A STATISTICAL SUBSPACE METHOD FOR BLIND CHANNEL IDENTIFICATION IN OFDM COMMUNICATIONS

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ABSTRACT

This paper presents a subspace method for blind channel estimation based on a special output correlation matrix. The approach relies on the i.i.d. assumption of the data sequence and uses the cyclic prefix redundancy present in OFDM systems or single-carrier systems with frequency domain equalization. This method has an important feature that allows channels to be longer than the cyclic prefix. In addition, unknown frame synchronization can be accommodated. There is no constraint on the zero locations of the channel and the performance is asymptotically independent of white noise. The method is compared with a simple correlation approach and a deterministic subspace method.

1. INTRODUCTION

For high speed communications, OFDM (Orthogonal Frequency Division Multiplexing) has recently emerged as a viable scheme that allows a simple channel equalization through frequency domain multiplication rather than time-domain deconvolution. Channel estimation can even be avoided if differential PSK modulation is used [6]. However, differential demodulation suffers a typical 3 dB SNR loss compared with coherent demodulation which requires channel knowledge. To estimate the channel, both training and blind identification methods can be applied. In broadcasting or multicasting systems, periodic training data frames can significantly reduce the overall system channel capacity. Thus, blind channel identification becomes an attractive alternative.

In this paper, we present a blind channel estimation algorithm which exploits only the transmission redundancy induced by the cyclic prefix (CP) in an OFDM system. The method also applies to single-carrier (SC) systems that also use CP for frequency domain equalization. The pros and cons of OFDM and SC communications in terms of carrier offset sensitivity, peak-to-average ratio and other factors can be found in [1][6]. Our method exploits second-order statistics of the received signal under the assumption that the source sequences is independent and identically distributed (i.i.d.). Channels longer than the CP can also be identified, which is a merit useful for dispersive channels under-accommodated by the CP guard interval. OFDM symbol synchronization requirement can be relaxed by over-estimating the channel response padded with zeros at both ends. The additional zeros in the channel response have little effect on the algorithm and the estimation performance is asymptotically independent of the white noise. We will compare our method with two statistical approaches

reported in [4] and a deterministic (without the i.i.d. assumption) subspace method in [5], all of which assume that symbol synchronization is achieved and that the unknown channel response is shorter than the CP.

2. SYSTEM MODEL AND PROBLEM FORMULATION

In an OFDM transmitter, channel-encoded serial data stream is first segmented into length- N blocks often referred to as OFDM symbols. We denote the k^{th} OFDM symbol by $\mathbf{s}(k) = [s_1(k), s_2(k), \dots, s_N(k)]^T$ on which IDFT is applied. The last D samples of the N -IDFT outputs are repeated in front of the N -IDFT outputs as the so-called cyclic prefix (CP). This forms a $(N + D)$ sample frame which will be transmitted in serial. The N -point IDFT can be described as $F \cdot \mathbf{s}(k)$ where F is a N -by- N IDFT matrix. Denoting F_D as the sub-matrix consisting of the last D rows of the F , we can write the transmitted k^{th} frame as $\mathbf{x}(k) = [F_D^T, F^T]^T \mathbf{s}(k)$. In an SC transmitter, however, no IDFT is performed and the CP is just the repetition of the last D data symbols.

The received baseband signal is sampled at the baud rate. The sampled data are segmented into frames of size- $(N + D)$ with (or without) the knowledge of the transmitted frame boundary. The dispersive channel is of order L_h , which is denoted as $\mathbf{h} = [h_0, h_1, \dots, h_{L_h}]^T$. We assume, for notational simplicity, that the OFDM symbol size N is an integer multiplication of the length of CP (i.e., $N = MD$), and we would like to represent the received and transmitted frames in sub-blocks of size- D whose boundaries are aligned with the CP block. Since we will use correlation between adjacent frames, we write two successive received frames as:

$$\begin{bmatrix} \mathbf{y}_0(k) \\ \mathbf{y}_1(k) \\ \vdots \\ \mathbf{y}_M(k) \\ \mathbf{y}_0(k+1) \\ \mathbf{y}_1(k+1) \\ \vdots \\ \mathbf{y}_M(k+1) \end{bmatrix} = H_2 \begin{bmatrix} \mathbf{x}_{M-L+1}(k-1) \\ \vdots \\ \mathbf{x}_M(k-1) \\ \mathbf{x}_1(k) \\ \vdots \\ \mathbf{x}_M(k) \\ \mathbf{x}_1(k+1) \\ \vdots \\ \mathbf{x}_M(k+1) \end{bmatrix} + \tilde{\mathbf{n}}_2 \quad (1)$$

$$\triangleq \tilde{\mathbf{y}}_2(k) = \begin{bmatrix} \tilde{\mathbf{Y}}_1(k) \\ \tilde{\mathbf{Y}}_1(k+1) \end{bmatrix} \quad \triangleq \tilde{\mathbf{x}}_2$$

where $\mathbf{x}_i(k)$ ($\mathbf{y}_i(k)$) denotes the i^{th} size- D sub-block of the

$$\underbrace{\left[\begin{array}{c|cc} \dots & 0 & h_{L_h} \\ \vdots & \ddots & \vdots \\ \vdots & \vdots & h_{L_h} \\ \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots \end{array} \right]} = [C_L \dots C_0] \quad (3)$$

In the case that the frame boundary has an unknown lag of $pD - q$ where, without loss of generality, we can always choose $D > q > 0$ and vary p . If $p = 0, q \neq 0$ (i.e., ahead of the true boundary by $q < D$), we can accommodate this q factor by over-estimating the channel from both ends as explained below. The sub-block boundaries are kept in line with the CP block so that we can still use the data model of (1). But now, the partition lines in (3) will move to the right by q columns, i.e., in the new C_0 , h_0 retreats to the $(-q)^{th}$ diagonal (i.e., below the main diagonal by q). However, we still estimate C_0 as a “full” lower triangular matrix and C_L as “full” upper triangular. Therefore, the channel order is over-estimated as LD . In order to cover the whole channel, the number of C_i blocks needs to be $L = \lceil \frac{L_h + q}{D} \rceil \leq \lceil \frac{L_h - 1}{D} \rceil + 1$. By setting $L = \lceil \frac{L_h - 1}{D} \rceil + 1$, we can guarantee the accommodation of any unknown q .

$$\hat{Y}_1 = \vec{Y}_2[pD + 1 : (p + M)D].$$

3. ALGORITHMS BASED ON CORRELATIONS

IDFT and DFT satisfy the perfect reconstruction property, i.e., $FF^H = I$ where $F^H = F^{-1}$ is the DFT matrix. For single-carrier systems, we simply have $F = I_{N \times N}$. Under the assumption of i.i.d. source, we can verify that

$$E\{\vec{X}_2\vec{X}_2^H\} = I \quad (4)$$

A very simple correlation method is proposed in [4] where the frame boundary is assumed known and the channel is shorter than the CP, i.e., $L_h < D$. Only two matrices C_0 and C_1 exist in this case. Observing from (1) and (2), we have the following correlation which holds asymptotically between the first and last sub-block of the same received frame (assuming white noise):

$$E[\mathbf{y}_0(k)\mathbf{y}_M^H(k)] = \mathbf{C}_0\mathbf{C}_0^H \quad (5)$$

In the same paper [4], another approach is proposed where C_0 is estimated from the Cholesky decomposition of $C_0 C_0^H$. Instead of one column of $C_0 C_0^H$, more information is used. The performance still depends on a good conditioning of matrix $C_0 C_0^H$ which requires a large h_0 as well. In implementation, since the empirical expectation $E\{\mathbf{y}_0(k)\mathbf{y}_M^H(k)\}$ is often not positive definite as it only approximates $C_0 C_0^H$ asymptotically, some remedies have to be implemented to enforce its positive-definiteness.

If the channel is longer than the CP (i.e., $L_h \geq D$), C_0 contains only part of the channel ($h_0, \dots, h_{D-1}$). In this case, not only correlations between $\mathbf{y}_0(k)$ and $\mathbf{y}_M(k)$ of the same received frame, but also between $\mathbf{y}_0(k)$ and sub-blocks in the next frames must be used. Assuming known L , we can observe from (1) and (2) that

$$E\{\mathbf{y}_0(k)[\mathbf{y}_M^H(k), \mathbf{y}_0^H(k+1), \dots, \mathbf{y}_{L-1}^H(k+1)] \\ = [C_0 C_0^H, C_0 C_1^H, \dots, C_0 C_L^H]$$

Using the lower triangular structure of C_0 , we can now estimate the entire channel from the first row of the above correlation matrix which is $h_0(h_0^*, \dots, h_{L_D-1}^*)$. The Cholesky decomposition approach can not be used here since only C_0 is lower triangular. When frame boundary is unknown such as the $q \neq 0, p = 0$ case, we need to detect q since now the $(q+1)^{th}$ row contains the channel.

3.3. A New Algorithm Based on Correlations

To alleviate the channel estimation sensitivity to h_0 , we exploit more correlation information between sub-blocks involving two received frames. From (1) and (2), we can observe that

$$E\{\tilde{Y}_1(k) \begin{bmatrix} \tilde{Y}_M(k) \\ \tilde{Y}_1(k+1) \end{bmatrix}^H\} = \underbrace{\begin{bmatrix} C_0 C_0^H & C_0 C_1^H & \dots & C_0 C_L^H & \mathbf{0} \dots \\ * & C_1 C_1^H & \dots & C_1 C_L^H & \mathbf{0} \dots \\ \vdots & \vdots & \ddots & \vdots & \mathbf{0} \dots \\ * & * & \dots & C_L C_L^H & \mathbf{0} \dots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}}_{\triangleq R}$$

where “*” denotes correlation blocks whose exact expressions depend on the specific value of L and M . They can have the form of $C_i C_j^H$, for example, when $L = 1$. In this case, we might also want to use them. But we shall not pursue this tangent for now. Instead, We shall only exploit the upper triangular part of R . As in the simple correlation method, the correlations of white noise is asymptotically zero since they are between blocks that are at least M blocks apart. We extract the upper triangular part of R and add to its Hermitian to form a matrix as an outer-product of the form CC^H where

$$C = [C_0^T, C_1^T, \dots, C_L^T, \mathbf{0}, \dots]^T.$$

Note that C is Toeplitz and it is always of full column rank- D no matter how many preceding (or trailing) zeros are at both ends of the channel. Now a small h_0 will not cause any problem and the unknown boundary q does not need to be detected because we over-estimate the channel at both ends. In addition, the zero location of the channel will not cause ill-conditioning of C either. In implementation, we will not compute and include those zero blocks in C if we know an upper bound of L since too many parameter constraints will degrade the estimation accuracy.

Denote R_L as the $(L+1)D$ -by- $(L+1)D$ Hermitian matrix formed from R that asymptotically equals to the non-zero part of CC^H . From the Toeplitz structure of C , a subspace method can then be adopted as in [3][2]. From the null space of R_L which consists of LD vectors of length $(L+1)D$, a Hankel matrix G_i , ($i = 1 \dots LD$) of size LD -by- $(D+1)$ is formed for each null vector. The eigenvector corresponding to the smallest eigenvalue of $\sum_{i=1}^{LD} G_i G_i^H$ is an estimation of the channel up to a complex scalar. We may be able to use a minimum (and possibly more reliable) noise subspace instead of all LD null vectors to estimate the channel.

When the frame boundary is in error by more than one sub-block (i.e., $p > 0$), we will label received data as successive sub-blocks $\tilde{y}_i$ without assigning any frame index. We observe the following correlations between proper sub-blocks in $\tilde{Y}_1 = [\tilde{y}_0^T, \dots, \tilde{y}_M^T]^T$ and $\tilde{Y}_2 = [\tilde{y}_M^T, \dots, \tilde{y}_{2M}^T, \tilde{y}_{2M+1}^T, \dots, \tilde{y}_{3M+1}^T]^T$, i.e.,

$$E\{\tilde{Y}_1 \tilde{Y}_2^H\} = \tilde{R}$$

where (assuming $p < L$ in the representation below)

$$\tilde{R} \triangleq [\tilde{R}_1, \tilde{R}_2]$$

$$\begin{bmatrix} C_p C_p^H & \dots & C_p C_L^H & \mathbf{0} & & \\ * & \ddots & \vdots & \vdots & & \\ & * & C_L C_L^H & \vdots & & \\ & & & \ddots & C_0 C_0^H & \dots & C_0 C_{p-1}^H & & \\ & & & & * & \ddots & \vdots & & \\ & & & & * & * & C_{p-1} C_{p-1}^H & & \\ & & & & & & C_{p-1} C_p^H & \dots & C_{p-1} C_L^H & \mathbf{0} \dots \end{bmatrix}$$

We see that the upper triangular part of $\tilde{R}_1$ resembles that of an outer-product structure of a circularly shifted version of C , which is,

$$\tilde{C} = [C_p^T, \dots, C_L^T, \mathbf{0}, \dots, \mathbf{0}, C_0^T, \dots, C_{p-1}^T]^T \quad (6)$$

except that the $(L-p+1)D$ -by- pD upper right corner block of $\tilde{R}$ consists of asymptotic zeros instead of what is should be in $\tilde{C}\tilde{C}^H$. In order to obtain the desired outer-product, we can replace that corner with the Hermitian of the non-zero part of $\tilde{R}_2$ after we detect p or equivalently, the location of C_0 . There are structured information in $\tilde{R}_1$ that can be useful in detecting p . For example, there usually exist consecutive zeros between $h_{L_h} h_{L_h}^*$ (last non-zero diagonal entry of the $C_L C_L^H$ block) and $h_0 h_0^*$ (first non-zero diagonal entry of the $C_0 C_0^H$ block). Additionally, only $C_0 C_0^H$ has all the correlation blocks directly above it being zeros.

We even can choose not to estimate p by estimating the channel at its maximum allowed order $L_h = (M-1)D$. More specifically, by adding the Hermitian of $E\{\tilde{y}_{i-1}[\tilde{y}_{2M}^H \dots \tilde{y}_{2M+i-2}^H]\}$ back to $\tilde{R}_1[1 : (i-1)D, (i-1)D+1 : iD]$ for all $i = 2, \dots, M+1$, we will get the desired outer-product of $\tilde{C}\tilde{C}^H$ because either the added or the original correlation block (or both) is asymptotically a zero block. Thus, we can estimate a circularly shifted channel with order $L_h = (M-1)D$ to avoid any attempt to estimate the unknown boundary factor. However, if too many coefficients are estimated, the estimation accuracy will be degraded. Moreover, the h_0 position may still be needed to finalize the channel identification since a circularly shifted channel is not good enough for equalization.

3.4. Comparison with a Deterministic Method

While our method is based on i.i.d. assumption, another subspace method was proposed where the i.i.d. assumption is relaxed to requiring “sufficient” number of OFDM frames [5] for persistent excitation. The algorithm was originally for the synchronized case with $L_h < D$ (i.e., $L = 1$), but it can be extended to the $L_h > D$ case as follows. Note from (1) that one frame corresponds to a $(M+1)D$ -by- $(M+L)D$ matrix (the upper half of H_2). By stacking at least $L+1$ consecutively received frame, a large matrix $\mathcal{H}$ of size $(L+1)(M+1)D$ -by- $(L+M+LM)D$ is formed which has more rows than columns and will be full column rank. As long as the source autocorrelation matrix is of full column rank which requires the collection of at least $(L+M+LM)D$ frames, the null subspace of the correlation

matrix can be employed to estimate the channel. Now, a correlation matrix of size $(L+1)(M+1)D$ -by- $(L+M+LM)D$ as opposed to size $(L+1)D$ -by- $(L+1)D$ (i.e., R_L in our method) is computed. The eigen-decomposition is more complex too. The performance of this deterministic subspace method is not affected by the numerical value of h_0 either while $q \neq 0$ is also allowed. However if $p > 0$, one frame will correspond to $\mathcal{H}_2[pD+1 : (p+M)D, :]$. It can be shown that more received frames (at least $M+L$, instead of $L+1$) have to be stacked in order to result in a “tall” channel matrix. Even if the prohibitively high computation burden can be tolerated, p still remains known because the structure of that channel matrix must be known in order to use the subspace approach.

4. SIMULATIONS

We consider the case of direct OFDM broadcasting system. The wireless channel we use is generated according to the “Typical Urban” profile specified by the European COST207 project, the symbol rate is set to be 4M Baud/s and a stationary channel is assumed (i.e., no Doppler shift). The modulation is 16-QAM. Channel vector is normalized to norm 1, so the input SNR is $10 \log(\sigma_s^2/\sigma_n^2)$. Our measurement is the channel mean-squared estimation error $MSE \triangleq \|\hat{\mathbf{h}} - \mathbf{h}\|^2$ averaged over 100 Monte Carlo runs.

We first consider the synchronized case (i.e., $p = q = 0$) with $D = 8$ and $M = 4$. To compare our method to those in [4] and [5], we truncated channels to $L_h = 7$ such that h_0 is the second biggest coefficient. Even for this h_0 , our method always out-performs the simple correlation. It also out-performed the deterministic method under low SNR or small number of frames.

Figure 1: Synchronized Case with Large h_0 and $L_h < D$

In Figure 2, we include one more preceding channel coefficient which is still about 1/8 of the peak value. Not surprisingly, the simple correlation method provides errant results.

Next in Figure 3, we consider the case where $D = 4$ but $L_h = 12$, i.e., the channel is longer than the CP. We also

Figure 2: Synchronized Case with Small h_0 and $L_h = D$ include an unknown lag of $q = 3$. To compare with the deterministic subspace method, we assume $p = 0$. Due to the unknown q , 5 sub-blocks are needed to cover the channel (i.e., $L = 4$). Here, we actually estimated a channel of order $LD = 16$.

Figure 3: Unknown Boundary Case with $L_h > D$

5. REMARKS

Due to the i.i.d. assumption, our subspace method still require many frames to obtain good statistical information, while the deterministic subspace method also needs sufficient frames. Hence, they may be applicable to only stationary multicasting systems. The CP can be viewed as a special block code, provides an alternative for blind channel estimation in a single-carrier system if a multichannel setting (e.g., oversampling or antenna array) does not exist. This is akin to transmitter-induced cyclostationarity proposed in [7]. In fact, the repetition coding and interleaving scheme in [7] correspond to the $L_h < D$ and $M = 1$ case.

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