Antenna and Node Selection for Multi-Antenna Relay Networks in Correlated Channels

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SUMMARY  In this paper, we investigate the antenna and node selection issues for amplify-and-forward (AF) and decode-and-forward (DF) multi-antenna relay networks in correlated channels. Based on the channel statistics, optimal selection criteria for antenna and relay node are derived jointly, aiming to maximize the ergodic capacity. Instantaneous channel knowledge-based selection schemes, motivated by traditional antenna selection algorithms, are investigated as well. It is shown that the proposed node selection schemes derived from antenna selection on relay nodes are feasible and effective in relay systems. Statistical selection shows considerable capacity gain compared to full complexity scheme and random selection strategy in AF mode, while instantaneous selection performs better in DF relaying. Furthermore, the proposed schemes are shown to be robust to channel estimation errors due to their correlation-oriented nature.

key words: multi-antenna relay, antenna selection, node selection, correlated channel

1. Introduction

Driven by the demands for high quality services and high date rate coverage in future wireless communication systems, various fundamental changes on transmission technologies and network architectures are developed. Multi-input multi-output (MIMO) techniques are well studied to promise significant improvements in terms of spectral efficiency and link reliability [1]–[3] at the expense of increased hardware and signal processing complexity. However, the link reliability is prone to be undermined by the degradation of channel matrix rank. Therefore, the relaying technology for exploiting system resources through the cooperation of relay nodes is being considered [4]. The application of relays to the traditional MIMO system is able to increase the cooperative diversity [5], [6] and can be regarded as a rank-improvement strategy for the traditional point-to-point MIMO link [7].

To realize relay transmission, two main protocols have been developed for relay systems. One is the amplify-and-forward (AF) [5], [8], [9] mode, a simple case where the relay node only forwards the received signals without decoding process. AF has low complexity but introduces performance penalty at higher signal-to-noise ratio (SNR). The other one is the decode-and-forward (DF) protocol [10], [11], where the relay node retransmits the decoded signals to the destination, which has higher hardware and signal processing complexity but avoids propagating the noise produced at the relay node. Correspondingly, various relay antenna configurations, compatible with the above protocols, have been considered. Earlier work mainly focuses on the single-antenna relay system, where each relay node only carries a single antenna, and multiple relays could construct a virtual antenna array (VAA) [12] that offers diversity gain similar to that offered by the traditional MIMO system [13]. Some recent work has involved the scenario of multi-antenna relay system, for the additional performance gain and space-time process flexibility brought by the multiple antennas on the relay node [14], [15]. Benefits of multi-antenna relay networks have been demonstrated in [16].

Since the cooperative efficiency and hardware cost are major factors that may limit the performance of multi-antenna relay systems, the antenna selection as well as the relay node selection are becoming important issues for further development. In [17], the BER performance of antenna selection is studied for a two-hop AF MIMO relay system. [18] discusses several single-node selection strategies in DF relaying networks, in which terminals obtain help from only one relay. In [19], several node selection algorithms are proposed for receiver cooperative downlink systems. Nosratinia in [20] investigates the node selection in a peer-to-peer cooperative network. Both antenna and node selection strategies are exploited in a multi-relay deployment in [21].

However, all the aforementioned research is only based on the assumption that the channel is rich enough. Therefore, one of the remaining issues is how to select relay antennas in correlated channels. Although some work studied the antenna selection algorithms in traditional one-hop systems [22], [23], these strategies could hardly be applied in relay links because the capacity of relay channel does not depend on any single hop but the whole relay link. Another problem is that the previous antenna selection operations are isolated with the relay selection. In fact, results derived from the antenna selection often contain useful information that could be referred by the relay selection operations. In other words, a combination of antenna and relay selection is worth exploring to save the algorithm complexity in relay systems.

Motivated by these observations, we propose a way of combining the antenna and relay selection in correlated MIMO fading channels. Based on statistical and instantaneous channel knowledge respectively, this paper investi-
gates relay node selection strategies derived from antenna selection analysis in both AF and DF relaying. In each case, we first consider the problem of selecting a receive and transmit antenna subset at the relay for single-relay systems. Then the joint antenna and node selection for multi-relay networks is studied. Unlike the aforementioned work, this paper derives optimal criteria for the selection problem basing on the statistical channel knowledge of correlated MIMO Rayleigh fading channels, which is essential since the delay in the feedback path may lead to the performance degradation on instantaneous channel knowledge-based system. The statistics of the channel fluctuates on a much slower time scale, which can considerably reduce the system overheads. In addition, the instantaneous channel knowledge-based selection mechanism, motivated by the traditional MIMO antenna selection strategies, is also studied so as to seek more insights for the proposed selection criteria. Furthermore, robustness of the proposed selection schemes to channel estimation errors is also analyzed. Note that the large scale path-loss between nodes is not considered in the paper, since we intend to explore the essence of the selection criteria in correlated channels. However, the selection issue based on both the antenna correlation as well as the received signal power will be an interesting future topic.

The rest of this paper is organized as follows. In Sect. 2, the spatial multiplexing-based multi-antenna relay system model is described. In Sect. 3 and Sect. 4, antenna and node selection schemes in AF and DF relaying are studied respectively. Simulation results and analysis are presented in Sect. 5. Finally, conclusions are drawn in Sect. 6.

2. System Model

The relay system under consideration focuses on a two-hop relaying network in which half-duplex constraints are imposed on relay nodes and thus the two-hop transmission is implemented in two different slots. As shown in Fig. 1, a source node and a single destination node both with $M$ antennas are taken into account. Totally $K$ relay nodes, each carrying $N$ antennas, are supposed to locate in the middle of the transmission link. Each of these relays is equipped with a uniform linear array (ULA) and with only $M$ RF chains, hence $M$ out of $N$ available antennas need to be selected for the transmission. For simplicity, the direct link between the source and destination node is ignored, which means the destination only relies on the relaying signals. This assumption simplifies the analysis and is pragmatic in the case when the destination node is at the cell edge or is highly shadowed. Additionally, a power constraint is imposed on both the source and relays.

2.1 Correlated Channel Model with Antenna Selection

We adopt the widely used Kroncker correlation model for the channel, which accurately reflects many practical channels of interest. It is assumed that no transmitter-side correlation exists at the first hop for simplicity while the correlation for receive antennas at the second hop is not considered either, which is reasonable since the destination node is typically located in a rich scattering environment. Let the first and second hop channel matrices be denoted by $H$ and $A$ respectively. According to the correlated model, these channel matrices are given by

$$\mathbf{H} = \mathbf{R}^\dagger \mathbf{H}_w,$$

$$\mathbf{A} = \mathbf{A}_w \mathbf{T}^\dagger,$$

where entries in $\mathbf{H}_w$ and $\mathbf{A}_w$ are modeled with independent identically distributed (i.i.d.) complex Gaussian random variables with $CN(0,1)$. $\mathbf{R}$ and $\mathbf{T}$ are $N \times N$ matrices representing the spatial correlation for receive and transmit antennas on the relay respectively. These correlation matrices are determined by the angles of arrival (AOA) and angles of departure (AOD). Assuming that the AOD is $\theta_{\text{R}} = \theta_{\text{R}} + \hat{\theta}_{\text{R}}$ with $\theta_{\text{R}} \sim N(0, \sigma_{\theta_{\text{R}}}^2)$ and AOA is $\theta_{\text{T}} = \theta_{\text{T}} + \hat{\theta}_{\text{T}}$ with $\theta_{\text{T}} \sim N(0, \sigma_{\theta_{\text{T}}}^2)$, simple approximation forms for the correlation matrices are given as [24]:

$$\mathbf{R}_{mn} \approx e^{-\frac{j \lambda d (n-m) \cos(\theta_{\text{R}})}{2}} e^{-\frac{j \lambda d (n-m) \sin(\hat{\theta}_{\text{R}})}{2}} = e^{-\frac{j \lambda d n \cos(\theta_{\text{R}})}{2}},$$

$$\mathbf{T}_{mn} \approx e^{-\frac{j \lambda d (n-m) \cos(\theta_{\text{T}})}{2}} e^{-\frac{j \lambda d n \sin(\hat{\theta}_{\text{T}})}{2}} = e^{-\frac{j \lambda d n \cos(\theta_{\text{T}})}{2}},$$

where $d$ denotes the relative relay antenna spacing with respect to the carrier wavelength $\lambda$. $m$ and $n$ represent the entry indices of $\mathbf{R}$ and $\mathbf{T}$.

Define $\mathbf{P}$ to be the selection matrix that chooses $M$ rows from $\mathbf{H}$, and $\mathbf{Q}$ as the selection matrix which picks out $M$ columns from $\mathbf{A}$. Then the channel matrices after selection can be written as

$$\mathbf{H}_p = \mathbf{P} \mathbf{H} = \mathbf{R}_p^\dagger \mathbf{H}_p w,$$

$$\mathbf{A}_q = \mathbf{A} \mathbf{Q} = \mathbf{A}_w \mathbf{Q}_q \mathbf{T}_q^\dagger,$$

where $\mathbf{R}_p$ ($\mathbf{T}_q$) is an $M \times M$ sub-matrix of $\mathbf{R}$ ($\mathbf{T}$) formed by deleting $N - M$ rows (columns) and the corresponding columns (rows) from $\mathbf{R}$ ($\mathbf{T}$). And $\mathbf{H}_p w$ ($\mathbf{A}_w$) is an $M \times M$ matrix with i.i.d. complex Gaussian entries.

2.2 Cooperative Relaying with Node Selection

The source broadcasts an $M \times 1$ data vector $\mathbf{s}$ to all the potential relays in the first slot and the $M \times 1$ received data vector

![Fig. 1 Two hop multi-antenna relay networks.](image-url)
where $\mathbf{H}_{p,k}$ is the channel matrix between the source and the $k$th relay. The total transmit power $P_k$ is equally distributed among the $M$ antennas on the source and the covariance matrix of $s$ is $(P_k/M)\mathbf{I}_M$. $\mathbf{n}_k$ is an $M \times 1$ i.i.d. zero-mean, circularly symmetric complex Gaussian random vector at the $k$th relay with covariance matrix $\sigma_{i,k}^2 \mathbf{I}_M$, where $\mathbf{I}_L$ is the $i \times i$ identity matrix.

In the second slot, the $k$th relay node processes the received data vector and forwards a corresponding transmit vector $\mathbf{x}_k$ to the destination. Suppose $L$ relay nodes are chosen from all $K$ relays, and a total transmit power $P_R$ is uniformly distributed among the selected $L$ relays. The signal vector received at the destination is expressed as

$$\mathbf{r} = \sum_{k=1}^{L} \mathbf{A}_{q,k} \mathbf{x}_k + \mathbf{z}$$

$$= \sum_{k=1}^{L} \mathbf{A}_{q,k} [\rho_k (\mathbf{H}_{p,k} \mathbf{s} + \mathbf{n}_k)] + \mathbf{z},$$

where $\mathbf{z}$ is an $M \times 1$ noise vector, with i.i.d. complex Gaussian entries $\sim \mathcal{CN}(0, \sigma_z^2)$.

For the AF relaying protocol, the transmit vector is given by $\mathbf{x}_k = \rho_k \mathbf{y}_k$, where $\rho_k$ is the power coefficient which helps implementing the power constraint $\mathbb{E} [\mathbf{x}_k^H \mathbf{x}_k] = P_R / L$ on the $k$th relay. $\mathbb{E}()$ denotes the expectation and $(\cdot)^H$ is the Hermitian transpose. Hence the power coefficient for the multi-antenna relay system is given by

$$\rho_k = \frac{P_R M}{L \cdot [P_S \cdot \text{trace}(\mathbf{H}_{p,k}^H \mathbf{H}_{p,k}) + (M\sigma_{i,k}^2)]^{1/2}}.$$  \hfill (9)

In this paper, $\sigma_{i,1} = \sigma_{i,2} = \cdots = \sigma_{i,K} = \sigma_d$ is assumed for simplicity. For the DF relaying mode, the $k$th relay would first decode the received vector $\mathbf{y}_k$ with a certain receiver structure and then form a data vector $\mathbf{x}_k$ to be forwarded to the destination.

3. Antenna and Node Selection Strategies in AF Relaying

In this section, antenna and relay node selection strategies are studied for the AF relaying protocol, with both statistical and instantaneous channel knowledge considered. Note that spatial multiplexing receivers are assumed to be adopted at all the transmission nodes. The source and destination node are assumed to know the channel state information (CSI) so as to operate the selection algorithms and send signaling to relays.

3.1 Statistical Selection in AF Relaying

In this section, we first consider the antenna selection on a single-relay deployment. And then the relay node selection, based on the conclusion drawn from the case of single-relay antenna selection, is discussed.

3.1.1 Antenna Selection for Single-Relay Scenario

The VBLAST-MMSE [21] receiver is assumed to be adopted at the destination. According to (7) and (8), the er
godic capacity of the single-relay scenario without antenna selection can be written as

$$C = \mathbb{E} \left\{ \log \frac{\mathbf{I}_M + \frac{P_S}{M \sigma^2} (\rho \mathbf{A}^H \mathbf{A})^H \mathbf{I}_M \rho \mathbf{A}^H \mathbf{A})^H}{1} \right\}.$$

Consequently, the optimal ergodic capacity after antenna selection on the relay is given by

$$C_S = \max_{\mathbf{P}, \mathbf{Q}} \mathbb{E} \left\{ \log \frac{\mathbf{I}_M + \frac{P_S}{M \sigma^2} (\rho \mathbf{A}^H \mathbf{A})^H \mathbf{I}_M \rho \mathbf{A}^H \mathbf{A})^H + \rho \mathbf{A}^H \mathbf{Q} \right\},$$

where $\mathbf{V}$ and $\mathbf{G}$ are defined as

$$\mathbf{V} = \rho \mathbf{A}_{q,w}^H \mathbf{T}_{q,w} \mathbf{R}_{q,w}^H \mathbf{H}_{p,w},$$

$$\mathbf{G} = \mathbf{I}_M + \rho^2 \mathbf{A}_{q,w}^H \mathbf{T}_{q,w} \mathbf{R}_{q,w}^H \mathbf{A}_{q,w}.$$  \hfill (13)

The aim is to find the best statistical channel knowledge-based selection matrix pair $\mathbf{P}$ and $\mathbf{Q}$, or equivalently the correlation matrix pair $\mathbf{R}_p$ and $\mathbf{R}_q$, in order to maximize (11). Since it is difficult to solve the above optimization issue involving the ergodic capacity, we refer to the following lower bound [25]:

$$C = \mathbb{E} \left\{ \log \frac{\mathbf{I}_N + \frac{1}{N_L} \mathbf{H}^H \mathbf{H}}{1 + \frac{1}{N_L} \exp \left( \frac{1}{N_L} \mathbb{E} \left\{ \log \left| \mathbf{H}^H \mathbf{H} \right| \right\} \right)} \right\}.$$  \hfill (14)

where $N_r$ and $N_t$ are the numbers of the receive and transmit antennas respectively. $\Gamma$ is the average SNR and it has been proved to be very tight in the high SNR regime under the assumption of full rank channel matrix and $N_t \leq N_r$. Applying the lower bound to (11), we have

$$C_S \geq M \log \left( 1 + \frac{P_S}{M \sigma^2} \exp \left( \frac{1}{M} \right. \right. \right.$$

$$\times \mathbb{E} \log \left( \frac{\rho^2 \mathbf{A}_{q,w}^H \mathbf{T}_{q,w} \mathbf{R}_{q,w}^H \mathbf{H}_{p,w} \mathbf{R}_p^H \mathbf{T}_q \mathbf{A}_{q,w}}{\mathbf{I}_M + \rho^2 \mathbf{A}_{q,w}^H \mathbf{T}_{q,w} \mathbf{R}_{q,w}^H \mathbf{A}_{q,w}} \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right)$$
\[
\times \left( \log \left| I_M + \rho^2 A_{qw} T_q^\dagger T_q^* A_{qw} \right| \right)^{\frac{1}{2}} \right),
\]
where
\[
\Omega = \exp \left( \mathbb{E} \left\{ \log \left| \rho^2 H_{pw} H_{pw}^* A_{qw} \right| \right\} \right). \tag{16}
\]
By utilizing Jensen’s inequality on the concave function \( \log \left| I_M + \rho^2 A_{qw} T_q^\dagger T_q^* A_{qw} \right| \), the capacity can be further lower bounded as
\[
C_S \geq M \log \left( 1 + \frac{P_s}{M \sigma^2} \Omega \exp \left( \frac{-1}{M} \right) \right)
\times \left( \log \left| I_M + \frac{P_s \text{Mtrace}(T_q^*)}{P_s \text{Mtrace}(R_p) + M^2 \sigma^2} I_M \right| \right)
= M \log \left( 1 + \frac{P_s(1 + M \sigma^2)}{M \sigma^2 [P_s + M(P_R + \sigma^2)]} |R_p T_q|^2 \right). \tag{17}
\]
Define \( \gamma_{AS} = |R_p T_q| \) and the capacity is maximized when \( \gamma_{AS} \) is maximized according to (17). Therefore, the parameter \( \gamma_{AS} \) can be regarded as the statistical criterion for the antenna selection in the single-relay scenario.

3.1.2 Node Selection for Multi-Relay Scenario (L=1)

Motivated by the conclusion drawn from the single-relay antenna selection, an extension to the best-one relay node selection (i.e. \( L=1 \)) is straightforward. To choose the best relay for transmission according to the knowledge of channel statistics, we can simply pick the relay with the maximal \( \gamma_{AS} \), which can be expressed as
\[
RN = \arg\max_k \left| [R_{p,k} T_{q,k}] \right|, \tag{18}
\]
where \( RN \) represents the selected relay node. This criterion hints that the node selection is combined with the operation of single-relay antenna selection, since the individual antenna selection on each relay node needs to be finished first so that (18) can be manipulated.

3.1.3 Node Selection for Multi-Relay Scenario (L>1)

When more than one relay node assist the transmission, the capacity without node selection or antenna selection in (10) is rewritten as
\[
C = \mathbb{E} \left\{ \log \left| I_M + \frac{P_s}{M \sigma^2} \sum_{k=1}^K \rho_k A_k H_k \right| \right\} \\
\times \left( \sum_{k=1}^K \rho_k A_k H_k \right)^\dagger \left( I_M + \sum_{k=1}^K \rho_k^2 A_k A_k^\dagger \right)^{-1} \right). \tag{19}
\]
Without loss of generality, assume the number of selected relay nodes \( L=2 \) and the individual antenna selection for each relay is finished before the node selection, the ergodic capacity after selection is
\[
C_S = \max_{k_1, k_2 \in K} \mathbb{E} \left\{ \log \left| I_M + \frac{P_s}{M \sigma^2} \right. \right.
\times \left( \rho_1 A_{q,k_1} H_{p,k_1} + \rho_2 A_{q,k_2} H_{p,k_2} \right)
\times \left( \rho_1 A_{q,k_1} H_{p,k_1} + \rho_2 A_{q,k_2} H_{p,k_2} \right)^\dagger
\left. \times \left( I_M + \rho_1^2 A_{q,k_1} A_{q,k_1}^\dagger + \rho_2^2 A_{q,k_2} A_{q,k_2}^\dagger \right)^{-1} \right\}
\]
\[
= \max_{k_1, k_2 \in K} \mathbb{E} \left\{ \log \left| I_M + \frac{P_s}{M \sigma^2} V_2 G_{eq}^\dagger V_2^\dagger \right| \right\}, \tag{20}
\]
where
\[
V_2 = \frac{\rho_1^2 A_{q,k_1} H_{p,k_1} T_{q,k_1}^\dagger}{p_{q,k_1} T_{q,k_1}^\dagger} R_{p,k_1}^\dagger H_{pw,k_1}.
\tag{21}
\]
\[
G_{eq} = I_M + \rho_1^2 A_{q,k_1} T_{q,k_1}^\dagger R_{p,k_1} H_{pw,k_1} + \rho_2^2 A_{q,k_2} T_{q,k_2}^\dagger R_{p,k_2} H_{pw,k_2}.
\tag{22}
\]
Define \( H_p = R_{p,k}^\dagger H_{pw,k} \) and \( V_2 = \rho_{eq} A_{q,h} H_p \) as in the single-relay scenario, \( V_2 V_2^\dagger \) can be expressed as
\[
V_2 V_2^\dagger = \frac{\rho_{eq}^2 A_{q,h} T_{q,k}^\dagger R_{eq,h} H_{pw,h}^\dagger R_{eq,h}^\dagger A_{q,w}^\dagger}{p_{eq}^2 T_{q,k}^\dagger R_{eq,h}^\dagger A_{q,w}^\dagger}. \tag{23}
\]
where the equivalent form \( T_{eq,k}^2 = \left( R_{p,k} T_{q,k} + R_{p,k} T_{q,k}^\dagger \right)^2 \) and \( \mathbb{E}[p_{eq}] = P_R/[2(M \sigma^2)] \). (See Appendix A for proof.)

Assume \( (R_{p,k} T_{q,k} + R_{p,k} T_{q,k}^\dagger) \) is a full rank matrix. We follow the derivation steps in the case of antenna selection, and the capacity in (20) becomes
\[
C_S = \max_{k_1, k_2 \in K} \mathbb{E} \left\{ \log \left| I_M + \frac{P_s}{M \sigma^2} \right. \right.
\times A_{qw}(R_{p,k_1} T_{q,k_1} + R_{p,k_2} T_{q,k_2})^\dagger H_{pw}
\times H_{pw}^\dagger(R_{p,k_1} T_{q,k_1} + R_{p,k_2} T_{q,k_2})^\dagger A_{qw}
\times \left( I_M + \rho_1^2 A_{q,k_1} A_{q,k_1}^\dagger + \rho_2^2 A_{q,k_2} A_{q,k_2}^\dagger \right)^{-1} \right\}
\]
\[
\geq M \log \left( 1 + \frac{P_s (P_s + M \sigma^2)}{M \sigma^2 [P_s + M (P_R + \sigma^2)]} \right)
\times |R_{p,k_1} T_{q,k_1} + R_{p,k_2} T_{q,k_2}| \right). \tag{24}
\]
where
\[
\Theta = \exp \left( \mathbb{E}[\log |V_{eq} H_{pw,h}^\dagger A_{q,w,q_{eq}}|] \right). \tag{25}
\]
The detailed derivation of (24) is contained in Appendix B. Consequently, the criterion expression of node selection when \( L=2 \) can be defined as \( \gamma_{NS} = |R_{p,k_1} T_{q,k_1} + R_{p,k_2} T_{q,k_2}| \) and (24) is maximized when the best relay node set with maximal \( \gamma_{NS} \) is chosen. Apparently, if \( L \) relays are to be picked out of all potential ones, selection criterion should be as follows:
Algorithm 1 Instant Antenna Selection for AF Relay

\[
\begin{align*}
\text{1: } & \text{ procedure } \text{ant}\text{-}\text{sel}(H, N, N_\text{s}) \\
\text{2: } & \Theta \leftarrow \{1, 2, ..., N\} \\
\text{3: } & \text{cor} \leftarrow 0 \\
\text{4: } & \text{while } |\Theta| > N_\text{s} \text{ do} \\
\text{5: } & \text{for } i \leftarrow 1, N - 1 \text{ do} \\
\text{6: } & \text{for } j \leftarrow i + 1, N \text{ do} \\
\text{7: } & \text{if } \text{cor} < \langle h_i, h_j \rangle \text{ then} \\
\text{8: } & \text{cor} \leftarrow \langle h_i, h_j \rangle \\
\text{9: } & i \leftarrow i \\
\text{10: } & j \leftarrow j \\
\text{11: } & \text{end if} \\
\text{12: } & \text{end for} \\
\text{13: } & \text{end for} \\
\text{14: } & \text{if } \|h_i\|^2 > \|h_j\|^2 \text{ then} \\
\text{15: } & \Theta \leftarrow \Theta - \{i\} \\
\text{16: } & \text{else} \\
\text{17: } & \Theta \leftarrow \Theta - \{j\} \\
\text{18: } & \text{end if} \\
\text{19: } & \text{end while} \\
\text{20: } & \text{end procedure}
\end{align*}
\]

(26)

This criterion is consistent with (18) when the number of selected relay nodes \(L\) equals one.

3.2 Instantaneous Selection in AF Relaying

3.2.1 Antenna Selection for Single-Relay Scenario

The instant selection harnesses the instantaneous channel state information, rather than the channel statistics as mentioned in the last subsection, to carry out the selection process. According to the correlation model assumption, the antenna selection procedure can be handled basing on the correlation information amid antennas where high correlated antennas usually carry similar data information.

The correlation-based selection scheme is conducted individually on the two hops. And the process carried out on the first hop is described as Algorithm 1.

For antenna selection on the second hop, we need only change the above row processing into column processing (with channel matrix \(H\) substituted by \(A\), and vector \(h\) by \(a\)). Note that in Algorithm 1, line 7 to 11 implicate that the antenna with lower power is eliminated when two antennas are highly correlated.

Though the discussed selection strategy necessitates the instantaneous channel knowledge feed back in the second hop, the number of its operations \(2 \sum_{i=1}^{N_\text{s} - 1} \binom{N_\text{s}}{i}\) is much less than \(\binom{N_\text{s}}{N_\text{s}}^2\) of the exhaustive search method. On the other hand, the separate selection operations do not necessarily warrant the uncorrelated property of the whole link channel matrix (i.e. \(A\bar{H}\)), thus the performance tends to undergo a degradation.

3.2.2 Node Selection for Multi-Relay Scenario

Analogous to the development from antenna selection to node selection of statistical scheme, the criterion of choosing the best relay with most uncorrelated antennas can be expressed as

\[
RN = \arg \min_k \left\{ \text{mean} \left( \max_{i,j} \left( \langle h_{i,k}, h_{j,k} \rangle \right) \right),
\max_{i,j} \left( \langle a_{i,k}, a_{j,k} \rangle \right) \right\},
\]

(27)

where \(RN\) is the selected relay node and \(\langle , \rangle\) denotes the inner product operator. The item \(\left( \langle h_{i,k}, h_{j,k} \rangle \right)\) indicates that the antenna pair of the first hop with the highest correlation value is chosen as the worst antenna pair, and so does the item \(\left( \langle a_{i,k}, a_{j,k} \rangle \right)\) of the second hop. The expression \(\text{mean}(\cdot)\) hints that the relay with better correlation condition is selected. In other words, it is only necessary to feed back the correlation value of each relay, calculated in the stage of antenna selection, instead of the whole channel matrix to the source. Since all the correlation values of different antenna pairs have already been calculated, the node selection would not ask for a recalculation, which can considerably save the computational complexity.

When a couple of relays, instead of a single one, are to be chosen from the relay set, a straightforward way can be derived as a simple extension to the case when \(L=1\). We only need first to figure out the correlation degree of each relay, namely the value of \(\text{mean} \left( \max \left( \langle h_{i,k}, h_{j,k} \rangle \right), \max \left( \langle a_{i,k}, a_{j,k} \rangle \right) \right)\) for \(k \in \{1, 2, ..., K\}\), and then choose the \(L\) relays with relatively lower correlation degrees.

4. Antenna and Node Selection Strategies in DF Relaying

4.1 Statistical Selection in DF Relaying

4.1.1 Antenna Selection for Single-Relay Scenario

In the DF relaying, the ergodic capacity of the two hops without antenna selection would be written separately as

\[
C_{1\text{-hop}} = \mathbb{E} \left\{ \log \left( I_M + \frac{P_S}{M\sigma^2} HH^T \right) \right\},
\]

(28)

\[
C_{2\text{-hop}} = \mathbb{E} \left\{ \log \left( I_N + \frac{P_S}{N\sigma^2} AA^T \right) \right\}.
\]

(29)

According to the nature of the DF relaying, the capacity of the whole link is

\[
C = \mathbb{E} \left\{ \min \left( \log \left( I_M + \frac{P_S}{M\sigma^2} HH^T \right), \log \left( I_N + \frac{P_S}{N\sigma^2} AA^T \right) \right) \right\}.
\]
log \left| I_N + \frac{P_g}{N \sigma^2} A A^t \right|). \quad (30)

For the single-relay deployment, the optimal ergodic capacity after antenna selection in the first hop is

\[
C_{S,1-hop} = \max_P \mathbb{E} \left\{ \log \left| I_M + \frac{P_S}{M \sigma^2} (PH)(PH)^t \right| \right\}. \quad (31)
\]

Hence the aim is to find the best instantaneous channel knowledge-based selection matrix pair $P$, or equivalently the correlation matrix pair $R_p$, in order to maximize (31). Similar to the derivation in AF relaying, we apply the lower bound to (31) and have

\[
C_{S,1-hop} \geq M \log \left( 1 + \frac{P_S}{M \sigma^2} \right) \times \mathbb{E} \left\{ \log \left| R_p^{1/2} H_p H_p^t R_p^{1/2} \right| \right\} = M \log \left( 1 + \frac{P_S}{M \sigma^2} \right) \times \mathbb{E} \left\{ \log \left| H_p H_p^t \right| \right\}. \quad (32)
\]

Define $\gamma_{AS,1-hop} = \left| R_p \right|$ and the capacity is maximized when $\gamma_{AS,1-hop}$ is maximized according to (32). Similarly, the selection criterion of the second hop can also be derived as $\gamma_{AS,2-hop} = \left| T_q \right|$, and the antennas of a single relay can be chosen via these two individual criteria.

4.1.2 Node Selection for Multi-Relay Scenario

When signals are decoded on the DF relay, multi-relay transmission without coherent combination at the destination is not desired. We can simply pick the relay according to the following transmission according to the knowledge of channel statistics. We can simply pick the relay according to the following criterion of the second hop can also be derived as

\[
\gamma_{AS,2-hop} = \max_{\Theta} \text{arg max} \left\{ \left| R_p \right|^2 \right\}. \quad (33)
\]

where $\text{min}()$ is used due to the bottleneck effect of DF relaying.

4.2 Instantaneous Selection in DF Relaying

The antenna selection strategy in this scenario is the same as Algorithm 1, while the separate selection operations here, unlike those in the AF relaying mode, can better contribute to the link capacity due to the nature of DF mode. With the same reason mentioned in the above scenario, we only consider the best node selection in this deployment.

The node selection scheme in this case is different from that in the AF scenario in that we need not only consider the correlation degrees of the antennas, but also the balance of the two hops. Therefore, we should first find out the worst antenna pair of each relay in both two hops, as described in Sect. 3.2.2, and then choose the worst antenna, i.e. with the lowest power, from the worst antenna pair of each relay. Hence, the best one would be the relay whose worst antenna has the highest power. With the parameters assumption in Algorithm 1, the main steps of the algorithm are shown as Algorithm 2.

5. Simulation Results

Simulation results with the derived selection criteria are provided in this section. Assume a uniform linear antenna array is equipped at each relay node. Both the single-relay case ($M=2, N=4, K=1$) and the multi-relay scenario ($M=2, N=4, K=3, L=1$ and 2) are considered and a Monte-Carlo simulation with 2000 channel realizations is performed. Related parameters described in Sect. 2 are given in Table 1. The following schemes are investigated in the simulation:

- AAU – All antenna used
- ANU – All node used
- RAN – Random selection
- ICS – Instant correlation selection
- SOS – Statistical optimal selection
- IOS – Instant optimal selection.

Note that ICS and SOS are namely the schemes investigated in the above sections. The scheme of IOS simply chooses
5.1.1 Antenna Selection for AF Relay

5.1.2 Node Selection for AF Relay

Table 3 compares the criterion parameter and ergodic capacity of different relay node selection schemes in the multi-relay scenario \((K=3, L=2)\), including the transmission with relay subset \([1,3]\), subset \([2,3]\), all node used (ANU), RAN, ICS, SOS (i.e. subset \([1,2]\)) and IOS. It can be seen that the SOS gives a 0.64 bps/Hz selection gain compared to the worst relay subset \([1,3]\) and a 0.2 bps/Hz selection gain compared to the transmission with ANU. The ICS scheme also performs worse than SOS for the same reason in the antenna selection scenario. For comparison, the best-one relay selection \((K=3, L=1)\) is also simulated. Capacity CDF plot of these two cases \((L=1)\) and \((L=2)\) are shown in Fig. 3 and Fig. 4. It is observed that SOS has the similar performance with ANU when \(L=1\), while the former outperforms the latter one when \(L=2\), which indicates that a moderate augmentation of the relay number with node selection can give rise to the increase of the system capacity.

Figure 5 shows the ergodic capacity with diverse node selection schemes versus PNR. The performance deviation among these schemes is consistent with the above discussion. Note that the selection gain between SOS and ANU increases when PNR rises. This is for the reason that the capacity lower bound utilized in the selection criteria derivation is tighter in high SNR regime, which indicates that the higher SNR results in the more precise selection. Besides, the selection with SOS would concentrate the increased power on the “good” relay subset while ANU only allocates power uniformly among all relays.
node selection schemes in DF multi-relay scenario. Capacity achieved by the relay subset [2] and [3] is also drawn in the figure, indicating that the SOS chooses relay subset [1] for transmission. It is shown that with the step of the power detection for the worst antenna in Algorithm 2, the ICS scheme bears higher capacity than that of the SOS scheme, which is different with the AF mode.

5.3 Impact of Channel Estimation Errors

Due to the noise and time-varying nature, channel estimates used in selection algorithm are imperfect, which would lead to incorrect selection and erode the system capacity. Denote \( \hat{H} \) and \( \hat{A} \) to be the channel estimation matrices of the first hop and second hop respectively, and the corresponding estimation error matrices are represented by \( H_e \) and \( A_e \). Hence the channel matrix can be modeled as \( \mathbf{H} = \hat{\mathbf{H}} + \mathbf{R}^T H_e \) and \( \mathbf{A} = \hat{\mathbf{A}} + \mathbf{A}_e T^T \), where \( \hat{\mathbf{H}} = \mathbf{R}^T \hat{\mathbf{H}} \) and \( \hat{\mathbf{A}} = \hat{\mathbf{A}}_e T^T [26] \). In addition, \( \hat{\mathbf{H}}_w \) and \( \mathbf{H}_e \) are uncorrelated and their entries are
Since the simulation results also show that the AF relaying with node selection does not necessarily outperform the scheme with all nodes used, future work will focus on the discussion of the optimal number of relay nodes employed in the AF statistical selection. Furthermore, the application of various space-time coding schemes in multi-antenna relay systems will be another interesting topic.

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Since $V_2$ is the linear combination of matrix $\rho_k A_q H_{pk}(k = 1, 2)$, $V_2$ can consequently be defined as the equivalent form $\rho_{eq} A_q H_{eq}$, in which $H_{eq}$ only contains the receive antenna correlation and $A_q$ only with the transmit correlation.

Therefore, according to (9) and (21)

$$E[V_2 V_2^\dagger] = E\left(\rho_{k1} A_{qw,k1} T_q^\dagger R_q^\dagger H_{pw,k1} + \rho_{k2} A_{qw,k2} T_q^\dagger R_q^\dagger H_{pw,k2}\right) \times \left(\rho_{k1} A_{qw,k1} T_q^\dagger R_q^\dagger H_{pw,k1} + \rho_{k2} A_{qw,k2} T_q^\dagger R_q^\dagger H_{pw,k2}\right)^\dagger$$

$$= \frac{P_R}{2(P_S + M \sigma^2)} \times \left[\text{trace}\left(R_{p,k1} T_{q,k1} + R_{p,k2} T_{q,k2}\right)\right] I_M.$$  \hspace{1cm} (A-1)

By utilizing Jensen’s inequality, the capacity is further lower bounded as

$$C_S \geq M \log \left(1 + \frac{P_S (P_S + M \sigma^2) \Theta}{M \sigma^2 [P_S + M (P_R + \sigma^2)]}\right) \times \left|\text{trace}\left(R_{p,k1} T_{q,k1} + R_{p,k2} T_{q,k2}\right)\right|^\dagger,$$  \hspace{1cm} (A-4)

where $\Theta = \exp \left(E[\log |\rho_{eq} H_{pw} H_{pw}^\dagger A_q A_q^\dagger|]\right)$. \hspace{1cm} $\blacksquare$

**Appendix B: Derivation of (24)**

Applying the lower bound in (14),

$$C_S = \max_{k_1,k_2} E \left[\log |I_M + \frac{P_S}{M \sigma^2} \rho_{eq}^{k_1} A_{qw,k1} T_{q,k1} + R_{p,k2} T_{q,k2}\right]^\dagger H_{pw} \times H_{pw}^\dagger R_{p,k1} T_{q,k1} + R_{p,k2} T_{q,k2}^\dagger A_q^\dagger \times \left(I_M + \rho_{k1}^2 A_{q,k1} A_{q,k1}^\dagger + \rho_{k2}^2 A_{q,k2} A_{q,k2}^\dagger\right)^{-1}\right]\right)$$

$$\geq M \log \left(1 + \frac{P_S}{M \sigma^2} \exp \left(\frac{1}{M}\right) \times E \left[\log |\rho_{eq}^{k_1} A_{qw,k1} R_{p,k1} T_{q,k1} + R_{p,k2} T_{q,k2}\right]^\dagger H_{pw} \times H_{pw}^\dagger R_{p,k1} T_{q,k1} + R_{p,k2} T_{q,k2}^\dagger A_q^\dagger \times \left(I_M + \rho_{k1}^2 A_{q,k1} A_{q,k1}^\dagger + \rho_{k2}^2 A_{q,k2} A_{q,k2}^\dagger\right)^{-1}\right]\right)$$

$$= M \log \left(1 + \frac{P_S}{M \sigma^2} \left|R_{p,k1} T_{q,k1} + R_{p,k2} T_{q,k2}\right|^\dagger \Theta \times \left(\frac{1}{M} E \left[\log |I_M + \rho_{k1}^2 A_{q,k1} A_{q,k1}^\dagger + \rho_{k2}^2 A_{q,k2} A_{q,k2}^\dagger\right]\right)\right).$$  \hspace{1cm} (A-3)
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