CODE-TIMING SYNCHRONIZATION IN DS-CDMA SYSTEMS USING SPACE-TIME DIVERSITY

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ABSTRACT

The synchronization of a desired user transmitting a known training sequence in a direct-sequence asynchronous code-division multiple-access (DS-CDMA) system is addressed. It is assumed that the receiver consists of an arbitrary antenna array and works in a near-far, frequency-nonselective, slowly fading channel. The estimator that we propose is an approximation of the maximum likelihood (ML) estimator for a signal model in which the contribution of all the interfering components (e.g., multiple-access interference, external interference and noise) is modeled as a Gaussian term with an unknown and arbitrary space-time correlation matrix. The main contribution of this paper is the fact that the estimator makes efficient use of the structure of the signals in the two domains. Its performance is compared with the Cramér-Rao Bound and with the performance of other methods that also employ an antenna array but only exploit the structure of the signals in one of the two domains, while using the other only as a means of path diversity. It is shown that the use of the temporal and spatial structures is necessary to achieve synchronization in heavily loaded systems or in the presence of directional external interference.

1. INTRODUCTION

The multiple-access interference (MAI) in a near-far environment usually necessitates the use of multichip detectors. However, most multichip detectors require precise knowledge of the users’ code timings, powers and carrier phases. Therefore, accurate code synchronization (acquisition and tracking) is an essential task for the correct performance of a DS-CDMA system in a near-far environment. As a result, several near-far resistant timing estimators have been proposed in the literature for a single-antenna receiver (see [1], [2] and references therein).

Although it is well known that detection performance in DS-CDMA systems can be greatly improved through the use of antenna arrays, the synchronization problem using multiple antennas has not been fully investigated. In this paper we propose a method for estimating the timing of a certain user that transmits a known training sequence. We will focus on the code synchronization because estimates of the other parameters are easily obtained once the code-timing is acquired. It is assumed that the receiver consists of an arbitrary antenna array that operates in a frequency-nonselective slowly fading channel [3]. The method could also be extended to frequency-selective slowly fading channels, but we will restrict ourselves to the former case for simplicity. Note that it is reasonable and usual to assume that during initial acquisition the users transmit a certain preamble. Once a reliable estimate of the timing is formed, the estimator can be switched to a decision-directed mode. The fact that the method estimates the parameters of only one user while retaining near-far resistance is also of interest, because it leads to decentralized implementations and dramatically reduces the complexity with respect to methods that estimate the parameters of all users simultaneously (see e.g. [1]).

Following an approach that has already been applied successfully to this and other problems, all signals, except that of the desired user, are modeled as a Gaussian component with arbitrary and unknown correlation matrix. This idea has been used for the problem at hand in [2], [4]-[6], for Doppler estimation in radar systems in [7] and for time-delay estimation in navigation systems in [8]. The significance of this paper is that we consider a space-time correlation matrix, which allows both the temporal (provided by the codes) and spatial structure (provided by the antenna array) of the received signals to be exploited. The method proposed herein extends and outperforms those presented in previous works. The approaches in [2] and [4] correspond to single-antenna receivers, and therefore they only exploit the temporal structure of the signals. An extension of these techniques for a multiple-sensor receiver can be found in [5]. However, it reduces to several single-sensor estimators applied in parallel to several independent channels. Hence the effect on the antenna array is only to increase the signal-to-noise ratio (SNR) by a factor equal to the number of sensors (i.e., maximal ratio combining). On the other hand, only the spatial structure is used effectively in [6], so a very large number of antennas may be needed to achieve near-far resistance. It will be shown that the use of the spatial and temporal structure of the interference is indispensable in achieving code synchronization in some scenarios, and this can be accomplished with a small number of antennas.
2. SIGNAL MODEL

Consider an asynchronous DS-CDMA system with $K$ users and an arbitrary receiving antenna array of $L$ sensors, which satisfies the narrow-band array condition. For the case of slow flat-fading, the received complex signal (after down-conversion and chip-matched filtering) at the $l$th sensor is

$$y_l(t) = \sum_{k=1}^{K} c_{l,k} s_k(t - \tau_k) + n_l(t) \quad l = 1, 2...L \quad (1)$$

where $c_{l,k}$ is the complex fading coefficient for the $k$th user at the $l$th sensor (includes the effects of the propagation, the transmitted power and the carrier phase), $\tau_k$ is the delay associated with the $k$th user, and $n_l(t)$ represents the thermal noise and all other external interferences. Since the fading is slow (i.e. the coherence time of the channel is much smaller than the symbol period [3]), for the estimator derivation we will assume that the fading coefficients are constant during an observation interval of $M+1$ symbols. The term corresponding to the $k$th user is:

$$s_k(t) = \sum_{m=0}^{M} d_k(m) c_k(t - mT) \quad (2)$$

$$c_k(t) = \sum_{n=0}^{N-1} g_k(n) p(t - nT_c) \quad (3)$$

where the symbols $d_k(m)$ are transmitted at a rate $\frac{1}{T_c}$ and constitute an iid sequence with variance $\sigma_i^2$. The length of the chip sequence $g_k(n)$ is $N = \frac{1}{T_c}$, the chip rate is $\frac{1}{T_c}$ and $p(t)$ represents the chip-shaping waveform. The signals in (1) are sampled at the chip rate (the technique presented herein can be readily extended if the signals are oversampled), and each set of $N$ samples is stacked into a column vector$^1$:

$$y_l(m) = [y((mN + 1)T_c) \ldots y((mN + N)T_c)]^T \quad (4)$$

The sampling is completely asynchronous, and the single condition is that previous bit synchronization of the desired user has been achieved, i.e. $\tau_1 \in [0, T)$ (without loss of generality we assume that the first user is the desired one). If the length of the chip-shaping waveform is $T_c$ (this is exact with rectangular chip-shaping pulses and a good approximation with other pulse types) only two consecutive symbols from each user contribute to $y_l(m)$, so the contribution of the $k$th user to this vector is:

$$y_{l,k}(m) = c_{l,k} A^{(k)}(\tau_k) d_k(m) \quad (5)$$

where

$$d_k(m) = [d_k(m) \ldots d_k(m-1)]^T \quad (6)$$

$$A^{(k)}(\tau_k) = \begin{bmatrix} a^{(k)}(\tau_k) & a^{(k)}(\tau_k) \end{bmatrix} \quad (7)$$

$$a^{(k)}(\tau_k) = c_k(iT_c - \tau_k) \quad i = 1...N \quad (8)$$

$$a^{(k)}(\tau_k) = c_k(iT_c + T - \tau_k) \quad i = 1...N \quad (9)$$

Therefore, we can write the received $N \times 1$ vector at the $l$th sensor as (for simplicity $A(\tau) = A^{(1)}(\tau_1)$)

$$y_l(m) = \alpha_{l,1} A(\tau_1) d_1(m) + e_l(m) \quad m = 0, 1...M - 1 \quad (10)$$

The vector $e_l(m)$ includes the MAI, the thermal noise and possible external interferences.

3. GAUSSIAN ASSUMPTION

If the temporal vectors received from every antenna are stacked into a $LN \times 1$ vector:

$$y(m) = [y_1^T(m) \ y_2^T(m) \ldots \ y_L^T(m)]^T \quad (11)$$

then equation (10) can be rewritten in a compact form as

$$y(m) = (\alpha \otimes A(\tau_1)) d_1(m) + e(m) \quad (12)$$

where $\otimes$ denotes the Kronecker product, $e(m)$ is formed identically to $y(m)$ and

$$\alpha = [\alpha_{1,1} \alpha_{2,1} \ldots \alpha_{L,1}]^T \quad (13)$$

We model $e(m)$ as a zero-mean circular complex Gaussian $LN \times 1$ vector, independent of the desired user signal, with an arbitrary and unknown covariance matrix, and independent for different samples:

$$E\{e(m)\} = 0 \quad E\{e(m)e(i)^H\} = Q\delta_{m,i} \quad (14)$$

The problem is to estimate $\tau_1$ from $\{y_i(m)\}_{m=0}^{M-1}$ assuming that $\{c_1(n)\}_{n=0}^{N-1}$ and $\{d_1(m)\}_{m=0}^{M-1}$ are known, that is to say, the spreading sequence and a training bit sequence for the desired user are available.

It is well known that the assumption that the MAI is Gaussian in a sample-by-sample basis is a misconception [9] that leads to non near-far resistant estimators. However, the estimator proposed herein does not suffer from the same misconception because it retains the structure of the MAI in the matrix $Q$, and so it is near-far resistant. Actually, it is the fact that an unknown correlation matrix $Q$ is considered for the equivalent noise that makes the estimator able to attenuate any interfering signal that exhibits a certain structure in the temporal and/or spatial domains. In this paper we present the estimator that results from an arbitrary matrix $Q$, in contrast to previous work that has solved the problem stated herein for simplified structures of that matrix. As outlined in the introduction, in [6] it is assumed that the space-time correlation matrix can be decomposed as $Q = Q_r \otimes I_N$, where $Q_r$ is an arbitrary $L \times L$ matrix that corresponds to the spatial correlation of the interference. A dual decomposition is considered in [5]. In this case, the matrix $Q$ is expressed as $Q = I_L \otimes Q_r$, where $Q_r$ is an $N \times N$ matrix representing the temporal structure of the interference.

4. MAXIMUM LIKELIHOOD ESTIMATOR

The estimator of the parameters of the desired user is derived by applying the ML principle to the signal model described above. The negative log-likelihood function is proportional to:

$$\Lambda_1(\tau_1, \alpha, Q) = \ln |Q| + \text{Tr} \{Q^{-1} C\} \quad (15)$$
The cost function to be minimized can be written as
\[ C = \frac{1}{M} \sum_{m=1}^{M} (y(m) - D \mathbf{d}_1(m)) (y(m) - D \mathbf{d}_1(m))^H \]

Concentrating (15) with respect to \( Q \) yields (within constants)
\[ \Lambda_2(\tau_1, \alpha) = \ln |I_{LN} + \hat{W}^{-1} (\mathbf{D} - \hat{\mathbf{D}}) \hat{R}_{\alpha \alpha} (\mathbf{D} - \hat{\mathbf{D}})^H| \]  
(16)

where \( \hat{\mathbf{D}} = [\hat{\mathbf{d}}_\alpha \quad \hat{\mathbf{d}}_\lambda] = \hat{R}_{\alpha} \hat{R}_{\alpha}^{-1} \) is an unstructured estimate of \( \mathbf{D} \) and we have defined
\[ \hat{R}_{yy} = \frac{1}{M} \sum_{m=1}^{M} y(m) y^H(m) \]  
(17)
\[ \hat{R}_{\alpha \lambda} = \frac{1}{M} \sum_{m=1}^{M} \mathbf{d}_\lambda(m) \mathbf{d}_\lambda^H(m) \]  
(18)
\[ \hat{R}_{y \alpha} = \frac{1}{M} \sum_{m=1}^{M} y(m) \mathbf{d}_\alpha^H(m) \]  
(19)
\[ \hat{W} = \hat{R}_{yy} - \hat{R}_{y \alpha} \hat{R}_{\alpha \alpha}^{-1} \hat{R}_{y \alpha}^H \]  
(20)

Note that matrix \( \hat{W} \) is an unstructured estimate of the correlation matrix \( Q \). Minimizing \( \Lambda_2 \) with respect to the unknown parameters requires a multidimensional search over the parameter space. Instead, we determine approximate ML estimates by: i) keeping only the first term of the Taylor expansion of \( \Lambda_2 \), ii) replacing \( \hat{R}_{\alpha \lambda} \) by its asymptotic value, which is \( \sigma_\lambda^2 \mathbf{I} \). It can be proved that these approximations do not affect the asymptotic properties of the estimator. The cost function to be minimized can be written as
\[ \Lambda_3(\tau_1, \alpha) = -2 \text{Re} \left\{ \alpha^H \mathbf{p}_+ (\tau_1) \right\} + \alpha^H \mathbf{F}_+ (\tau_1) \alpha - 2 \text{Re} \left\{ \alpha^H \mathbf{p}_- (\tau_1) \right\} + \alpha^H \mathbf{F}_- (\tau_1) \alpha \]  
(21)
where we have defined\(^3\)
\[ \mathbf{p}_\pm (\tau_1) = \text{mat}_{x \pm \tau_1} \left\{ \hat{W}^{-1} \hat{d}_\pm \right\} \mathbf{a}_\pm (\tau_1) \]  
(22)
\[ \mathbf{F}_\pm (\tau_1) = (\mathbf{I}_L \otimes \mathbf{a}_\pm (\tau_1))^H \hat{W}^{-1} (\mathbf{I}_L \otimes \mathbf{a}_\pm (\tau_1)) \]  
(23)

At this point the minimization of (21) with respect to \( \alpha \) is immediate and yields
\[ \alpha_{ML} = (\mathbf{F}_1 (\tau_1) + \mathbf{F}_2 (\tau_1))^{-1} \left( \mathbf{p}_+ (\tau_1) + \mathbf{p}_- (\tau_1) \right) \]  
(24)

After substituting (24) into (21), the timing estimator is obtained as
\[ \hat{\tau}_1 = \arg \max_{\tau_1} \left( \mathbf{p}_+ (\tau_1) + \mathbf{p}_- (\tau_1) \right)^H \cdot (\mathbf{F}_\pm (\tau_1) + \mathbf{F}_\pm (\tau_1))^{-1} \left( \mathbf{p}_+ (\tau_1) + \mathbf{p}_- (\tau_1) \right) \]  
(25)

In order that \( \hat{W} \) be non-singular we need \( M \geq LN \), which may result in too large of a training sequence. However, if \( M \) is much bigger than the dimension of the signal subspace \( (D_x) \) of \( \hat{W} \) it is possible to obtain a non-singular estimate of \( Q \) with fewer than \( LN \) symbols. Thus, in our simulations we will use a parametric estimate obtained as the matrix with a \( D_x \)-dimensional signal subspace that is closest in the sense of the Frobenius norm to \( \hat{W} \) (a similar method is used in [4]). This procedure not only avoids the previous bound on \( M \) for the application of the estimator, but also improves the performance for all values of \( M \). This comes at the price of an increased computational load and requires knowledge/estimation of \( D_x \). A simpler alternative is to apply diagonal loading to the \( \hat{W} \), but simulation results have shown that the eigenanalysis method performs slightly better. Note that the advantage of exploiting the spatial structure of the signals is that every antenna adds \( N \) degrees of freedom to the system, whereas each user occupies only two degrees and each external punctual interferer occupies between 1 and \( N \) depending on its bandwidth.

5. CRAMER-RAO BOUND

It can be proved that the estimator presented above is consistent and asymptotically efficient as long as the signal to which it is applied corresponds to the model used in the derivation. However, it will be shown that the performance is also close to the CRB when applied in some real scenarios. The CRB matrix is block diagonal with respect to the noise \( \{ \eta \} \) and the signal parameters \( \eta = [\text{Re} \{ \alpha^T \}, \text{Im} \{ \alpha^T \}, \tau_1^T] \). The CRB for \( \eta \) is

\[ \text{CRB}^{-1}(\eta) = 2M \sigma_\eta^2 \text{Re} \left\{ \begin{bmatrix} \mathbf{F}_1 & j\mathbf{F}_1^H & \mathbf{F}_2 \alpha \\ -j\mathbf{F}_1^H & \mathbf{F}_1 & -j\mathbf{F}_2 \alpha \\ \alpha^H \mathbf{F}_1^H & j\alpha^H \mathbf{F}_1^H & \alpha^H \mathbf{F}_2 \alpha \end{bmatrix} \right\} \]

where
\[ \mathbf{F}_1 = (\mathbf{I}_L \otimes \mathbf{a}_+^H (\tau_1)) \mathbf{Q}^{-1} (\mathbf{I}_L \otimes \mathbf{a}_+ (\tau_1)) + (\mathbf{I}_L \otimes \mathbf{a}_-^H (\tau_1)) \mathbf{Q}^{-1} (\mathbf{I}_L \otimes \mathbf{a}_- (\tau_1)) \]  
(26)
\[ \mathbf{F}_2 = (\mathbf{I}_L \otimes \mathbf{a}_+^H (\tau_1)) \mathbf{Q}^{-1} (\mathbf{I}_L \otimes \mathbf{b}_+ (\tau_1)) + (\mathbf{I}_L \otimes \mathbf{a}_-^H (\tau_1)) \mathbf{Q}^{-1} (\mathbf{I}_L \otimes \mathbf{b}_- (\tau_1)) \]  
(27)
\[ \mathbf{F}_3 = (\mathbf{I}_L \otimes \mathbf{b}_+^H (\tau_1)) \mathbf{Q}^{-1} (\mathbf{I}_L \otimes \mathbf{b}_+ (\tau_1)) + (\mathbf{I}_L \otimes \mathbf{b}_-^H (\tau_1)) \mathbf{Q}^{-1} (\mathbf{I}_L \otimes \mathbf{b}_- (\tau_1)) \]  
(28)
\[ \mathbf{b} \pm (\tau_1) = \frac{\partial \mathbf{a} \pm (\tau_1)}{\partial \tau_1} \]  
(29)

6. SIMULATION RESULTS

In this section we compare the performance of our estimator, referred to as “space-time diversity”, with two of the estimators proposed to date that give the best results. Namely, we consider the methods presented in [5] and [6], which we will refer to as “time-diversity” and “space-diversity”, respectively. Two performance measures are analyzed: the acquisition probability \( (P_a) \) and the root mean squared error (RMSE) given correct acquisition. We define a correct acquisition to have occurred when the delay estimate is within a half-chip of the true value. A method is considered
for the estimator derivation and corresponds to a Doppler frequency \( f_d \) described in [1]. In this case, all the sources have a Gaussian angular spread equal to 5 degrees, the power of each source is distributed among its rays according to a Laplacian distribution, and the Doppler spectrum has the conventional U-shape with \( f_d T = 2 \cdot 10^{-3} \) [3] (e.g. this corresponds to a system with a 900 MHz carrier frequency, 50 kb/s data rate and 120 km/h speed).

We first consider the effect of the length of the training sequence \( M \). The results are shown in Fig 1. The estimator proposed in this paper is the only one that attains the CRB for the static channel, even though the Gaussian assumption is only an approximate one. This is achieved for lengths of the training sequence larger than 250 bits. For smaller values, there is a very slight degradation with respect to the CRB, which causes the difference between the RMSE and the CRB in the next figures. As expected, the performance of all the estimators deteriorates in the mobile channel, where the RMSE can not be further reduced by increasing \( M \). This impairment should not be interpreted as a failure of the estimators, but only as the effect of working in a much more complex scenario, and it will be visible in all the following results. Also in the mobile channel the use of “space-time div.” outperforms the other two approaches, and “time div.” presents the largest RMSE. In Figures 2 and 3 we investigate the effect of varying the number of users. This has special interest for a base station that uses spatial-division multiple access (SDMA), and therefore it may have more users than the length of the codes. Again the use of “space-time div.” gives better results, both in RMSE and \( P_{ac} \), than the other two methods. “Space div.” and “time div.” experience a serious deterioration when the number of users exceeds the length of the code (i.e. \( K > N \)), and they completely fail when \( K > 2N \). On the other hand, using the space-time estimator the number of users may be increased beyond twice the code length without an excessive degradation. For instance, note that for \( K = 40 \) users and the static channel the probability of acquisition remains virtually equal to 1, and only goes down to 0.82 for the mobile channel. Next, the effect of a wide-band external interference is analyzed (see Figures 4 and 5). The “space-time div.” and “space div.” estimators present an adequate performance when the signal-to-interference ratio (SIR) is small (i.e. \( > -10\) dB). Their performance is nearly insensitive to the SIR except for extremely low SIR in the mobile channel. Despite everything, the former gives better results than the latter in all the cases. Moreover, the “space div.” estimator is not (for the system parameters that we have considered) a near-far resistant estimator. In Figure 6 the near-far resistance of the different estimators is compared. The near-far ratio is defined as the ratio between the mean power of each interfering user and that of the desired user. The estimator proposed in this paper performs satisfactorily in the mobile channel up to a NFR equal to 35 dB, which is an improvement of 8 dB and 18 dB over the “time div.” and “space div.” estimators, respectively.

7. CONCLUDING REMARKS

A code-timing synchronization technique for DS-CDMA systems that operates in near-far slowly fading channels and employs an arbitrary antenna array for reception has been derived by applying the ML principle. It is assumed that the desired user transmits a known training sequence, and all other received components are modeled as a term with unknown space-time correlation. This approach fully exploits the spatial and temporal structure of the interfering

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**Figure 1:** RMSE vs Length of the training sequence \( M \). \( K = 10, N = 15, \text{EbNo} = 10\) dB, NFR = 10 dB

**Figure 2:** RMSE vs Number of users \( K \). \( M = 80, N = 15, \text{EbNo} = 10\) dB, NFR = 10 dB
signals in order to cancel them. As a result, the proposed technique outperforms existing synchronization methods for reasonable lengths of the training sequence and is indispensable for the correct acquisition and tracking of the synchronization parameters in heavily loaded systems and/or in the presence of external interference.

8. REFERENCES


