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Discussion of Rate in Quasi-Orthogonal Space-Time Block Codes and Code Design
for Rank-Deficient Correlated Channels

THESIS

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To
my parents

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Abstract

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In this thesis we discuss coded modulation schemes designed for multiple antenna wireless channels without information of the channel at the transmitter. We focus on space-time block codes and explain the benefits and limitations of primary designs. This thesis has two parts. In the first part, which comes mainly in Chapter three, we discuss a modified version of space-time codes called quasi-orthogonal space-time block codes. Using a correct definition of rate, we show that there is no quasi-orthogonal space-time block code with full rate and diversity for more than four transmit antennas.

In the second part of this thesis, we discuss correlated multi-antenna systems. The transmit diversity of any scheme in these channels is limited to the rank of the correlation matrix. When the rank of the correlation matrix is not full, we cannot get diversity equal to the number of transmit antennas. We prove that even at that case we cannot get a rate more than one while achieving the maximum diversity of the channel. In addition we introduce a systematic way to achieve this maximum diversity, while keeping other good properties of the code such as rate and decoding complexity. Also the designs will have the benefit of unit peak to average ratio, as well as simple decoding property.

Professor Hamid Jafarkhani
Thesis Committee Chair

Chapter 1

Introduction

1.1 Overview

Fading in wireless channels causes loss in performance. To overcome this bad effect, communication engineers came up with techniques to decrease the probability of having an overall weak channel. These techniques are mainly called diversity. In general, diversity means using different dimensions of the channel, e.g. space, time, frequency, and so on, to improve the equivalent channel seen by the receiver. In this thesis we will specifically deal with one of the realizations of space diversity techniques termed as space-time codes. A space-time code [1] is in general any modulation scheme which is designed for a multiple transmitter wireless system that tries to achieve antenna (space) diversity. The very first designs of space-time codes were in the form of trellis coded modulation, and suffered from exponential decoding complexity as the num-

ber of transmit antennas increased. After a while, Alamouti [2] proposed a simple transmitter diversity scheme which benefitted both full diversity of a two-transmit antenna channel as well as simple Maximum-Likelihood(ML) decoding. The good properties of this code inspired Tarokh *et al.* [3] to inspect the existence of similar designs for more numbers of transmit antennas. In the case of complex codes, i.e. modulation schemes using complex constellation members, the authors proposed a structured modulation scheme, called Orthogonal Space-time Block Code that could send on average one symbol in every two time slots, and achieved full diversity as well as simple ML decoding. They presented examples for three and four transmit antennas with average rate of $\frac{3}{4}$, or in case of real constellations they presented rate 1 codes for any number of transmit antennas. It was shown in that paper that for complex constellations there is no square rate-1 code, i.e. a code for which the time length of the block equals the number of transmit antennas like that of Alamouti, for more than two transmit antennas. Meaning that Alamouti code was the only square full-rate complex orthogonal space time block code. Later in [4] it was shown that the same applies to non-square designs. Then Jafarkhani [5] proposed another scheme that was inspired by Alamouti code and could achieve full rate by pairwise decodability for four transmit antennas.¹ A modified version of this code which used rotated constellations could achieve maximum diversity of the channel in addition to the described properties of the original design [6]. Chapter 3 of this thesis is dedicated

¹In Section 2.3 we discuss in detail what the exact meaning of pairwise decodability is.

to investigating if Jafarkhani's idea can be extended to bigger number of transmit antennas.

1.2 Correlated multiple antenna channels

Codes proposed in the literature in the area of MIMO channels are designed to satisfy the criteria mentioned in [1]. In that paper authors assumed that path gains are independently identically distributed. Recently in [7], Bolcskei *et al.* inspected the performance of modulation schemes in channels with spatial correlation. Basically they expand the pairwise error probability and average it on the distribution of the channel coefficients. They propose a closed form distance matrix and design criteria the same as the one in [1]. When correlation of the antennas is such that it causes rank deficiency in the distance matrix, we do not get full diversity. Direct usage of space-time codes is not the best we can do at this case. Because space-time codes are full rank codes and they are meant to achieve diversity equal to the number of transmit antennas. We know that when we increase the number of antennas we have to relax some of good properties of these codes to achieve full rank. Meaning that for more than three transmit antennas we should either relax simple decoding property or the full rate. However we are not going to get full-diversity in the case of a rank-deficient correlation, disregarding the rank of the code. Therefore the usage of space-time codes in these channels is accompanied by paying high costs in terms of decoding complexity and rate. In Chapter 4 of this thesis we try to design codes that

achieve maximum diversity of the code as well as keeping other good properties of a code such as simple decoding and rate. In addition we propose our scheme in such a way that it only uses the minimum number of transmit antennas needed to achieve maximum diversity.

Chapter 2

Space-time block codes

2.1 System model and design criterion

We consider a communication system with N transmit and M receive antennas. The fading coefficient \mathbf{h}_{ij} , which forms channel matrix \mathbf{H} , is the complex path gain from transmit antenna i to receive antenna j . We assume that the coefficients are independently Normally distributed with unit variance. Channel matrix \mathbf{H} is assumed to be known to the receiver, but not at the transmitter. We also assume that \mathbf{H} remains constant within a block of T symbols. With these assumptions, we have the following expression for the received vector \mathbf{r} :

$$\mathbf{r} = \mathbf{C}\mathbf{H} + \mathbf{N} \tag{2.1}$$

where \mathbf{N} is an independent zero mean Gaussian noise vector, with variance normalized to give out the desired signal to noise ratio. The matrix \mathbf{C} stands for a $T \times N$ block of data

$$\mathbf{C} = \begin{pmatrix} C_{1,1} & C_{1,2} & \dots & C_{1,N} \\ C_{2,1} & C_{2,2} & \dots & C_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ C_{T,1} & C_{T,2} & \dots & C_{T,N} \end{pmatrix} \quad (2.2)$$

In order to come up with a design criterion, first we need to quantify the effects of mistaking two codewords with each other. In the case of a space-time code, a codeword is a $T \times N$ matrix given by Equation (2.2). Let us assume that we transmit a codeword \mathbf{C} as shown by the above equation. An error occurs if the decoder mistakenly decides that we have transmitted another codeword, for example \mathbf{E} .

$$\mathbf{E} = \begin{pmatrix} E_{1,1} & E_{1,2} & \dots & E_{1,N} \\ E_{2,1} & E_{2,2} & \dots & E_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ E_{T,1} & E_{T,2} & \dots & E_{T,N} \end{pmatrix} \quad (2.3)$$

If the codebook (the set of all codewords) only contains \mathbf{C} and \mathbf{E} , we denote the pairwise error probability of transmitting \mathbf{C} and detecting it as \mathbf{E} by $P(\mathbf{C} \rightarrow \mathbf{E})$. Note that in general when the codebook contains I codewords, using the union bound, the

probability of error when we transmit \mathbf{C} is upper bounded by

$$P(\text{error}|\mathbf{C} \text{ is sent}) = \bigcup_{\mathbf{E}_i \neq \mathbf{C}} P(\mathbf{C} \rightarrow \mathbf{E}_i) \quad (2.4)$$

The overall error probability can be calculated (bounded) by conditioning on the transmitted codeword. In what follows we calculate the pairwise error probability $P(\mathbf{C} \rightarrow \mathbf{E})$ and use it to define the design criteria. In order to calculate the pairwise error probability, first we assume a fixed known channel matrix \mathbf{H} and then calculate the average error by computing the expected value over the distribution of the \mathbf{H} . The average symbol transmission power from each antenna is $E_S = \frac{1}{N}$ and the variance of a noise sample is $E[|n_i|^2] = N_0 = \frac{1}{SNR}$. We consider the distribution of the received signal for a known codeword \mathbf{C} and channel matrix \mathbf{H} , i.e. $f(\mathbf{C}, \mathbf{H})$. Note that the linear combination of independent Gaussian random variables is a Gaussian random variable. Since we assume a Gaussian noise \mathbf{N} with independent components, for a fixed \mathbf{C} and \mathbf{H} , the received vector \mathbf{r} is also a Gaussian random variable. Therefore,

$$f(\mathbf{C}, \mathbf{H}) = \frac{1}{(\pi N_0)^{\frac{M \times M}{2}}} \exp \left\{ \frac{-\text{Tr}[(\mathbf{r} - \mathbf{C} \cdot \mathbf{H})^H (\mathbf{r} - \mathbf{C} \cdot \mathbf{H})]}{N_0} \right\} = \frac{1}{(\pi N_0)^{\frac{M \times M}{2}}} \exp \left\{ \frac{\|\mathbf{r} - \mathbf{C} \cdot \mathbf{H}\|_F^2}{N_0} \right\} \quad (2.5)$$

where $\text{Tr}(\mathbf{A})$ is the trace of a matrix \mathbf{A} and $\|\cdot\|_F$ is the Frobenius norm operator.

Maximum-likelihood decoding decides in favor of a codeword that maximizes $f(\mathbf{C}, \mathbf{H})$.

Let us assume that we transmit \mathbf{C} . The received vector will be $\mathbf{r} = \mathbf{C} \cdot \mathbf{H} + \mathbf{N}$ and given the channel matrix \mathbf{H} , the pairwise error probability is calculated by

$$P(\mathbf{C} \rightarrow \mathbf{E}|\mathbf{H}) = \mathbf{P}(\|\mathbf{r} - \mathbf{C} \cdot \mathbf{H}\|_F^2 - \|\mathbf{r} - \mathbf{E} \cdot \mathbf{H}\|_F^2 > 0) \quad (2.6)$$

We rewrite Equation (2.6) to calculate the pairwise error probability as follows:

$$\begin{aligned}
& P(\mathbf{C} \rightarrow \mathbf{E}|\mathbf{H}) \\
& = P(\text{Tr}[(\mathbf{r}-\mathbf{C}\cdot\mathbf{H})^{\mathbf{H}}(\mathbf{r}-\mathbf{C}\cdot\mathbf{H})-(\mathbf{r}-\mathbf{E}\cdot\mathbf{H})^{\mathbf{H}}(\mathbf{r}-\mathbf{E}\cdot\mathbf{H})] > 0) \\
& = P(\text{Tr}[(\mathbf{C} - \mathbf{E})\cdot\mathbf{H} + \mathbf{N})^{\mathbf{H}}((\mathbf{C} - \mathbf{E})\cdot\mathbf{H} + \mathbf{N})] - \mathbf{N}^{\mathbf{H}}\cdot\mathbf{N} < 0) \\
& = P(\text{Tr}[(\mathbf{H}^{\mathbf{H}}\cdot(\mathbf{C} - \mathbf{E})^{\mathbf{H}}(\mathbf{C} - \mathbf{E})\cdot\mathbf{H}) - \mathbf{X}] < 0) \\
& = P(\mathbf{X} > \|\mathbf{C} - \mathbf{E}\cdot\mathbf{H}\|_F^2) \tag{2.7}
\end{aligned}$$

where $\mathbf{X} = \text{Tr}[(\mathbf{H}^{\mathbf{H}}\cdot(\mathbf{C} - \mathbf{E})^{\mathbf{H}}\cdot\mathbf{N} + \mathbf{N}\cdot(\mathbf{C} - \mathbf{E})\cdot\mathbf{H})]$ is a zero mean Gaussian random variable with variance $2N_0\|\mathbf{C} - \mathbf{E}\cdot\mathbf{H}\|_F^2$. Therefore, one can calculate the pairwise error probability using the Q function.

$$P(\mathbf{C} \rightarrow \mathbf{E}|\mathbf{H}) = Q\left(\frac{\|\mathbf{C} - \mathbf{E}\cdot\mathbf{H}\|_F^2}{\sqrt{2N_0\|\mathbf{C} - \mathbf{E}\cdot\mathbf{H}\|_F^2}}\right) = Q\left(\sqrt{\frac{1}{2N_0}}\|\mathbf{C} - \mathbf{E}\cdot\mathbf{H}\|_F\right) \tag{2.8}$$

Therefore, it remains to calculate $\|\mathbf{C} - \mathbf{E}\cdot\mathbf{H}\|_F$ to derive the conditional pairwise error probability. Let us define the matrix $\mathbf{A}(\mathbf{C}, \mathbf{E}) = (\mathbf{C} - \mathbf{E})^{\mathbf{H}}\cdot(\mathbf{C} - \mathbf{E})$. We know that eigenvalues of \mathbf{A} are nonnegative real numbers $\lambda_1, \lambda_2, \dots, \lambda_N$. Using the singular value decomposition theorem, we have

$$\mathbf{A}(\mathbf{C}, \mathbf{E}) = \mathbf{V}^{\mathbf{H}}\cdot\mathbf{\Lambda}\cdot\mathbf{V} \tag{2.9}$$

where $\mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_N)$. Therefore, $\|\mathbf{C} - \mathbf{E}\cdot\mathbf{H}\|_F^2 = \text{Tr}[\mathbf{H}^{\mathbf{H}}\cdot\mathbf{A}(\mathbf{C}, \mathbf{E})\cdot\mathbf{H}] = \text{Tr}[\mathbf{H}^{\mathbf{H}}\cdot\mathbf{V}^{\mathbf{H}}\cdot\mathbf{\Lambda}\cdot\mathbf{V}\cdot\mathbf{H}]$. Since the elements of \mathbf{H} are independent Gaussian random variables, the elements of $\mathbf{V}\cdot\mathbf{H}$ are also Gaussian. We denote the (n,m)th element of $\mathbf{V}\cdot\mathbf{H}$ by β_{mn} . Therefore,

$$\begin{aligned}
\| (\mathbf{C} - \mathbf{E}) \cdot \mathbf{H} \|_F^2 &= \text{Tr} \left[\begin{pmatrix} \beta_{1,1}^* & \beta_{1,2}^* & \dots & \beta_{1,N}^* \\ \beta_{2,1}^* & \beta_{2,2}^* & \dots & \beta_{2,N}^* \\ \dots & \dots & \dots & \dots \\ \beta_{M,1}^* & \beta_{M,2}^* & \dots & \beta_{M,N}^* \end{pmatrix} \right. \\
&\quad \left. \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_N \end{pmatrix} \cdot \begin{pmatrix} \beta_{1,1} & \beta_{2,1} & \dots & \beta_{M,1} \\ \beta_{1,2} & \beta_{2,2} & \dots & \beta_{M,2} \\ \dots & \dots & \dots & \dots \\ \beta_{1,N} & \beta_{2,N} & \dots & \beta_{M,N} \end{pmatrix} \right] = \\
&\quad \text{Tr} \left[\begin{pmatrix} \sum_{n=1}^N \lambda_n |\beta_{n,1}|^2 & \dots & \dots \\ \dots & \sum_{n=1}^N \lambda_n |\beta_{n,2}|^2 & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots & \sum_{n=1}^N \lambda_n |\beta_{n,M}|^2 \end{pmatrix} \right] \tag{2.10}
\end{aligned}$$

Calculating the trace in the last equation results in

$$\| (\mathbf{C} - \mathbf{E}) \cdot \mathbf{H} \|_F^2 = \sum_{m=1}^M \sum_{n=1}^N \lambda_n |\beta_{n,m}|^2 \tag{2.11}$$

Applying Equation (2.11) in Equation (2.8) results in

$$P(\mathbf{C} \rightarrow \mathbf{E} | \mathbf{H}) = Q \left(\sqrt{\frac{1}{2N_0} \sum_{m=1}^M \sum_{n=1}^N \lambda_n |\beta_{n,m}|^2} \right) \tag{2.12}$$

Using the famous upper bound for Q function saying $Q(x) \leq \frac{1}{2} e^{-\frac{x^2}{2}}$ we can calculate

an upper bound on the conditional pairwise error probability as follows

$$P(\mathbf{C} \rightarrow \mathbf{E}|\mathbf{H}) \leq \frac{1}{2} \exp \left(-\frac{1}{4N_0} \sum_{m=1}^M \sum_{n=1}^N \lambda_n |\beta_{n,m}|^2 \right) \quad (2.13)$$

Knowing that $|\beta_{n,m}|$ is Rayleigh distributed, we can calculate the expected value of the pairwise error probability.

$$P(\mathbf{C} \rightarrow \mathbf{E}) = E[P(\mathbf{C} \rightarrow \mathbf{E}|\mathbf{H})] \leq \frac{1}{\prod_{n=1}^N [1 + (\lambda_n/4N_0)]^M} \quad (2.14)$$

We assume that $\mathbf{A}(\mathbf{C},\mathbf{E})$ is of rank r , i.e there are r nonzero eigenvalues for \mathbf{A} . At high SNRs, we can neglect the 1 in the denominator of the Inequality(2.14). Remembering $N_0 = \frac{1}{SNR}$ and SNR we can write the following upper bound:

$$P(\mathbf{C} \rightarrow \mathbf{E}) \leq \frac{4^{rM}}{(\prod_{n=1}^r \lambda_n)^M SNR^{rM}} \quad (2.15)$$

From the above upper bound, we can define the diversity gain of the code as the exponent of SNR, i.e. rM , and coding gain as the multiplication of nonzero eigenvalues. Therefore a good design criterion to guarantee full diversity is to make sure that for all possible codewords \mathbf{C} and \mathbf{E} the matrix $\mathbf{A}(\mathbf{C},\mathbf{E})$ is full rank [1]. Then to increase the coding gain for a full diversity code, an additional good design criterion is to maximize the minimum determinant of matrices $\mathbf{A}(\mathbf{C},\mathbf{E})$ for all $\mathbf{C} \neq \mathbf{E}$. The above two criteria for designing space-time codes are called rank and determinant criterion respectively.

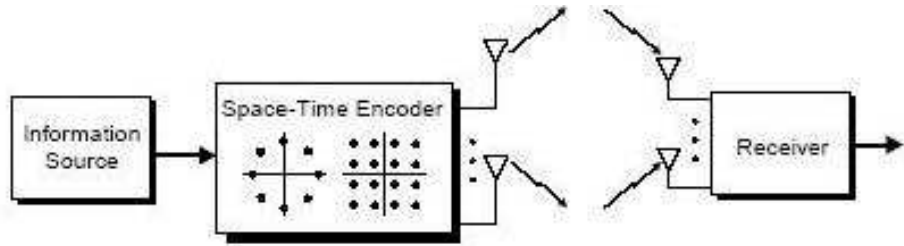


Figure 2.1: *Space – time coding block diagram*

2.2 Alamouti code and orthogonal space time block codes

We start our discussion of space-time block coding with a simple example. Let us assume a system with $N = 2$ transmit antennas and one receive antenna.

Figure 2.1 shows the encoder block diagram of such an example for a system called Alamouti code [2], [3]. To transmit b bits/channel use, we use a modulation scheme that maps every b bits to one symbol from a constellation with $2b$ symbols. The constellation can be any real or complex constellation, for example PAM, PSK, QAM, and so on. First, the transmitter picks two symbols from the constellation using a block of $2b$ bits. If s_1 and s_2 are the selected symbols for a block of $2b$ bits, the transmitter sends s_1 from antenna one and s_2 from antenna two at time one. Then at time two, it transmits $-s_2^*$ and s_1^* from antennas one and two, respectively.

Therefore, the transmitted codeword is

$$\mathbf{C} = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix} \quad (2.16)$$

To check the diversity order of the code, we need to calculate the rank of all possible difference matrices $\mathbf{A}(\mathbf{C}, \mathbf{E})$ for all $\mathbf{C} \neq \mathbf{E}$. Let us consider a different pair of symbols (e_1, e_2) and the corresponding codeword

$$\mathbf{E} = \begin{pmatrix} e_1 & e_2 \\ -e_2^* & e_1^* \end{pmatrix} \quad (2.17)$$

Then $\det[\mathbf{A}(\mathbf{C}, \mathbf{E})] = |s_1 - e_1|^2 + |s_2 - e_2|^2$ is zero if and only if $s_1 = e_1$ and $s_2 = e_2$.

Therefore, $\mathbf{A}(\mathbf{C}, \mathbf{E})$ is always full rank when $\mathbf{C} \neq \mathbf{E}$ and the Alamouti code satisfies the determinant criterion. It provides a diversity of $2M$ for M receive antennas and therefore is a full diversity code. Note that the code transmits one symbol, b bits, per time slot. This is the maximum number of possible symbols for a full diversity code [1]. Let us assume that the path gains from transmit antennas one and two to the receive antenna are $h_{11} = \alpha_1$ and $h_{21} = \alpha_2$, respectively. Then, based on our model in Equation (2.1), the decoder receives signals r_1 and r_2 respectively at times one and two such that

$$\begin{aligned} r_1 &= \alpha_1 s_1 + \alpha_2 s_2 + n_1 \\ r_2 &= -\alpha_1 s_2^* + \alpha_2 s_1^* + n_2 \end{aligned} \quad (2.18)$$

For a coherent detection scheme where the receiver knows the channel path gains α_1

and α_2 , the maximum-likelihood detection requires minimizing the decision metric

$$|r_1 - \alpha_1 s_1 - \alpha_2 s_2|^2 + |r_2 + \alpha_1 s_2^* - \alpha_2 s_1^*|^2 \quad (2.19)$$

for all possible values of s_1 and s_2 . Such a decoding requires a full search over all possible pairs (s_1, s_2) and in general its complexity grows exponentially by the number of transmit antennas. Expanding the cost function (2.19), we can decompose the expression to

$$|s_1|^2 \sum_{n=1}^2 |\alpha_n|^2 - [r_1 \alpha_1^* s_1^* + r_1^* \alpha_1 s_1 + r_2 \alpha_2^* s_1 + r_2 \alpha_2^* s_1^*], \quad (2.20)$$

which is only a function of s_1 , and the other one

$$|s_2|^2 \sum_{n=1}^2 |\alpha_n|^2 - [r_2 \alpha_2^* s_2^* + r_2^* \alpha_2 s_1 + r_1 \alpha_1^* s_2 + r_1 \alpha_1^* s_2^*], \quad (2.21)$$

that is only a function of s_2 . Therefore in the receiver we can minimize these sub-functions separately which results in separate decoding of the two symbols. So the decoding complexity of the code increases linearly, instead of exponentially, by the number of transmit antennas. In addition, if all the constellation symbols have equal energy, the first term in both sub-functions is the same for all possible values of s_1 and s_2 and can be removed from the cost sub-functions. Note that we use maximum ratio combining for the maximum-likelihood decoding with more than one receive antenna. In this case all the above formulas are valid when every cost function is the sum of the corresponding cost functions for each receive antenna.

The scheme presented above provided us two important properties of simple decoding and full diversity. A question that may come to mind is: Is it possible to

design similar codes for more number of transmit antennas? In [3] authors answered this question. Basically they have shown that the good properties mentioned about Alamouti scheme come from the fact that in that code we have

$$\mathbf{C}^H \cdot \mathbf{C} = (|s_1|^2 + |s_2|^2)I_2 \quad (2.22)$$

It is easily seen that every matrix with this property can be a generator of a code that provides full diversity as well as simple decoding. The next natural step is to investigate the possibility of similar generator matrices for more than two transmit antennas. For real numbers, matrices satisfying Equation (2.22) are called orthogonal designs. These matrices have been studied completely by Hurwitz and Radon [8]. It has been shown in [8] that square orthogonal designs only exist for $N=2,4,8$, where N is the dimension of the square matrix. This means that the original designs offered in mathematical literature only provide an orthogonal code for the case of $N=2,4,8$ transmit antennas. Therefore authors of [3] generalized the meaning of an orthogonal design to non-square matrices. They showed for every N , there exists T where the $T \times N$ matrix \mathbf{C} with real entries $s_1, -s_1, s_2, -s_2, \dots, s_K, -s_K$ such that

$$\mathbf{C}^T \cdot \mathbf{C} = \kappa(s_1^2 + s_2^2 + \dots s_K^2)I_N, \quad (2.23)$$

where I_N is the $N \times N$ identity matrix and κ is a constant. Defining rate of the code as $R = \frac{K}{T}$ they have shown that the matrix found to satisfy the equation has the advantage of full rate or $R=1$. In addition they went further and generalized their definition to include complex matrices. They defined a generalized complex orthogonal design as

a $T \times N$ matrix \mathbf{G} with complex entries $\pm s_1, \pm s_1^*, \pm s_2, \pm s_2^*, \dots, \pm s_K, \pm s_K^*$ such that

$$\mathbf{C}^H \mathbf{C} = \kappa(|s_1|^2 + |s_2|^2 + \dots + |s_K|^2)I_N, \quad (2.24)$$

A matrix that satisfies the above requirement can be the generator matrix for a complex code that provides full diversity as well as simple maximum-likelihood decoding. Unfortunately it was shown that such codes are limited to rate $\frac{3}{4}$ for more than two antennas [4]. In the original paper a scheme was proposed that guaranteed rate $\frac{1}{2}$ for any number of transmit antennas.

2.3 Quasi-orthogonal space-time block code

In the last section we noticed the two good properties of orthogonal space-time block codes (OSTBC's), i.e. simple maximum-likelihood decoding and full diversity. Unfortunately there is a shortcoming with this class of codes which is the lack of full rate. A lot of effort had been made to find orthogonal designs with highest rates in a systematic way [9]. However, unfortunately the innate limitation of $\frac{3}{4}$ [4] for rate had prevented designers from more efforts. Therefore to reach a higher rate, one should change the structure of the orthogonal code, e.g. relaxing one of the properties of an orthogonal design. For example we can think of designing a full-diversity rate one code that does not have the property of single maximum likelihood decoding. Increasing the order of complexity of decoding by one from separate decoding we get to pairwise decoding; meaning that each two symbols should be detected independent

of other pairs. The first such design was offered by Jafarkhani in [5] as the following matrix.

$$\mathbf{C} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{pmatrix} \quad (2.25)$$

The minimum rank of the $\mathbf{A}(\mathbf{C}, \mathbf{E})$ is two for $\mathbf{C} \neq \mathbf{E}$ matrices. The design was called a *Quasi Orthogonal Space-Time Block Code*. The reason was that in the new design each column of the generator matrix was orthogonal to all the others except one. Remembering the fact that in the orthogonal designs each column was orthogonal to all others tells us why this name was chosen by the author. Later in [6] Su *et al.* used rotated constellation for two of the symbols and achieved diversity of four in their design.

Let us now investigate the concept of pairwise decoding. Similar to decoding formulas in previous section, the maximum-likelihood decoding for the QOSTBC is

$$\min \mathbf{H}^H \cdot \mathbf{C}^H \cdot \mathbf{C} \cdot \mathbf{H} - \mathbf{H}^H \cdot \mathbf{C}^H \cdot \mathbf{r} - \mathbf{r}^H \cdot \mathbf{C} \cdot \mathbf{H} \quad (2.26)$$

where \mathbf{C} is derived by replacing x_k by s_k in (2.25). Simple algebraic manipulation shows that maximum-likelihood decoding is equal to minimizing the following sum:

$$f_{14}(s_1, s_4) + f_{23}(s_2, s_3) \quad (2.27)$$

where

$$\begin{aligned}
f_{14}(s_1, s_4) &= \sum_{m=1}^M [(|s_1|^2 + |s_4|^2)(\sum_{n=1}^4 |\alpha_{n,m}|^2) + \\
& 2\text{Re}\{(-\alpha_{1,m}r_{1,m}^* - \alpha_{2,m}^*r_{2,m} - \alpha_{3,m}^*r_{3,m} - \alpha_{4,m}r_{4,m}^*)s_1 + \\
& 2\text{Re}\{(-\alpha_{4,m}r_{1,m}^* + \alpha_{3,m}^*r_{2,m} + \alpha_{2,m}^*r_{3,m} - \alpha_{1,m}r_{4,m}^*)s_4\} \\
& 4 + \text{Re}\{\alpha_{1,m}\alpha_{4,m}^* - \alpha_{2,m}^*\alpha_{3,m}\}\text{Re}\{s_1s_4^*\}],
\end{aligned} \tag{2.28}$$

and

$$\begin{aligned}
f_{23}(s_2, s_3) &= \sum_{m=1}^M [(|s_2|^2 + |s_3|^2)(\sum_{n=1}^4 |\alpha_{n,m}|^2) + \\
& 2\text{Re}\{(-\alpha_{2,m}r_{1,m}^* + \alpha_{1,m}^*r_{2,m} - \alpha_{4,m}^*r_{3,m} + \alpha_{3,m}r_{4,m}^*)s_1 + \\
& 2\text{Re}\{(-\alpha_{3,m}r_{1,m}^* - \alpha_{4,m}^*r_{2,m} + \alpha_{1,m}^*r_{3,m} + \alpha_{1,m}r_{4,m}^*)s_4\} \\
& + 4\text{Re}\{\alpha_{2,m}\alpha_{3,m}^* - \alpha_{1,m}^*\alpha_{4,m}\}\text{Re}\{s_2s_3^*\}].
\end{aligned} \tag{2.29}$$

Since $f_{14}(s_1, s_4)$ is independent of (s_2, s_3) and $f_{23}(s_2, s_3)$ is independent of (s_1, s_4) , the pairs (s_1, s_4) and (s_2, s_3) can be decoded separately. Therefore, we can say this scheme is pairwise decodable.

Chapter 3

Full-diversity rate one QOSTBC's are limited to 4 transmit antennas

3.1 Introduction

As was discussed in the last chapter due to rate limitations in OSTBC's efforts have been made, and eventually in [5] a new class of codes named "Quasi-Orthogonal Space-time Block Codes" (QOSTBCs) was introduced where, for four transmit antennas full rate was achieved but at the expense of increasing the complexity from single decoding to pairwise decoding. Next, it was shown that by using rotated constellations it is also possible to achieve full diversity as well as full rate [6]. Searching for QOSTBCs for more number of antennas is naturally the next step. Recently there has been some effort on finding upper bounds for QOSTBCs [10], or proposing full

rate QOSTBCs [11].

Previous work on the rate bound of QOSTBC only considered the square code matrix, i.e. schemes having time delay equal to the number of transmit antennas. Also they lacked the diversity discussion which is of major concern for any subclass of STBCs. Therefore we were motivated to investigate the problem in a more general framework. Also we figured a misconception in the interpretation of the term *rate* in the context of block codes. In this chapter we first discuss the original meaning of rate in a block code and then based on that show that rate one full-diversity, QOSTBC does not exist for more than four transmit antennas, disregarding any time delay. The system model will be the same as the previous chapter. In [1], it is proved that for a code, using a constellation with 2^b members, it is possible to have a full-diversity code, with a rate of at most b bits per channel use. Equivalently the best we can do when we have full diversity is to send one average constellation member per time slot. In Theorem 3.3.1 of that paper, the authors use the term 2^b as the number of different symbols we could use for each entry of the code matrix. Also in the proof they do not assume that the constellation we use should be the same for all code entries. Therefore the result of the theorem is that *maximum rate of a full-diversity modulation scheme is when bit rate is equal to the \log_2 of the number of symbols we can send in the channel at each time from each transmit antenna* . In original OSTBCs or QOSTBCs, code rate is an indication of bit rate. That is, for example for a rate one code, if we consider each element of the code matrix, bit rate

is equal to \log_2 of the number of choices we have for that specific element. Now let us consider the following code matrix [13]:

$$\begin{pmatrix} x_1^R + jx_3^R & x_2^R + jx_4^R & -x_1^I + jx_3^I & -x_2^I + jx_4^I \\ -x_2^R + jx_4^R & x_1^R - jx_3^R & x_2^I + jx_4^I & -x_1^I - jx_3^I \\ -x_1^I + jx_3^I & -x_2^I + jx_4^I & x_1^R + jx_3^R & x_2^R + jx_4^R \\ x_2^I + jx_4^I & -x_1^I - jx_3^I & -x_2^R + jx_4^R & x_1^R - jx_3^R \end{pmatrix} \quad (3.1)$$

where the transmitted symbols are $x_q = x_q^R + x_q^I$, and superscripts $()^R$ and $()^I$ denote the real and imaginary parts of a complex element respectively. Suppose that x_1 and x_2 are chosen from 4-QAM, and x_3 and x_4 are chosen from a rotated version of 4-QAM.¹ Real and imaginary part of a 4-QAM signal can take 2 different values. However a signal from a rotated 4-QAM constellation, necessarily takes 3 different numbers either as its real or imaginary part. Therefore each entry of this matrix can get 6 different complex values. So equivalently, this block code is using a constellation of 6 members. Therefore, for this code, the upper bound predicted by the aforementioned theorem when providing full diversity is \log_2^6 . Also any other block code that uses a combination of symbols as its entries, is susceptible to expand its constellation and therefore have a different potential rate upper bound. In other words, in these codes, the original code rate in [1] is not equivalent to "bit rate" and therefore is not as relevant. The discussion here was not meant to suppress the works [11]-[13]. Since those designs use transformation to gain other good properties of the

¹We have to use rotated constellation to get full diversity.

code, e.g. single ML decoding. However we need to point out that we should not think of those codes as full-rate anymore; there is still room to get higher rates. For example when we use linear combination of two symbols at each entry of our code, the upper bound predicted is 2 symbols per channel use, for a full-diversity code. Since for every transmitted signal we have a choice of $2^b \times 2^b$ different combinations. This upper bound had been achieved in [14].²

Note that codes that use rotated constellation for specific symbols of their code matrix [6], or those using expanded block matrices [15],[16] but not expanded symbol members, do not fall into this category. That is, in these schemes, we have still 2^b choices for each transmitted entry of the code.

In this work, we now consider block codes that have only negative, conjugate, or negative conjugate of the original symbols in their entries as defined in [3]. As we know these transforms do not expand cardinality of the well known constellations e.g. PSK, QAM, PAM, and so on. The reason we emphasize on cardinality is because when talking about rate, we do not care about the choices of each transmitted signal, but we do care about the number of choices we have for that specific signal.

²Of course we shall not mix the concept of spatial multiplexing discussed in that paper with the term rate defined here.

3.2 Proof of non-existence

Recalling the system model from Equation (2.1), we consider the following maximum-likelihood (ML) decoding metric [1]:

$$\text{Tr}\{[\mathbf{r} - \mathbf{C} \cdot \mathbf{H}]^H \cdot [\mathbf{r} - \mathbf{C} \cdot \mathbf{H}]\}. \quad (3.2)$$

where Tr is the trace operator.

The codeword that minimizes (3.2) is the most likely codeword. If we expand this expression, the only term that may cause multiplication of different entries of \mathbf{C} together and therefore increase decoding complexity is $\text{Tr}\{\mathbf{H}^H \cdot \mathbf{C}^H \cdot \mathbf{C} \cdot \mathbf{H}\}$. Since realizations of \mathbf{H} include all complex numbers, it can be shown that pairwise decodability of $\text{Tr}\{\mathbf{H}^H \cdot \mathbf{C}^H \cdot \mathbf{C} \cdot \mathbf{H}\}$ is equivalent to that of $\mathbf{C}^H \cdot \mathbf{C}$. So from now on, when we say block code \mathbf{C} , is pairwise decodable, we mean all entries of $\mathbf{C}^H \cdot \mathbf{C}$ are made of specific fixed symbol pairs multiplied together, or any function of single symbols. Therefore when we search for symbols in the constellation to minimize the metric, we have an algorithm with complexity order of s^2 , where s is the cardinality of the constellation set being used.

The entries of $\mathbf{C}^H \cdot \mathbf{C}$ are complex inner product of columns of matrix \mathbf{C} with one another. For example any diagonal entry of $\mathbf{C}^H \cdot \mathbf{C}$ is produced by inner product of a column by itself. Conditioned that entries of \mathbf{C} are x_q , complex conjugate, negative, or negative conjugate of that, these diagonal entries of $\mathbf{C}^H \cdot \mathbf{C}$ are not multiplication of two different symbols. So for \mathbf{C} to be pairwise decodable, all the inner products

of different columns of \mathbf{C} - which make non-diagonal entries of $\mathbf{C}^H \cdot \mathbf{C}$ - should only produce zero or sum of multiplication of specific pairs, or functions of single symbols. These pairs should be made of same fixed symbols wherever they appear, ignoring the conjugate or sign.

Theorem: A rate 1, pairwise decodable matrix for five or more transmit antennas that provides full diversity does not exist.

Below comes some lemmas to help us prove this theorem.

Lemma 1: For a rate one code matrix, to have full diversity, each symbol should appear exactly once at every column of the code matrix. That is we should have no zero entry, or symbol that appears more than once.

Proof: Assume the contrary; that in one of the columns of the rate one full-diversity code matrix \mathbf{C} we do not see a specific symbol namely x_i . Now remember the rank criterion from [1], in which we should consider the rank of all difference matrices $\mathbf{D}=\mathbf{C}-\mathbf{E}$. The difference matrix that we are interested in is the one in which all symbols in \mathbf{C} and \mathbf{E} are the same except for x_i . So the difference matrix will have one zero column (the column in \mathbf{C} that does not include symbol x_i) and therefore \mathbf{D} cannot be full rank. As a result the code matrix \mathbf{C} does not provide full diversity. Therefore in every column we should have at least one at every symbol. Also to have full rate, the number of time slots (rows) should be equal to the number of symbols; so having at least one from all the symbols in a column is equivalent to having them only once. □

Lemma 2: For a rate one code, to have full diversity, no symbol should appear in the same row in two different columns.

Proof: Suppose the contrary, i.e. assume that for symbol x_i , there are two columns that have this symbol (or its conjugate) in the same row. Again we construct a difference matrix \mathbf{D} , the same as what we created in the proof of Lemma 1. In this difference matrix all the entries are zero except those that include symbol x_i in \mathbf{C} . Therefore considering the fact that at every column and specifically in the two aforementioned columns, there is only one x_i , we have two columns (vectors) that are in the form of $\mathbf{u}_1 = \alpha \mathbf{u}_2$, where α is $\pm \frac{x_i}{x_i^*}$ or $\pm \frac{x_i^*}{x_i}$ or ± 1 . Therefore the two columns are linearly dependent and the matrix including these two columns cannot be full rank. So the code does not provide full diversity. Note that in this case even if we use rotated constellations for a symbol in one of the two columns, the result will not change. This is due to the fact that the two vectors (columns) of matrix will be still in the form of $\mathbf{u}_1 = \beta \mathbf{u}_2$ and this causes decrease in rank. □

Corollary: To have both pairwise decoding and full-rank property, non-diagonal entries of $\mathbf{C}^H \cdot \mathbf{C}$ should be in the form of pairs and not functions of single symbols. Recall that non-diagonal entries of this matrix are the probable cause of non-pairwise decodability.

Lemma 3: In a rate one code, which is a matrix made of x_q 's or their conjugates

or negatives, we cannot have three orthogonal columns. By orthogonality we mean that the complex inner product of the two columns be zero for all complex values of x_q 's.

Proof: Existence of these three columns is equivalent to having a rate one orthogonal space time block code for 3 transmit antennas, which is shown to be impossible in [1],[3]. □

Lemma 4: In $\mathbf{C}^H \cdot \mathbf{C}$, where \mathbf{C} is a full-diversity, rate one pairwise decodable matrix, any pair should appear at most in 4 entries, and these entries, if any, should not be in the same row or column.

Proof: First we should notice that, a pair cannot appear twice in a row or in a column of $\mathbf{C}^H \cdot \mathbf{C}$. Since every row or column of $\mathbf{C}^H \cdot \mathbf{C}$ is made by inner product of one column of \mathbf{C} by all other columns, the claimed expression is equivalent to having a symbol of that pair in same rows of two different columns of \mathbf{C} , and is in contradiction with Lemma 3.

Now we show why there should be at most 4 entries, holding each specific pair. Because of the hermitian property of $\mathbf{C}^H \cdot \mathbf{C}$, having a pair in an entry of this matrix is equivalent to having it in the symmetric entry of the very matrix. Also we know that no pair comes in a diagonal entry; therefore, we should have even number of pairs in $\mathbf{C}^H \cdot \mathbf{C}$. As a result, having any pair in more than four entries, means having it at least six times. It is impossible to have six entries in a 5×5 matrix that are not in the same

column or row. □

In the next two independent lemmas we investigate the properties of vanishing and non-vanishing pairs. They will be used in the body of the main proof.

Lemma 5: To have a vanishing pair in the inner product of two different columns that have no zeros as their entry, it is necessary that the number of overall conjugate signs in two columns, of each two symbol indices in the pair should be equal to 1. In addition the number of negative signs among them should be an odd number. That is to say, if a pair of symbols, namely x_i and x_j in an inner product are to vanish, then we should have exactly one x_i symbol out of the two having a conjugate sign, and one x_j symbol out of the two that has a conjugate sign. Also, one or three of the overall four symbols in two columns should have negative signs.

Proof: A vanishing pair in two columns makes an *Alamouti* code [2], and the claimed property is easily inspected in the list of all forms of these 2×2 codes in [15]. □

Lemma 6: For every three columns made of four symbols, that are chosen from \mathbf{C} , if one column is orthogonal to the other two, then those two columns should create two pairs in their inner product.

Proof: First we notice that because of Lemma 3, those two columns, i.e. the second and third columns, cannot be orthogonal and therefore their inner product gives us at least one pair. The reason why it should create the other pair is discussed below.

Let us first assume that these columns are made of $x_1, x_2, x_3,$ and x_4 . We know that the first column is orthogonal to the second and third columns. We show that any two symbols that are in the same row in column two and three, make a pair in the inner product of the two columns and cannot be vanishing pairs. Pick one of the rows, that includes x_i in column one, x_j in column two, and x_k in column three, where i, j, k are distinct and are chosen from $\{1, 2, 3, 4\}$. Our goal is to show that x_j and x_k are not vanishing pairs. Because of the orthogonality of columns one and two, x_i and x_j are vanishing pairs in the inner product of these two columns. Therefore The number of overall x_j s in columns one and two that have conjugate sign is equal to 1. Also because of orthogonality of columns one and three, x_j should be a vanishing pair in the inner product of these columns; which means the overall number of x_j 's in columns one and three that have conjugate sign is equal to 1. From these two statements we conclude that the number of x_j 's that have a conjugate sign is equal in columns two and three. So the overall number of conjugate signs for this symbol in the last two columns cannot be equal to 1. Therefore, because of Lemma 5 x_j cannot be at any vanishing pair in the inner product of the last two columns. The same reasoning applies to any other symbol in the last two columns. This proves that all the symbols give non-vanishing pairs and since the columns were chosen from \mathbf{C} they should provide two pairs. □

Proof of the theorem: In Lemma 4 we proved that if there exists a rate one, full-diversity, pairwise decodable $T \times 5$ code \mathbf{C} , any pair should at most appear four times

in $\mathbf{C}^H \cdot \mathbf{C}$. Recalling that every entry of $\mathbf{C}^H \cdot \mathbf{C}$ is caused by inner product of two columns of \mathbf{C} , the statement in Lemma 4 is equivalent to saying that each pair is appeared in at most two inner products of distinct columns of \mathbf{C} . Distinctness of the columns comes from the fact that pairs are seen in distinct rows and columns in $\mathbf{C}^H \cdot \mathbf{C}$. Let us choose a pair that appears in two inner products. This should be possible, because if not, all pairs are seen at most in one inner product, and then we can remove two columns from \mathbf{C} in such a way that in the inner product of the remaining three at least two pairs are not seen. Therefore the three subcolumns including the rows where these pairs appear should be orthogonal, and this contradicts Lemma 3.

Since we should have a pair that appears in two inner products, without loss of generality, we assume that $x_1 x_{T+1}$ is that specific pair. This is always possible with a simple permutation of the symbol indices of symbols. Consider the four columns that are involved in the inner products producing this pair. Name them as \mathbf{u}_1 , \mathbf{u}_2 , \mathbf{u}_3 , and \mathbf{u}_4 , in such a way that the mentioned inner products giving out two pairs be $\langle \mathbf{u}_1, \mathbf{u}_2 \rangle$ and $\langle \mathbf{u}_3, \mathbf{u}_4 \rangle$. Designate all the rows that have x_1 and x_{T+1} among these four columns. There are two in \mathbf{u}_1 and \mathbf{u}_2 and two in \mathbf{u}_3 and \mathbf{u}_4 . By extracting the symbols that are located in these rows in the four columns, make four subcolumns that each of them has four rows. Now consider the third subcolumn and look at those rows that do not have x_1 and x_{T+1} in them. Let us call these symbols x_i and x_j . Since $x_1 x_{T+1}$ pair should be vanishing in $\langle \mathbf{u}_1, \mathbf{u}_3 \rangle$ and $\langle \mathbf{u}_2, \mathbf{u}_3 \rangle$, we conclude that the first and second subcolumns should also have x_i and x_j as their other two symbols. (They

already had x_1 and x_{T+1} .) In a similar manner we can show that the same symbols should be the other two symbols in the fourth column.

So far we have four subcolumns made of x_1 , x_{T+1} , x_i , and x_j that should be naturally in different rows (Lemma 3). Also since x_1 and x_{T+1} are non-vanishing only in $\langle \mathbf{u}_1, \mathbf{u}_2 \rangle$ and $\langle \mathbf{u}_3, \mathbf{u}_4 \rangle$, we should necessarily have $\langle \mathbf{u}_1, \mathbf{u}_3 \rangle$ and $\langle \mathbf{u}_1, \mathbf{u}_4 \rangle$ equal to zero. A similar argument is true for $\langle \mathbf{u}_2, \mathbf{u}_3 \rangle$ and $\langle \mathbf{u}_2, \mathbf{u}_4 \rangle$. Then applying Lemma 6, we find that x_i and x_j also make a non-vanishing pair in $\langle \mathbf{u}_1, \mathbf{u}_2 \rangle$ and $\langle \mathbf{u}_3, \mathbf{u}_4 \rangle$. Therefore because of Lemma 4 they will be vanishing at any other inner product. Now let us go back to the original matrix and consider the fifth column. Make a new subcolumn \mathbf{u}_5 from that, which is extracted from the same rows as the previous four subcolumns. Because of Lemma 3, none of the symbols in this subcolumn can be from $\{x_1, x_{T+1}, x_i, \text{ and } x_j\}$. So let us call them x_a, x_b, x_c , and x_d . Since all the first four symbols are vanishing in inner products with \mathbf{u}_5 (they already had their two non-vanishing pairs), we should be able to find 5 other subcolumns namely $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$, and \mathbf{v}_5 from the original columns of the matrix to cancel out terms including x_a, x_b, x_c , and x_d in the inner product of column number five with the other four. Obviously $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, and \mathbf{v}_4 are made of x_a, x_b, x_c , and x_d .

Considering any two vanishing pairs from $(\mathbf{u}_1, \mathbf{v}_1)$ and $(\mathbf{u}_5, \mathbf{v}_5)$,we have two orthogonal columns made of 4 symbols. Considering same pairs in $(\mathbf{u}_2, \mathbf{v}_2)$ and $(\mathbf{u}_5, \mathbf{v}_5)$ we have the same result. The four symbols making those two pairs in the very discussion are the same in subcolumns one and two and Lemma 6 can be applied. Note

that the reason we can apply the discussion to this specific pair of subcolumns is because of the fact that the location of symbols in \mathbf{v}_i subcolumns is designated by those in \mathbf{u}_i 's and as we recall, we had the exact properties for the corresponding \mathbf{u} subcolumns. Applying Lemma 6, we conclude that the inner product of \mathbf{v}_1 and \mathbf{v}_2 causes two non-vanishing pairs. Same discussion applies to \mathbf{v}_3 and \mathbf{v}_4 . Therefore, x_a , x_b , x_c , and x_d will have exactly two non-vanishing pairs and they appear in $\langle \mathbf{v}_1, \mathbf{v}_2 \rangle$ and $\langle \mathbf{v}_3, \mathbf{v}_4 \rangle$. So $\langle \mathbf{v}_1, \mathbf{v}_3 \rangle$ and $\langle \mathbf{v}_1, \mathbf{v}_4 \rangle$ should not produce any pairs and are equal to zero. Therefore, we come to the following useful conclusion:

The three columns made of concatenation of $(\mathbf{u}_5, \mathbf{v}_5)$, $(\mathbf{u}_1, \mathbf{v}_1)$, and $(\mathbf{u}_3, \mathbf{v}_3)$ are orthogonal to each other and this contradicts Lemma 3.

So far we have proved that there is no rate one, full-diversity pairwise decodable code matrix for 5 transmit antennas. For more than 5 transmit antennas, we prove the theorem by contrary. Suppose we had such a code. If we remove columns until we only have 5 of them, we will have a rate one full-diversity code for 5 transmit antennas. This is contradiction and proves the theorem. \square

3.3 Inspecting other codes

In this section we relax the condition of using single symbol at each entry of the code matrix. That is, we include schemes sending combination of symbols at each time slot from every transmit antennas. We first find properties of such designs, and then

try to approximate an upper bound for their rate. Then we show a trivial example to achieve that approximate bound, for all numbers of transmit antennas. From the discussion in this section it will be clear that while using combination of symbols, the upper bound in rate is not one anymore. This can be predicted directly from the theorem in [1].

To inspect the properties of a pairwise decodable block code \mathbf{C} we decompose the code to the following format[10]:

$$\mathbf{C} = \sum_{k=1}^{2K} x_k^R \mathbf{A}_{2k} + x_k^I \mathbf{A}_{2k-1} \quad (3.3)$$

Without loss of generality, we can assume that pairs that should be decoded together are x_i and x_{K+i} for $i=1,2,\dots,K$. Therefore only these couples should show up in $\mathbf{C}^H \mathbf{C}$, and others should vanish. This means:

$$\mathbf{A}_k^H \mathbf{A}_l + \mathbf{A}_l^H \mathbf{A}_k = 0 \quad (3.4)$$

$$1 \leq k, l \leq 4K, l \neq k, |k - l| \neq 2K$$

We are looking for the maximum possible number of matrices $\{\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_{4K}\}$ with specific number of columns (as N transmit antennas) and arbitrary number of rows, i.e. T, that have the above property as well as providing the full diversity. A trivial example of such matrices can be made by using the \mathbf{A}_i 's used in an orthogonal space-time block code for the first K symbols and using the rotated version for the next K symbols. That is, if $\mathbf{C}(x_1, x_2, \dots, x_K)$ is a rate $\frac{K}{T}$ OSTBC, then $\mathbf{C}(x_1 + \hat{x}_{K+1}, x_2 + \hat{x}_{K+2}, \dots, x_K + \hat{x}_{2K})$ is a full-diversity pairwise decodable matrix with rate $\frac{2K}{T}$ sym-

bols per channel use. Bellow comes an example of the case with $T=2K$, that leads to a pairwise decodable code with average transmission of one symbol per channel use.

$$\mathbf{C} = \begin{pmatrix} x_1 + \hat{x}_5 & x_2 + \hat{x}_6 & x_3 + \hat{x}_7 & 0 & x_4 + \hat{x}_8 & 0 & 0 & 0 \\ -x_2^* - \hat{x}_6^* & x_1^* + \hat{x}_5^* & 0 & x_3 + \hat{x}_6 & 0 & x_4 + \hat{x}_8 & 0 & 0 \\ x_3^* + \hat{x}_7^* & 0 & -x_1^* - \hat{x}_5^* & x_2 + \hat{x}_6 & 0 & 0 & x_4 + \hat{x}_8 & 0 \\ 0 & x_3^* + \hat{x}_7^* & -x_2^* + \hat{x}_6^* & x_1 + \hat{x}_5 & 0 & 0 & 0 & x_4 + \hat{x}_8 \\ x_4^* + \hat{x}_8^* & 0 & 0 & 0 & -x_1^* - \hat{x}_5^* & x_2 + \hat{x}_6 & -x_3 + \hat{x}_7 & 0 \\ 0 & x_4^* + \hat{x}_8^* & 0 & 0 & -x_2^* - \hat{x}_6^* & -x_1 - \hat{x}_5 & 0 & -x_3 - \hat{x}_7 \\ 0 & 0 & x_4^* + \hat{x}_8^* & 0 & -x_3^* - \hat{x}_7^* & 0 & x_1 + \hat{x}_5 & x_2 + \hat{x}_6 \\ 0 & 0 & 0 & x_4 + \hat{x}_8 & 0 & -x_3^* - \hat{x}_7^* & -x_2^* - \hat{x}_6^* & x_1^* + \hat{x}_5^* \end{pmatrix} \quad (3.5)$$

However, as we discussed earlier such codes can only be a full-rate code if the bit rate in these codes is equal to \log_2 of the number of choices we have for every entry of the code; a value which is not achieved due to lack of rate one OSTBCs for more than two transmit antennas.

Chapter 4

Code design for rank-deficient correlated channels

All the codes discussed in previous chapters, were designed based on the channel model where channel coefficients were independent. In [17] Gesbert *et al.* have proposed a model for describing spatial correlation in the channel. They show that correlation can be split into two factors, receive and transmit correlation. Based on this model, in [7] Bolcskei *et al.* studied the effect of spatial fading correlation on the performance of space-time codes. They show how diversity order of any space-time code, is affected by those two factors in correlated channels. The effect of receive correlation was shown to be multiplicative and independent of the code. On the other hand, the effect of transmit correlation was shown to be completely dependent on the code structure. Therefore we were motivated to investigate this dependence

and based on that, find a systematic way to design optimal codes for such channels. The rest of the chapter is organized as follows. In Section 4.2 we introduce the channel model and review the results of [7]. In Section 4.3 we find a design criterion for codes to achieve maximum diversity of rank deficient correlated channels and then inspect an upper bound for the rate of these codes. Finally, in Section 4.4 we describe how to design codes that achieve maximum channel diversity as well as having good rate and decoding complexity.

4.1 System model

We consider a communication system with N transmit and M receive antennas. The fading coefficient \mathbf{h}_{ij} is the complex path gain from transmit antenna i to receive antenna j . We assume that the coefficients are Normally distributed with unit variance. Channel matrix \mathbf{H} is assumed to be known to the receiver, but not at the transmitter. We also assume that \mathbf{H} remains constant within a block of T symbols. With these assumptions, the received vector will be

$$\mathbf{r} = \mathbf{C}\mathbf{H} + \mathbf{N} \quad (4.1)$$

where \mathbf{N} is an independent zero-mean Gaussian noise vector, with variance normalized to produce the desired SNR.

Also based on the assumptions of [7] for \mathbf{H} we have

$$\mathbf{H} = \mathbf{S}^{1/2} \mathbf{H}_w \mathbf{R}^{1/2} \quad (4.2)$$

where \mathbf{H}_w is an $N \times M$ i.i.d. complex Gaussian matrix with zero-mean and unit-variance entries. $\mathbf{S} = \mathbf{S}^{1/2} (\mathbf{S}^H)^{1/2}$ and $\mathbf{R} = \mathbf{R}^{1/2} (\mathbf{R}^H)^{1/2}$ are transmit and receive correlation matrices, respectively.

4.2 Code design criteria

To find a design criterion for correlated channels, one needs to find the pairwise error probability for these channels and then average it over the distribution of channel realizations. The first part of this process is the same as what we did in Chapter 2, Equation (2.8),

$$P(\mathbf{C} \rightarrow \mathbf{E} | \mathbf{H}) = Q \left(\sqrt{\frac{1}{2N_0}} \|(\mathbf{C} - \mathbf{E}) \cdot \mathbf{H}\|_F \right) \quad (4.3)$$

Also we can use the same upper bound for Q function to approximate this expression,

$$P(\mathbf{C} \rightarrow \mathbf{E} | \mathbf{H}) \leq \frac{1}{2} \exp \left(-\frac{1}{4N_0} \|(\mathbf{C} - \mathbf{E}) \cdot \mathbf{H}\|_F^2 \right). \quad (4.4)$$

The next step will be to take the expected value. The only difference in this case with that of Chapter 2 is the density function of elements in \mathbf{H} . It is shown in [7] based on the results in [18] that

$$P(\mathbf{C} \rightarrow \mathbf{E}) = E[P(\mathbf{C} \rightarrow \mathbf{E} | \mathbf{H})] \leq \frac{1}{\prod_{n=1}^N [1 + (\lambda_n / 4N_0)]^M} \quad (4.5)$$

where λ_n 's are eigenvalues of the following matrix:

$$[(\mathbf{C} - \mathbf{E})^* \mathbf{S}^T (\mathbf{C} - \mathbf{E})^T] \otimes \mathbf{R} \quad (4.6)$$

where \otimes represents Kronecker product. From (4.2) we can easily define diversity of a coded modulation scheme in a correlated fading environment as:

$$\min \text{rank}[(\mathbf{C} - \mathbf{E})^* \mathbf{S}^T (\mathbf{C} - \mathbf{E})^T]. \text{rank}(\mathbf{R}) \quad (4.7)$$

where minimum value is searched among all non-equal code matrices, \mathbf{C} and \mathbf{E} . To achieve the maximum diversity of such a channel, one should design the code matrix \mathbf{C} so that for all difference matrices $\mathbf{D} = \mathbf{C} - \mathbf{E}$ we have:

$$\text{rank}[(\mathbf{C} - \mathbf{E})^* \mathbf{S}^T (\mathbf{C} - \mathbf{E})^T] = \text{rank}(\mathbf{S}^T) \quad (4.8)$$

Consider a block code $\mathbf{C}(x_1, x_2, \dots, x_K)$ that sends x_i 's, their conjugates, negatives, or negative conjugates at each time slot. To have a code with unit PAPR this assumption is necessary. Otherwise we will have a code that sends combination of symbols at each transmission time from each antenna, and therefore the transmitted power varies depending on the selected symbols. Under the above condition we can write:

$$\mathbf{C} = \sum_{i=1}^K \text{Re}\{x_i\} \cdot \mathbf{A}_i + j \text{Im}\{x_i\} \cdot \mathbf{B}_i \quad (4.9)$$

and based on that we have:

$$(\mathbf{C} - \mathbf{E})^* \mathbf{S}^T (\mathbf{C} - \mathbf{E})^T$$

$$= (\mathbf{C} - \mathbf{E})^* \sum_{i=1}^K \text{Re}\{x_i - y_i\} \cdot \mathbf{S}^T \cdot \mathbf{A}_i^T + j \text{Im}\{x_i - y_i\} \cdot \mathbf{S}^T \cdot \mathbf{B}_i^T \quad (4.10)$$

where \mathbf{A}_i and \mathbf{B}_i matrices are made of 0 and ± 1 . In either of these matrices, a non-zero entry shows the place in \mathbf{C} where we have x_i .

In this section, we only discuss the necessary condition of achieving maximum diversity. In Section IV, we show how to provide the sufficient condition, once the necessary conditions are met. To find the desired necessary condition we assume a difference matrix $\mathbf{D} = \mathbf{C} - \mathbf{E}$ in which all corresponding symbols in \mathbf{C} and \mathbf{E} are the same except x_i and y_i . Also, the two constellation members x_i and y_i in these two matrices are intentionally chosen such that they have equal imaginary parts. For example let us choose two real constellation members.¹ Then we have:

$$\mathbf{D} = \mathbf{C} - \mathbf{E} = \text{Re}\{x_i - y_i\} \cdot \mathbf{A}_i \quad (4.11)$$

for which

$$(\mathbf{C} - \mathbf{E})^* \mathbf{S}^T (\mathbf{C} - \mathbf{E})^T = \text{Re}\{x_i - y_i\} \cdot (\mathbf{C} - \mathbf{E})^* \cdot \mathbf{S}^T \cdot \mathbf{A}_i^T \quad (4.12)$$

We know that the rank of the matrix in the right side of Equation (8) is always less than or equal to that of $\mathbf{S}^T \cdot \mathbf{A}_i$. Therefore, it is necessary that $\text{rank}[\mathbf{S}^T \cdot \mathbf{A}_i^T] = \text{rank}[\mathbf{S}^T] = \text{maximum achievable diversity}$, for every i .

We recall from linear algebra that when $\text{rank}[\mathbf{S}^T] = \alpha$ we can find a set of linearly

¹Since the rank criterion is over the entire difference matrices and besides we are looking for a necessary condition, the assumption above is allowed

independent vectors $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_\alpha\}$ that preserves linear independence property after it is transformed with \mathbf{S}^T . In other words $\{\mathbf{S}^T \cdot \mathbf{v}_1, \mathbf{S}^T \cdot \mathbf{v}_2, \dots, \mathbf{S}^T \cdot \mathbf{v}_\alpha\}$ is also a set of linearly independent vectors. The set of \mathbf{v}_i 's is actually a basis for the complement of the null space of \mathbf{S}^T . Noticing this fact, we can now design \mathbf{A}_i matrices satisfying the mentioned necessary condition. We simply choose α linearly independent vectors from the complement of the null space of \mathbf{S}^T . If we use these vectors as columns of \mathbf{A}_i^T , $\mathbf{S}^T \cdot \mathbf{A}_i^T$ will have rank equal to α . However, our code may not have unit PAPR. Since we may have summation of several symbols at each entry of our code. For example if $[1 \ 1 \ 0 \ 1]^T$ and $[1 \ 0 \ 1 \ 0]^T$ are used in \mathbf{A}_1^T matrix, we will have symbol x_1 several times in a row. That means if we do not want to have rate loss, i.e. send as many symbols as possible in a block, we will have other symbols in the same entry as we have x_1 . Whenever we have combination of more than one symbol in an entry of the code, we will not have unit PAPR. Therefore, to keep a unit PAPR we avoid using the basis vectors directly. Instead, we span these vectors to unit vectors to have both privileges of higher rate as well as unit PAPR. That is, we find unit vectors $\{\mathbf{u}_i\}$ that span the complement of the null space of \mathbf{S}^T . Obviously the number of these unit vectors may be more than rank of \mathbf{S}^T . In other words we may have namely β unit vectors $\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_\beta\}$, where $\beta \geq \alpha$. Since in the difference matrix \mathbf{D} , we can simply choose x_i and y_i to have same real parts, then applying the discussion above we get similar results for \mathbf{B}_i 's.

In [1] it was proved that for full-diversity codes, i.e. codes with difference matrix of

rank M_T , upper bound of rate is equal to 1. So one may think that in aforementioned codes, that are potentially of diversity less than M_T , we can have rates more than one symbol per time slot. Here we show that this is not possible:

Theorem: Any code designed to achieve the maximum diversity of a correlated channel cannot have rate more than 1, disregarding the rank of the transmit correlation matrix.

Proof: The proof is provided in the appendix .

4.3 Code design

In this section, we show how to design a code with maximum diversity. To design the code, first we try to find \mathbf{A}_i and \mathbf{B}_i matrices satisfying the necessary condition. Then we combine that result with a code already designed for fewer transmit antennas in the case of independent channel. The Orthogonal Space-Time Block Codes (OSTBCs) [3] designed for independent fading channels are full-diversity and have single decoding complexity. These codes were shown to have a rate less than one [3],[4] for three and more transmit antennas. Therefore, here we can use the Quasi-Orthogonal Space-time Block Code (QOSTBC) [5],[6] which has pairwise decodability property, as well as providing full diversity and higher rate. These two classes of codes, or any other code designed for independent fade channels can be used for our purpose. In what follows we show how to make the desired code:

1. Find the complement of the null space of \mathbf{S}^T , and span it to unit vectors. Let us assume \mathbf{S}^T as an $M_T \times M_T$ matrix with rank equal to α . Therefore the number of unit vectors spanning it, can be any number between α and M_T . Let us call that number β and assume that $\{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_\beta\}$ are those $M_T \times 1$ unit vectors. In the last section we explained why we need to choose unit vectors to provide better rate, conditioned on having unit PAPR.

2. Find a block code that provides full diversity for β transmit antennas in the independent fading case. Let us call it $\mathbf{G}(x_1, x_2, \dots, x_K)$

We claim that the following design achieves maximum diversity:

$$\mathbf{C} = \mathbf{G}(x_1, x_2, \dots, x_K) \cdot [\mathbf{u}_1 \mid \mathbf{u}_2 \mid \dots \mid \mathbf{u}_\beta]^T$$

The reason is:

$$\begin{aligned} & (\mathbf{C} - \mathbf{E})^* \mathbf{S}^T (\mathbf{C} - \mathbf{E})^T \\ &= (\mathbf{C} - \mathbf{E})^* \cdot [\mathbf{S}^T \cdot \mathbf{u}_1 \mid \mathbf{S}^T \cdot \mathbf{u}_2 \mid \dots \mid \mathbf{S}^T \cdot \mathbf{u}_\beta] \\ & \cdot \mathbf{G}(x_1 - y_1, x_2 - y_2, \dots, x_K - y_K)^T \\ &= \mathbf{G}(x_1^* - y_1^*, x_2^* - y_2^*, \dots, x_K^* - y_K^*) \\ & \cdot [\mathbf{u}_1 \mid \mathbf{u}_2 \mid \dots \mid \mathbf{u}_\beta]^H \cdot [\mathbf{S}^T \cdot \mathbf{u}_1 \mid \mathbf{S}^T \cdot \mathbf{u}_2 \mid \dots \mid \mathbf{S}^T \cdot \mathbf{u}_\beta] \\ & \cdot \mathbf{G}(x_1 - y_1, x_2 - y_2, \dots, x_K - y_K)^T \end{aligned}$$

The last expression is the product of 3 full-rank matrices, and one matrix (third one) of rank α . Therefore the product will be of rank α also, and we achieve maximum diversity.

As can be seen, to get the sufficient condition of maximum diversity from the necessary condition, we only need to find a full-rank code; a code that is designed for achieving full-diversity of an i.i.d MIMO channel with β transmit antennas.

One may think it is always possible to use a full-rank code designed for M_T transmit antennas, to achieve the maximum diversity allowed by the channel. Although it is always possible to achieve rank of \mathbf{S}^T in Equation (4.7) using $\mathbf{C} - \mathbf{E}$ of rank M_T , but using this method we will end up losing optimality in rate and decoding complexity. Since this way we are choosing a code optimized for a bigger number of transmit antenna.

After all we should mention that the analysis shown is necessary. Since we cannot separate uncorrelated antennas in a rank deficient channel. The choice of antennas is directly dependent to the decomposition of the correlation matrix. That is, only if there is a zero index among all the \mathbf{u}_i vectors, then we will not be using the antenna corresponding to that index. The same is true if we use the original basis of the complement of the null-space, i.e. $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_\alpha\}$.

4.4 Other codes

If we relax the unit PAPR condition and allow unitary transformation, we are able to design other good codes. To do that we only need to apply singular value decomposition on \mathbf{S}^T matrix. Then, we can use a space-time code for α (instead of β in our case) transmit antennas that is padded with zeros for the rest of antennas. We need to transform this code, with decomposition matrices in the transmitter and receiver. This way unitary transformations will cancel out, and we will have an uncorrelated equivalent channel. Except that channel gains are scaled by the eigenvalues of \mathbf{S}^T . In many communication systems, it is much preferred to have designs with smaller PAPR. If this issue can be ignored in a system, the scheme just mentioned is easier to deal with than the one discussed in previous sections.

4.5 Appendix

We want to show it is not possible to have a rate more than 1, while achieving maximum diversity of a correlated channel. Equivalently speaking, we want to show that K , the total number of symbols, cannot be more than the number of time slots T .

Let us create a $K \times T$ array of sets. Each set in its ij entry indicates the set of nonzero indices (rows) at column j in matrix \mathbf{A}_i . Since there is only one symbol at each entry of \mathbf{C} , there shall be no common non-zero entry in two different \mathbf{A}_i 's.

Therefore at every column of the aforementioned $K \times T$ array, all sets are exclusive and have no intersection. The dimension of the complement of the null space for \mathbf{S}^T is α . Thus we can find α vectors as a basis of this set, and β unit vectors spanning them. Where $\alpha \leq \beta \leq M_T$. We can name these unit vectors based on the index of their (one) nonzero row as $\{i_1, i_2, \dots, i_\beta\}$. So every vector from the complement of the null set can only have β nonzero indices (rows). Also for any of those β index positions, at any basis set for non-null space, there should be a vector having a non-zero element at that row. This statement accompanied by the fact that in all \mathbf{A}_i 's there are β linearly independent columns from complement of the null space of \mathbf{S}^T , implies that at every row of the array, the union of the sets includes all of those β indices $\{i_1, i_2, \dots, i_\beta\}$.

Up to now, we have an array that has two properties:

1. Any of the indices $\{i_1, i_2, \dots, i_\beta\}$ is repeated at least once in each row of the array.
2. No index is repeated twice, i.e. not repeated in two distinct sets, in each column of the array.

We have T columns, therefore because of the result no. 2 we can have at most T of each index and specifically of $\{i_1, i_2, \dots, i_\beta\}$. Also we have K rows, and because of the result no. 1 we should have at least K of those numbers. Thus K is upper bounded by T and we cannot have a rate more than 1.

Chapter 5

Conclusion

In this thesis we discussed coded modulation schemes designed for multiple antenna wireless channels without information of the channel at the transmitter. We focused on space-time block codes and explained the benefits and limitations of primary designs. This thesis had two parts. In the first, which came mainly in Chapter 3, we discussed a modified version of space-time codes called quasi-orthogonal space-time block codes. Based on the the definition we mentioned for rate, we showed that there is no quasi-orthogonal space-time block code with full rate and diversity for more than four transmit antennas.

Also in the second part of this thesis we discussed correlated multi-antenna systems. The transmit diversity of any scheme in these channels was shown to be limited to the rank of the correlation matrix. When the rank of the correlation matrix is not full, we cannot get diversity equal to the number of transmit antennas. We proved

that even at that case we cannot get a rate more than one while achieving the maximum diversity of the channel. In addition we introduced a systematic way to achieve this maximum diversity paying the least cost in terms of properties of the code, like decoding complexity and rate.

Bibliography

- [1] V. Tarokh, N. Seshadri, and A.R. Calderbank , “Space time codes for high data rate wireless communications: Performance Criterion and code construction,” *IEEE Trans. on Inform. Theory*, vol. 44, no. 2, March 1998.
- [2] S. M. Alamouti “A simple transmitter diversity scheme for wireless communications,” *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451-1458, October 1998.
- [3] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, “Space-time block codes from orthogonal designs,” *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456-1467, July 1999.
- [4] X. Liang and X. Xia, “On the Non-existence of Rate-One Generalized Complex Orthogonal Designs,” *IEEE Trans. on Inform. Theory*, vol. 49, no. 11, November 2003.
- [5] H. Jafarkhani, “A Quasi-Orthogonal Space-Time Blcok Code,” *IEEE Trans. comm.*, vol 49, no. 1, January 2001.

- [6] W. Su and X. Xia, "Quasi-orthogonal space-time block codes with full diversity," *IEEE Trans. on Inform. Theory*, vol. 50, no. 10, pp. 2331-2347, November 2004.
- [7] H. Blokskei, A. J. Paulraj, "Performance of space-time codes in the presence of spatial fading correlation," *Asilomar Conference on Signals, Systems and Computers, 2000.*, vol. 1, pp. 687-693, 29 October - 1 November 2000.
- [8] A.V. Geramita, J. Seberry "Orthogonal Designs, Quadratic Forms and Hadamard Forms and Hadamard Matrices," *Lecture Notes in Pure and Applied Mathematics*, vol 43, 1979.
- [9] W. Su, X. Xia, K. J. R. Liu "A systematic design of high-rate space-time block codes with full diversity," *IEEE Communications Letters*, vol. 8, no. 6, , pp. 380 - 382, June 2004.
- [10] Z. A. Khan and B. S. Rajan, "On the Maximal Rate of Square Quasi-Orthogonal Designs," *ISIT*, p. 254, 2004.
- [11] B. S. Rajan, H. Lee, and Z. A. Khan, "A rate-one full-diversity quasi-orthogonal design for eight Tx antennas," available at http://ece.iisc.ernet.in/~bsrajan/RLK_DRDOIISc_web.ps.
- [12] Z. A. Khan and B. S. Rajan, "Space-time block codes from co-ordinate interleaved orthogonal designs," *ISIT* , p. 275, 2002.

- [13] C. Yuen, Y. L. Guan, and T. T. Tjhung, "Construction of quasi-orthogonal STBC with minimum decoding complexity," *ISIT*, Chicago, June/July 2004.
- [14] H. Yao and G. W. Wornell, "Structured Space-Time Block Codes with Optimal Diversity-Multiplexing Tradeoff and Minimum Delay," *Proc. IEEE GLOBECOM*, December 2003.
- [15] H. Jafarkhani and N. Seshadri, "A super orthogonal space-time trellis code," *IEEE Trans. Inform. Theory*, vol. 49, no. 4, April 2003.
- [16] H. Jafarkhani and N. Hassanpour, "Super-Quasi-Orthogonal Space-Time Trellis Codes for Four Transmit Antennas," *IEEE Trans. on Wireless Comm.*, January 2005.
- [17] D. Gesbert, H. Bolcskei, and A. J. Paulraj, "Outdoor MIMO wireless channel: Models and performance prediction," *IEEE Trans. Comm.*, vol. 50, pp. 1926-1934, December 2002.
- [18] G. L. Turin, "The characteristic function of hermitian quadratic forms in complex normal variables," *Biometrika*, vol. 47, pp. 199-201, 1960.