fairness," In Proc. ACM SIGCOMM, September 1999.
- [24] N. Shacham, "Multipoint Communication by Hierarchically Encoded Data," In Proc. IEEE INFOCOM, May 1992.
- [25] T. Turetli, S. F. Parisis, J. Bolot, "Experiments with a Layered Transmission Scheme over the Internet," In Proc. IEEE INFOCOM, March 1998.
- [26] H. Tzeng, K. Siu, "On Max-Min Fairness Congestion Control for Multicast ABR Service in ATM," IEEE JSAC, April 1997.
- [27] L. Vicisano, L. Rizzo, J. Crowcroft, "TCP-Like Congestion Control for Layered Multicast Data Transfer," In Proc. IEEE INFOCOM, March 1999.
- [28] H. A. Wang, M. Schwartz, "Achieving Bounded Fairness for Multicast Traffic and TCP Traffic in the Internet," In Proc. ACM SIGCOMM, September 1998.
- [29] Y. R. Yang, M. S. Kim, S. S. Lam, "Optimal Partitioning of Multicast Receivers," In Proc. IEEE ICNP, November 2000.
- [30] H. Yousefi'zadeh, F. Fazel, H. Jafarkhani, "Hybrid Unicast and Multicast Flow Control: A Linear Optimization Approach," In Proc. IEEE/IEE High Speed Networks and Multimedia Communications (HSNMC), 2004. Extended Version Available at <http://www.ece.uci.edu/~hyousefi/pub.html/fcHSNMC.pdf>.
- [31] H. Yousefi'zadeh, H. Jafarkhani, A. Habibi "Layered Media Multicast Control (LMMC): Rate Allocation and Partitioning," Technical Report. Available at <http://newport.eecs.uci.edu/~hyousefi/pub.html/oraTD.pdf>.

<sup>3</sup>The function  $f : \mathcal{C} \rightarrow \mathcal{R}^n$  defined over the convex set  $\mathcal{C} \subseteq \mathcal{R}^n$  is called concave if  $\forall x_1, x_2 \in \mathcal{C}$  and  $0 \leq \alpha \leq 1$  the inequality  $f(\alpha x_1 + (1 - \alpha)x_2) \geq \alpha f(x_1) + (1 - \alpha)f(x_2)$  holds.