BURST SYNCHRONIZATION ON UNKNOWN FREQUENCY SELECTIVE
CHANNELS WITH CO-CHANNEL INTERFERENCE USING AN
ANTENNA ARRAY

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Abstract — In this work, burst oriented data transmission
over unknown frequency selective channels is considered.
The receiver is assumed to use multiple antennas and the
problem of estimating the start position of a data packet in
the presence of spatially correlated co-channel interference
is addressed. Each burst is assumed to contain a known
training sequence, and in the paper metrics for finding the
position of this training sequence are studied. More specifi-
cally, we examine the advantages of taking the spatial corre-
lation of the co-channel interference into account, as com-
pared with treating it as spatially white. Simulations and
experimental results for transmission of normal GSM bursts
in interference limited scenarios are presented.

I. INTRODUCTION

To meet the increasing demands on higher data rates, qual-
ity, coverage and capacity of wireless systems, the use of
antenna arrays has been proposed as a way to exploit the
spatial dimension more efficiently [1]. Among the possi-
bilities are improved range, diversity against fading, inter-
ference suppression, and spatially selective transmission to
reduce interference in the down link.

In this work, the spatial degrees of freedom are used for co-
channel interference (CCI) rejection. Interference rejecting
sequence estimators that take the spatial correlation of the
CCI into account may be found in [2–5]. Although there
exist blind methods that do not require dedicated training
symbols in order to operate, the use of training sequences
for channel identification and training of interference rejec-
tion combining algorithms appears to be necessary in com-
plicated multipath environments. A training sequence in
each burst is used to synchronize the receiver, and to (ini-
tially) determine the parameters of the interference rejecting
equalizer. It is clear that synchronization schemes that can
operate in the presence of strong CCI are needed.

In [6], several metrics for estimating the start position of a
data burst transmitted over an unknown frequency selective
channels are derived. This work is concerned with extend-
ing the data-aided maximum likelihood approach proposed
in [6] to the case with multiple antennas and to consider the
presence of CCI. The CCI usually has the same properties
as the signal of interest (such as the finite alphabet prop-
erty). However, as the digital sequences transmitted by the
interferers are in general unknown, the optimum solution to
the synchronization problem involves an exhaustive search
over all possible sequences. As this is deemed to be compu-
tationally cumbersome, we suggest a suboptimal, computa-
tionally simpler, approach in which the CCI and the additive
noise are modeled as a temporally white complex Gaussian
process. As mentioned above, such a modeling approach
has been proposed earlier for effective interference rejec-
tion combining (see, e.g., [2–5, 7]).

The difference between this work and part of what is pre-
sented in [6] is primarily the extension to the case with mul-
tiple antennas and spatially colored Gaussian noise of un-
known color. Two metrics are derived. The first metric takes
the spatial color of the CCI into account, whereas the sec-
ond metric neglects this color and models the CCI and noise
as spatially white. Although both metrics are functions of
the same matrix, performance in the presence of CCI is very
different. This is demonstrated by means of simulations and
on experimental data.

II. PRELIMINARIES

Consider a transmitter transmitting a modulated data stream
consisting of a known training sequence embedded in an
unknown data sequence. This is illustrated in Figure 1. A
discrete time model for the down-converted, filtered, and
sampled signal from each antenna is used. For simplicity,
frequency offset errors are neglected, and the scenario is as-
sumed to be time-invariant during the training period. As
in [6], the channels between the transmitter and the receiv-
ing antennas are frequency selective and are modeled as un-
known FIR filters. If the signal has some excess bandwidth,
the signal from each antenna is to be oversampled with re-
spect to the sampling symbol rate in order to obtain a suf-
cient statistic for estimation and detection. This is easily
included in the model by viewing each sampling phase as
an additional channel. The receive side is assumed to use $m$ antennas and the oversampling factor with respect to the symbol rate is denoted $q$.

The sampled sequences may then be arranged (see, e.g., [5] for details) as

$$\begin{align*}
(k) &= a(k-n) + (k),
\end{align*}$$

where $(k)$ is an $mq$ 1 column vector and

$$a(k) = [a(k) \ldots a(k-L)]^T.$$  

The $mq (L+1)$ matrix $\mathbf{a}$ represents the single-input-multiple-output (SIMO) channel for the user of interest, $a(k)$ are the symbols transmitted, and $(k)$ models both the CCI and noise. The start position of the frame and thus also of the training sequence is unknown, and this is included in the model above by introducing the unknown delay $n$. We will assume that the training sequence is embedded in the central part of the burst as to minimize the impact of time-variations during the frame. An example where this is the case is normal GSM bursts. Without loss of generality, the time-indexes can be ordered so that the training sequence, which is of length $N$ symbols, start at time 0. This means that $a(k)$ is known for time 0, 1, ..., $N-1$.

The problem studied is that of determining the sample position, $n$, in which the training sequence starts. An estimate of this position is necessary in order to train an interference rejecting equalizer. In general, the channel order, $L$, is a design parameter, chosen a priori in order to handle the maximum expected significant time-dispersion introduced by the physical channel and the transmit and receive filters. Although the estimate $n$ may be off a few positions, the sequence estimator may still be able to compensate for such an estimation error.

### III. DERIVATION OF METRICS

Under the assumption that the noise and CCI sequence, $(n)$, is wide-sense stationary, (1) may be rewritten as

$$\begin{align*}
(k+n) &= a(k+n) + (k),
\end{align*}$$

where the process $(n)$ now denotes the time-shifted noise process. The CCI contribution to $(n)$ may in general be modeled in the same way as the signal of interest, i.e., as a finite alphabet sequence filtered with an FIR filter. However, taking this structure into account will lead to a search over all finite alphabet sequences transmitted by the interferers. Instead of such a computationally demanding strategy, the CCI contribution is modeled together with the noise as complex Gaussian. This assumption is primarily a modeling assumption that leads to a metric that takes the spatial covariance of the CCI into account. Thus, $(k)$ is modeled as a zero-mean complex Gaussian process. For simplicity the process is assumed to be temporally white. However, as in, e.g., [2, 4, 5], CCI is accounted for by modeling the process as spatially colored.

#### Spatially Colored Noise

The maximum likelihood (ML) approach of [6] is followed. Since the training sequence, $a(k)$, is known for $k = 0, \ldots, N-1$, $a(k)$ may be formed for $k = L, \ldots, N-1$. Other temporal windows of $a(k)$ may also be used, see [8]. An alternative is to form the likelihood function with all observations available and to model the unknown data symbols as zeroes. This approach will not be considered here.

The negative log-likelihood function for $N = N - L$ consecutive observations of $(n)$ may under the assumption that the training sequence start in position $n$ be written as

$$\begin{align*}
\Lambda(n, \ ) &= - \sum_{i=1}^{N-L} \log f(n+l) - a(l); \\
&= - \log f(n+l) - a(l);
\end{align*}$$

where $f(\ ; \ )$ denotes the probability density function (pdf) of a complex Gaussian vector with zero mean and covariance. To arrive at this expression, the noise and CCI process is assumed to be temporally white. Making use of the expression for the complex Gaussian pdf [9], and neglecting irrelevant constants, yields

$$\begin{align*}
\Lambda(n, \ ) &= \log + \text{trace} \ (n, \ )^{-1} ,
\end{align*}$$

where denotes the determinant, $\text{det}$. $(n, \ ) = \mathbf{\hat{R}} (n) - \mathbf{\hat{R}} (n) \mathbf{\hat{R}}^* (n) + \mathbf{\hat{R}}^*.$

$(\ )^*$ denotes the complex conjugate transpose, and the sample covariances are defined as

$$\begin{align*}
\hat{R}^* (n) &= \frac{1}{N} \sum_{l=0}^{N-1} (l+n) a^* (l) \\
\mathbf{\hat{R}} (n) &= \frac{1}{N} \sum_{l=0}^{N-1} (l+n) a^* (l) \\
\hat{R} &= \frac{1}{N} \sum_{l=0}^{N-1} a (l) a^* (l).
\end{align*}$$

For notational convenience we will from here on occasionally omit the dependence on $n$.  

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Figure 1: A training sequence (TS) is inserted in the data stream so that the receiver can synchronize and estimate the parameters of an interference rejecting equalizer.
The negative log-likelihood function depends on the unknown channel and the unknown spatial covariance of the noise and CCI. The approach taken is to estimate these unknown parameters for each candidate position \( n \). The cost function in (4) is minimized with respect to \( n \), for each value of \( n \) and \( \hat{A}(n,\cdot) = \hat{A}(n,\cdot) \), under the assumption that \( (n,\cdot) \) is invertible, which it is with probability one if \( N \) is large. If a constant term is neglected, the concentrated cost function may be written as

\[
\Lambda(n, \cdot) = \log \hat{R} - \hat{R} \hat{R}^{-1}\hat{R}^* + \log \hat{R} - \hat{R} \hat{R}^{-1}\hat{R}^* ,
\]

where the second inequality holds with equality for \( \hat{R} = \hat{R} \hat{R}^{-1} \) for each value of \( n \). Thus, the maximum likelihood estimates of the parameters are given by

\[
\hat{a} = \hat{R} (n) \hat{R}^{-1} \quad \hat{u}_2 = \hat{R} (n) - \hat{R} (n) \hat{R}^{-1}\hat{R}^* (n) ,
\]

and the synchronization estimate is given by

\[
\hat{n} = \arg \min \Lambda(n) ,
\]

where the metric is

\[
\Lambda(n) = \log \hat{a} - \hat{u}_2 .
\]

The maximum likelihood estimate of the channels, \( \hat{a} \), is given as the least squares fit to the data, and the ML estimate of the spatial noise covariance, \( \hat{u}_2 \), is simply the sample covariance of the residuals. As \( n \) is varied over the synchronization window, the sample covariance of the residuals may be calculated for each candidate position \( n \). The position for which the determinant of the sample covariance of the residuals is minimum corresponds to the synchronization position.

Note that, as \( \hat{R} = \hat{R} \hat{R}^{-1} \), the metric in (7) may be rewritten as

\[
\Lambda(n) = \log \hat{R} - \hat{R} \hat{R}^{-1}\hat{R}^* + \log \hat{R} + \log \hat{R} - \hat{R} \hat{R}^{-1}\hat{R}^* ,
\]

where \( \hat{R} = \hat{R} \hat{R}^{-1} \) is the \( mq \) \( mq \) identity matrix. If the observations may be regarded as wide-sense stationary over the synchronization window, then the first term may be regarded as constant, i.e., \( \hat{R} (n) = \hat{R} \). An alternative estimator is then given by

\[
\hat{n} = \arg \min \log \hat{R} - \hat{R} \hat{R}^{-1}\hat{R}^* = \arg \min \log \hat{R} + \log \hat{R} - \hat{R} \hat{R}^{-1}\hat{R}^* ,
\]

since \( + = + \). Note that, for the case when \( L = 0 \), i.e., a flat channel with no time-dispersion and synchronized sampling, (8) is equivalent with maximizing the scalar function \( \hat{R} \hat{R}^{-1}\hat{R}^* \). It is easily seen that this metric is equivalent with using a linear least squares metric, i.e.,

\[
\hat{n} = \arg \min \min \sum_k (n + k) - \alpha(k) \hat{R} \hat{R}^{-1}\hat{R}^* ,
\]

which is the minimum mean squared error (MMSE) synchronization metric proposed in [3]. Here, \( \alpha \) denotes the Frobenius norm, \( \sqrt{\sum_{kk} (R A(n) R^{-1})^2} \).

Spatially White Noise

The metric derived in the previous section takes the spatial color of the CCI and noise into account. In this section, the metric for the case that the additive noise is modeled as spatially white is considered. Thus, such an estimator uses the knowledge \( = \sigma^2 \) for some scalar \( \sigma^2 \). The negative log-likelihood function may in this case be written as

\[
\Lambda(n, \cdot) = \sum_{k} (n + k) - \alpha(k) \hat{R} \hat{R}^{-1}\hat{R}^* ,
\]

where irrelevant constants have been neglected. This function may be minimized with respect to \( n \) for each \( n \). In fact, the minimizing argument coincides with the estimate for case with spatially colored noise, given in (5). The concentrated cost function then becomes

\[
\hat{n} = \arg \min \Lambda(n) ,
\]

where the metric in this case is given by

\[
\Lambda(n) = \log \hat{a} - \hat{u}_2 ,
\]

with \( \hat{a} \) defined in (6). Thus, if the noise is known to be spatially white, \( = \sigma^2 \), then the trace of the sample covariance matrix of the residuals is to be taken as metric. However, when the noise is modeled to have an unknown spatial color, the determinant of the same matrix is to be minimized. The metrics may also be expressed in terms of the eigenvalues of the sample covariance matrix of the residuals. The determinant-metric is equivalent with minimizing the geometric mean of the eigenvalues whereas the trace-metric is equivalent with minimizing the arithmetic mean.

The metric may also be simplified as follows. For wide-sense stationary scenarios, the approximation \( \hat{R} (n) \hat{R} \) may be used again. An alternative estimator is then given by

\[
\hat{n} = \arg \max \hat{R} (n) \hat{R} = \arg \max \hat{R} (n) ,
\]

where \( \hat{R} \hat{R}^{-1}\hat{R}^* \) may be recognized as a correlator, where \( \hat{R} \) is to be included as to decorrelate.
the delayed versions of the training sequence. The synchronization sequence is typically chosen so that the autocorrelation function is white-noise like, in which case \( \hat{R} \).

The metric may also be rewritten as

\[
\hat{n} = \arg\max_{n} \left( \hat{\Lambda} (n) \right)^2,
\]

which may be interpreted as finding the position that maximizes the energy in the channel estimates.

**IV. SPATIO-TEMPORAL PROCESSING**

So far, the CCI has been modeled as temporally white. For small antenna arrays and time-dispersive co-channel interference, there may not be enough spatial degrees of freedom to facilitate space-only interference rejection and synchronization in the presence of strong co-channel interference. It is then necessary to take the temporal correlation of the co-channel interference into account. The prediction error filter associated with a finite order linear predictor may then be used. The negative log-likelihood function may be concentrated with respect to the unknown, filtered channel and the parameters of the linear predictor. The maximum likelihood estimate of the spatial covariance of the prediction errors is then calculated in the same way, i.e., as the sample covariance matrix of the residuals, and may be used to find the synchronization position, see [4] for further details.

**V. MULTIPLE FRAMES**

Consider the extension to multiple frames. The transmission is assumed periodic in the sense that the time between any two frames is the same and is known exactly. If this is not the case, one has to resort to single frame synchronization. As in [6], metric averaging over several frames is considered. Suppose that data is available for \( M \) frames. Also, let us assume that the fading is independent from burst to burst so that both the channels and the spatial noise covariance, which will be a function of the co-channel interferers channels, are independent from burst to burst. Formulating the maximum likelihood estimator and concentrating it with respect to the \( M \) channels and the \( M \) spatial noise covariances leads to a cost function of the following form

\[
\hat{n} = \arg\min_{1} \sum_{n=1}^{M} \Lambda (n),
\]

where \( \Lambda (n) \) is one of the metrics in (7) or (9) calculated for the \( m \)th frame. A simpler approach is to use, not a rectangular window with \( M \) consecutive frames, but to update the metric with an exponential forgetting factor, i.e.,

\[
\hat{n} = \arg\min_{1} \sum_{1}^{\lambda} \Lambda (n).
\]

The forgetting factor, \( \lambda \), must as always be chosen as a compromise between tracking capability and steady state variance. Yet another approach, also proposed in [6], involves filtering the burst-wise estimates, e.g., by means of a feedback loop. Using data from several bursts requires a higher degree of stationarity and a longer training period.

**VI. NUMERICAL EXAMPLES**

In the numerical examples, reception of normal GSM bursts [11] was studied. An \( m = 4 \) element antenna array with symbol rate sampling, \( q = 1 \), was considered. Simulations with one co-channel interferer were done. The fading was independent from antenna to antenna, and the GSM typical urban (TU) channel model was used. Ideal frequency hopping was assumed, so that the channel realizations were independent from burst to burst, and the channels were assumed time-invariant during the bursts. The co-channel interferer transmitted a random bit stream and was modeled in the same way as the signal of interest. An \( L + 1 = 5 \) tap channel model was used, and a search window of length eleven symbols was used to locate the position of the \( N = 28 \) symbol long training sequence in each burst. Estimates of the channels, \( \hat{\Lambda} (n) \), and the spatial noise covariance, \( \hat{\Lambda} (\hat{n}) \), were calculated for the estimated synchronization position, \( \hat{n} \), and used in a 16 state sequence estimator implemented with the Viterbi algorithm to estimate the unknown data parts of the burst, see [2, 4] for details. In Figure 2, the average BER is shown for burst-to-burst synchronization for different signal to noise ratios and two different carrier to interference ratios (C/I), 100 dB and -10 dB. As can be seen, the trace-metric of (9) performs slightly better than the determinant-metric of (7) when there is no co-channel interference present (C/I = 100 dB). However, the performance degradation is very small. The reason for this degradation is that the spatial noise covariance contains \( m^2 \) real parameters that are jointly estimated with the synchronization positions, and this leads to slightly less accurate synchronization. For the case with a strong co-channel interferer (C/I = -10 dB), the advantage of using the determinant-metric as compared to the trace-metric is very large, as the receiver cannot synchronize from burst-to-burst using the trace-metric. Using data from multiple bursts was also considered. The synchronization position was fixed, and the metric from each frame was combined according to the formulation of (10) with a forgetting factor \( \lambda = 0.9 \). This improves performance, especially for the trace-metric, which relies on temporal correlation only.

**Experimental Results**

Performance was also investigated on data collected in a suburban environment in Düsseldorf, Germany, with a test bed for the air interface of a DCS-1800 base station [12]. The output from a dual polarized antenna array with four outputs for each polarization, \( m = 8 \), and symbol rate sampling, \( q = 1 \), was processed. One mobile transmitter and one interferer were present on the air simultaneously. The nominal DOA of the two transmitters were roughly the same.
in the experiment. Due to angular spreading, the channel realizations will be different.

In Figure 3, the BER for different estimated carrier to interferer ratios is shown. The trace-metric will provide very unreliable estimates of the synchronization position if burst-to-burst synchronization is used, and this explains the high BER. If the colored noise metric is used, the receiver can synchronize with data from a single burst even in this scenario. Also in this case, using data from multiple bursts improved performance, especially for the trace-metric.

Figure 2: Simulated performance using different synchronization metrics, four antennas, one co-channel interferer.

Figure 3: Experimental data. BER performance using different synchronization metrics.

VII. CONCLUDING REMARKS

Two different metrics for burst synchronization with antenna arrays were derived using the maximum likelihood approach proposed in [6]. Numerical examples and processing of experimental data illustrated that substantial performance gains may be achieved if the spatial correlation of the co-channel interference is taken into account, also when synchronizing the receiver.

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REFERENCES