Dynamic UAV Relay Positioning for the Ground-to-Air Uplink

Feng Jiang and A. Lee Swindlehurst
Department of Electrical Engineering and Computer Science
University of California, Irvine
Irvine, CA, 92697, USA
Email: {feng.jiang, swindle}@uci.edu

Abstract—In this paper we consider a collection of mobile single-antenna ground nodes communicating with a multi-antenna unmanned aerial vehicle (UAV) on a multiple-access ground-to-air wireless communications link. The UAV uses beamforming to mitigate the inter-user interference and achieve spatial division multiple access (SDMA). In addition, the UAV dynamically adjusts its heading in order to maximize a lower bound on either the ergodic sum rate of the uplink channel or the minimum ergodic rate of the worst-case user, using a Kalman filter to track the positions of the mobile ground nodes. We present simulation results that demonstrate the benefits of adapting the UAV heading in order to optimize the uplink communications performance.

I. INTRODUCTION

There is increasing interest in the use of relatively small, flexible unmanned aerial vehicles (UAVs) that fly at lower altitudes for providing relay services for mobile ad hoc networks with ground-based communication nodes [1]–[4]. We consider such an application in this paper, assuming a system with a multi-antenna unmanned aerial vehicle (UAV) flying over a collection of \( N \) single-antenna mobile ground nodes. The UAV acts as a relay, sending the messages from the co-channel users on the ground to some remote base station.

Different methods have been proposed in the literature to optimize the performance of UAV-assisted networks. In [1], a throughput maximization protocol for non-real time applications was proposed for a network with UAV relays in which the UAV first loads data from the source node and then flies to the destination node to deliver it. The authors in [2] investigate different metrics for ad hoc network connectivity and propose several approaches for improving the connectivity through deployment of a UAV. A swarm of single antenna UAVs are used as a virtual antenna array to relay data between a fixed ad hoc network on the ground in [3] and the performance of distributed OSTBC and beamforming are evaluated. In [4], a relay system with multi-antenna UAVs and multi-antenna mobile user terminals is investigated. The user terminals employ OSTBC to transmit data and the data transmissions are assumed to be interference free. Based on knowledge of the user terminal’s future position, a heading optimization approach is proposed which can maximize the uplink data rate of the network under the constraint that each user terminal’s data rate is above a threshold.

In this paper, we consider a model similar to [4], with several ground-based users communicating simultaneously with a multi-antenna UAV. However, different with [4], in this work, the co-channel interference among the user terminals are considered. The user terminals are assumed to transmit data with a single antenna, and the UAV uses beamforming to separate the co-channel data streams. We assume a correlated Rician fading channel between each ground node and the UAV, and we use a lower bound on the ergodic achievable rate to quantify the uplink performance of the relay network, assuming that the UAV uses a maximum signal-to-interference-plus-noise ratio (SINR) beamformer for interference mitigation. The UAV is assumed to fly with a constant velocity \( v_u \), and it adjusts its heading in discrete time steps (assuming a constraint on the maximum turning radius) in order to optimize the bound on the achievable rate. In particular, at time step \( n \), the UAV uses a Kalman filter to predict the future positions of the ground nodes at time \( n + 1 \), and the UAV computes its heading in order to optimize the bound based on the future estimates of the positions of the ground nodes.

The organization of this paper is as follows. Section II presents the signal and channel model used in the paper. Section III describes the model for the UAV and user node’s mobility, and a Kalman filter is constructed for prediction of the user node’s position. We then formulate the heading optimization problem and propose an adaptive heading adjustment algorithm in Section IV. Simulation results are provided in Section V and conclusions are summarized in Section VI.

II. SYSTEM MODEL

A. Signal Model

We assume a UAV configured with an array of \( M \) antennas, and a collection of \( N \) mobile ground nodes each equipped with a single antenna. For simplicity, each ground node is assumed to transmit with the same power \( P_t \), but this assumption is easily relaxed. The signal received at the UAV array can thus be written as

\[
y_n = \sum_{i=1}^{N} \sqrt{P_t} h_{i,n} x_{i,n} + n, \tag{1}
\]

where \( h_{i,n} \in \mathbb{C}^{M \times 1} \) is the channel vector between node \( i \) and the UAV, the data symbol \( x_{i,n} \) is a complex scalar with zero mean and unit magnitude, \( n \in \mathbb{C}^{M \times 1} \) is zero-mean additive Gaussian noise with \( \mathbb{E}\{nn^H\} = \sigma^2 I_M \) and \( I_M \) denotes an
\( M \times M \) identity matrix. The UAV isolates the data from the \( i \)th node by multiplying \( y_n \) with a beamformer \( \mathbf{w}_{i,n} \). Assuming the channels \( \mathbf{h}_{i,n}, i = 1, \ldots, N \) are known to the UAV, the vector \( \mathbf{w}_{i,n} \) that maximizes the \( SINR_i \) is given by [5]

\[
\mathbf{w}_{i,n} = Q_{i,n}^{-1}\mathbf{h}_{i,n},
\]

where \( Q_{i,n} = \sum_{j=1, j \neq i}^{N} P_j \mathbf{h}_{j,n}\mathbf{h}_{j,n}^H + \sigma^2 I_M \). The corresponding \( SINR_i \) can be calculated as

\[
SINR_i = P_i |\mathbf{h}_{i,n}^H Q_{i,n}^{-1} \mathbf{h}_{i,n}|^2.
\]

B. Channel Model

We assume a correlated Rician fading channel between each user node and the UAV with consideration of large-scale path loss:

\[
\mathbf{h}_{i,n} = \frac{\mathbf{h}_{i,n}}{d_{i,n}},
\]

where \( \mathbf{h}_{i,n} \) is the normalized channel gain, \( d_{i,n} \) is the distance between node \( i \) and the UAV during the \( n \)th time step, and \( \alpha \) is the path loss exponent. Define the three dimensional coordinates of the UAV and node \( i \) as \( (x_{i,n}, y_{i,n}, h_{i,n}) \) and \( (x_{i,n}, y_{i,n}, 0) \), so that \( d_{i,n} \) is computed by

\[
d_{i,n} = \sqrt{(x_{i,n} - x_{i,n})^2 + (y_{i,n} - y_{i,n})^2 + h_{i,n}^2}.
\]

For node \( i \), we write the Rician fading channel vector \( \mathbf{h}_{i,n} \) with two components [6], a line of sight (LOS) component \( \mathbf{h}_{i,n}^l \) and a Rayleigh fading component \( \mathbf{h}_{i,n}^r \):

\[
\mathbf{h}_{i,n} = \mathbf{h}_{i,n}^l + \mathbf{h}_{i,n}^r.
\]

The LOS response will depend on the heading of the UAV (which determines the orientation of the array) and the angle of arrival (AOA) of the signal. For example, assume a uniform linear array (ULA) with antennas separated by one-half wavelength, and that at time step \( n \), the AOA for the signal from the \( i \)th node is \( \theta_{i,n} \) relative to the ULA, then the LOS component \( \mathbf{h}_{i,n}^l \) would be modeled as

\[
\mathbf{h}_{i,n}^l = \frac{K}{\sqrt{1 + K}} \begin{bmatrix} e^{j\pi \sin(\theta_{i,n})} & \cdots & e^{j(M-1)\pi \sin(\theta_{i,n})} \end{bmatrix}^T,
\]

where \( K \) is the Rician \( K \)-factor. Since there is little multipath scattering near the UAV, any Rayleigh fading components will experience high spatial correlation at the receive end of the link. Thus, we model the spatial correlated Rayleigh component as

\[
\mathbf{h}_{i,n}^r = \sqrt{\frac{1}{1 + K}} \mathbf{R}_r g_{i,n},
\]

where \( g_{i,n} \in \mathbb{C}^{M \times 1} \) has i.i.d entries with distribution \( \mathcal{CN}(0, 1) \), and \( \mathbf{R}_r \) is the correlation matrix seen from the receiver. According to the measurement channel model for ULA in [7], \( \mathbf{R}_r \) can be written as

\[
\mathbf{R}_r = \mathbf{h}_{i,n}^l \mathbf{h}_{i,n}^l^H \odot \mathbf{B}(\theta_{i,n}, \sigma_\phi),
\]

where \( \odot \) denotes the elementwise product and the \( (l,m) \)th element of \( \mathbf{B}(\theta_{i,n}, \sigma_\phi) \) is defined as

\[
\left[ \mathbf{B}(\theta_{i,n}, \sigma_\phi) \right]_{l,m} = e^{-\frac{1}{2} \left[ (l-m)^2 \sigma_\phi^2 \cos(\theta_{i,n}) \right]^2},
\]

where \( \sigma_\phi \) denotes the standard deviation of the distribution of the node \( i \)'s rays direction around the nominal direction \( \theta_{i,n} \). The resulting distribution of \( \mathbf{h}_{i,n} \) is thus

\[
\mathbf{h}_{i,n} \sim \mathcal{CN}\left(\mathbf{h}_{i,n}^l, \frac{1}{K+1} \mathbf{R}_r\right).
\]

C. Mobility Model and Position Prediction

We adopt a first-order auto-regressive (AR) model for the dynamics of the ground-based nodes [8], and we assume the nodes provide their location to the UAVs at each time step. The UAV in turn uses a Kalman filter to predict the nodes positions at the next time step. Node \( i \)'s state at time step \( n+1 \) is defined as:

\[
\mathbf{s}_{i,n-1} = [x_{i,n-1}, y_{i,n-1}, v_{x,i,n-1}, v_{y,i,n-1}]^T,
\]

where \( v_{x,i,n-1}, v_{y,i,n-1} \) denote the velocities in the \( x \) direction and \( y \) direction respectively. According to the AR model, the state of node \( i \) at time step \( n \) is given by

\[
\mathbf{s}_{i,n} = \mathbf{T} \mathbf{s}_{i,n-1} + \mathbf{w}_{i,n},
\]

where \( \mathbf{w}_{i,n} \sim \mathcal{N}(0, \sigma_w^2 \mathbf{I}_4) \) represents the process noise and \( \Delta t \) denotes the time step length. The matrix \( \mathbf{T} \) reflects the transition between states \( \mathbf{s}_{i,n-1} \) and \( \mathbf{s}_{i,n} \) and the vector \( \mathbf{w}_{i,n} \) represents the randomness of the state \( \mathbf{s}_{i,n} \). Because of feedback errors and delay, the UAV’s knowledge of the user node’s position is subject to inaccuracy. The noisy observation of node \( i \)'s position is expressed as

\[
\mathbf{z}_{i,n} = \mathbf{F} \mathbf{s}_{i,n} + \mathbf{u}_{i,n},
\]

where \( \mathbf{u}_{i,n} \sim \mathcal{N}(0, \sigma_u^2 \mathbf{I}_2) \) represents the observation noise. We assume a standard implementation of the Kalman filter, as follows:

**Initialization**

\[
\mathbf{x}_{i,0} = \mathbf{F} \mathbf{s}_{i,0}, \quad \mathbf{P}_{i,0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.
\]

**Prediction**

\[
\hat{\mathbf{s}}_{i,n|n-1} = \mathbf{T} \mathbf{s}_{i,n-1|n-1},
\]

\[
\mathbf{P}_{i,n|n-1} = \mathbf{T} \mathbf{P}_{i,n-1|n-1} \mathbf{T}^T + \sigma_w^2 \mathbf{I}_4.
\]

**Kalman gain**

\[
\mathbf{K}_{i,n} = \mathbf{P}_{i,n|n-1} \mathbf{F}^T (\mathbf{F} \mathbf{P}_{i,n|n-1} \mathbf{F}^T + \sigma_u^2 \mathbf{I}_2)^{-1}.
\]

**Measurement update**

\[
\hat{\mathbf{s}}_{i,n|n} = \hat{\mathbf{s}}_{i,n|n-1} + \mathbf{K}_{i,n} (\mathbf{z}_{i,n} - \mathbf{F} \mathbf{s}_{i,n|n-1}),
\]

\[
\mathbf{P}_{i,n|n} = (\mathbf{I}_4 - \mathbf{K}_{i,n} \mathbf{F}) \mathbf{P}_{i,n|n-1}.
\]
III. HEADING OPTIMIZATION ALGORITHM

In this section we propose an adaptive heading algorithm that calculates the UAV heading at time step \( n - 1 \) so that the network’s performance at time step \( n \) will be optimized.

A. SDMA Scenario

Due to the randomness of the Rayleigh component \( \tilde{h}_{i,n}, i = 1, \ldots, N \), the \( SINR_{i,n} \) is a random variable. The average sum achievable rate of the network can be calculated by

\[
C_n = \sum_{i=1}^{N} E\{\log_2(1 + SINR_{i,n})\} \\
\approx \sum_{i=1}^{N} \log_2 \left( 1 + E\{SINR_{i,n}\} \right).
\]

(23)

The UAV heading \( \delta_n \) will impact \( C_n \) in two ways. First, it will change the distance between the user nodes and the UAV during time step \( n \), which will impact the received power. Second, changes in the heading will change the AOA of the LOS component, which impacts the ability of the beamformer to spatially separate the users. At time step \( n - 1 \), based on the noisy observation \( z_{i,n-1} \), the UAV uses the Kalman filter to predict \( (\hat{x}_{i,n}, \hat{y}_{i,n}) \) and hence \( E\{SINR_{i,n}\} \). The heading optimization problem can thus be formulated as

\[
\max_{\delta_n} \sum_{i=1}^{N} \log_2 \left( 1 + E\{SINR_{i,n}\} \right) \\
\text{subject to} \quad |\delta_n - \delta_{n-1}| \leq \Delta \delta,
\]

(24)

where \( \Delta \delta \) represents that maximum change in UAV heading possible for the given time step.

The mean of \( SINR_{i,n} \) is calculated by

\[
E\{SINR_{i,n}\} = E\{P_i \tilde{h}_{i,n}^H \tilde{E}(\mathbf{Q}^{-1}_{i,n}) \tilde{h}_{i,n} \} \\
= \frac{P_i}{\delta_{i,n}} \left( \frac{K}{K + 1} \tilde{h}_{i,n}^H \tilde{E}(\mathbf{Q}^{-1}_{i,n}) \tilde{h}_{i,n} + \frac{1}{K + 1} \text{tr}(\mathbf{R}_e \tilde{E}(\mathbf{Q}^{-1}_{i,n})) \right),
\]

(25)

where \( \text{tr}(\cdot) \) denotes the trace operator. Calculation of the term \( E\{\mathbf{Q}^{-1}_{i,n}\} \) is very complicated, so instead we use the following lower bound based on Jensen’s inequality:

\[
E\{SINR_{i,n}\} \geq \frac{P_i}{\delta_{i,n}} \left( \frac{K}{K + 1} \tilde{h}_{i,n}^H \tilde{E}(\mathbf{Q}_{i,n})^{-1} \tilde{h}_{i,n} + \frac{1}{K + 1} \text{tr}(\mathbf{R}_e \tilde{E}(\mathbf{Q}^{-1}_{i,n})) \right),
\]

(26)

where \( E\{\mathbf{Q}_{i,n}\} = \sum_{j=1,j \neq i}^{N} \frac{P_j}{\delta_{j,n}} \left( \frac{K}{K + 1} \tilde{h}_{j,n}^H \tilde{h}_{j,n} + \frac{1}{K + 1} \mathbf{R}_e \right) + \sigma^2 \mathbf{I}_M \). We denote the lower bound on the right side of equation (26) as \( E_i\{SINR_{i,n}\} \) and substitute it into (24), leading to a related optimization problem:

\[
\max_{\delta_n} \sum_{i=1}^{N} \log_2 \left( 1 + E_i\{SINR_{i,n}\} \right) \\
\text{subject to} \quad |\delta_n - \delta_{n-1}| \leq \Delta \delta.
\]

(27)

Problem (27) requires finding the maximum value of a single-variable function over a fixed interval \( \delta_n \in [\delta_{n-1} - \Delta \delta, \delta_{n-1} + \Delta \delta] \), and thus can be efficiently solved using a one-dimensional line search. Since problem (27) aims at maximizing the achievable sum rate of the system, the algorithm may lead to a large difference in achievable rates between the users. As an alternative, we may wish to guarantee fairness among the users, in which case we can also formulate the following max-min problem:

\[
\min_{\delta_n} \max_{i} \frac{1}{\log_2 \left( 1 + E_i\{SINR_{i,n}\} \right)} \\
\text{subject to} \quad |\delta_n - \delta_{n-1}| \leq \Delta \delta.
\]

(28)

To solve the above problem, we first approximate the maximum function in (28) as [9]

\[
\max_i \frac{1}{\log_2 \left( 1 + E_i\{SINR_{i,n}\} \right)} \approx \left( \sum_{i=1}^{N} \frac{1}{\log_2 \left( 1 + E_i\{SINR_{i,n}\} \right)} \right)^{\frac{1}{p}},
\]

(29)

where \( p \) is a large positive number. Note that when \( p \) is very large, the sum of \( p \)th powers of a set of \( N \) positive terms mainly depends on the largest among the \( N \) terms and the \( p \)th root of the sum provides a good approximation for the value of the largest term. Substituting (29) into (28) yields the following approximate max-min heading approach:

\[
\min_{\delta_n} \left( \sum_{i=1}^{N} \left( \frac{1}{\log_2 \left( 1 + E_i\{SINR_{i,n}\} \right)} \right)^p \right)^{\frac{1}{p}} \\
\text{subject to} \quad |\delta_n - \delta_{n-1}| \leq \Delta \delta.
\]

(30)

The objective function in (30) is a continuous function depending only on \( \delta_n \) and thus the line search method can still be used to find the solution.

The proposed Adaptive Heading Algorithm is summarized in the following steps:

1) Use the Kalman filter to predict the user positions \( (\hat{x}_{i,n}, \hat{y}_{i,n}) \) based on the noisy observations at time step \( n - 1 \), and construct the objective function based on the predicted positions.

2) Use a line search to find the solution of problem (27) or (30) for \( \delta_n \in [0, 2\pi] \), and denote the solution as \( \delta_n \).

Calculate the heading interval \( \mathcal{O}_n = [\delta_{n-1} - \Delta \delta, \delta_{n-1} + \Delta \delta] \). If \( \delta_n \in \mathcal{O}_n \), set \( \delta_n = \delta_n \), else set \( \delta_n = \arg \min_{\delta} |\delta - \delta_n| \), where \( \delta = \delta_{n-1} - \Delta \delta \) or \( \delta_{n-1} + \Delta \delta \).

3) UAV flies with heading \( \delta_n \) during time step \( n \).

Note that the line search in step 2 is over \([0, 2\pi]\) rather than just \([\delta_{n-1} - \Delta \delta, \delta_{n-1} + \Delta \delta]\), and that the boundary point closest to the unconstrained maximum is chosen rather than the boundary with the maximum predicted rate. Thus, the algorithm may temporarily choose a lower overall rate in pursuit of the global optimum, although this scenario is uncommon.

B. TDMA Scenario

We also consider a TDMA scenario in which the time step is divided into equal-length time slots and each node is
assigned a time slot for data transmission. After maximum ratio combining at the receiver, the signal-to-noise ratio (SNR) of user $i$ is given by

$$SNR_{i,n} = \frac{P_i}{\sigma^2} \|h_{i,n}\|^2,$$

(31)

whose mean can be calculated as

$$E\{SNR_{i,n}\} = \frac{P_i M}{d_{i,n}^{\alpha} \sigma^2}.$$

(32)

For the TDMA scenario, the max-sum rate problem is formulated as:

$$\max_{\delta_n} \frac{1}{N} \sum_{i=1}^{N} \log_2 \left(1 + \frac{P_i M}{d_{i,n}^{\alpha} \sigma^2}\right)$$

subject to $|\delta_n - \delta_{n-1}| \leq \Delta \delta$.

(33)

Similar to the SDMA case, a max-min formulation for the TDMA case is given by

$$\min_{\delta_n} \frac{1}{N} \left( \sum_{i=1}^{N} \log_2 \left(1 + \frac{P_i M}{d_{i,n}^{\alpha} \sigma^2}\right) \right)^{\frac{1}{N}}$$

subject to $|\delta_n - \delta_{n-1}| \leq \Delta \delta$.

(34)

Both (33) and (34) can be substituted in step 2 Adaptive Heading Algorithm and implemented as described there.

IV. SIMULATION RESULTS

A simulation example involving a UAV with an 8-element ULA and three user nodes was carried out to test the performance of the proposed algorithm. The time between UAV heading updates was set to $\Delta t = 1s$, and the simulation was conducted over $L = 300$ steps. In the simulation, we assume the users have the same initial velocity, and then move independently according to the perturbed constant velocity model described earlier. The initial velocity of the nodes is $10m/s$, and their initial positions are $(0, 25)$, $(240, 20)$, $(610, 30)$ respectively (measured in meters). The elements of the process and measurement noise vectors are assumed to be independent with variances given by $\sigma_w^2 = 0.5$ and $\sigma_u^2 = 0.1$, respectively. The user’s transmit power is set to $P_d = 35dB$ and the path loss exponent is $\alpha = 1$. When $L = 150$, all the nodes make a sharp turn and change their velocity according to $v_{150}^p/v_{150}^t = -1.8856$. The initial position of the UAV is $(x_{u_0}, y_{u_0}) = (50, 100)m$ and its altitude is assumed to be $h_u = 200m$. The speed of the UAV is $v_u = 50m/s$, and the maximum heading angle change is set to be either $\Delta \delta = \frac{\pi}{3}$ or $\frac{\pi}{12}$. The standard deviation of rays direction is $\sigma_\phi = 0.1$.

Figs. 1-2, 4-5 show the trajectories of the UAV and mobile nodes for the SDMA and the TDMA scenarios assuming either max-sum or max-min objective functions and $\Delta \delta = \frac{\pi}{3}$. The decision-making behavior of the UAV is evident from its ability to appropriately track the nodes as they dynamically change position. Due to the relatively high speed of the UAV, loop maneuvers are necessary to maintain an optimal position for the uplink communications signals. Compared with the max-sum rate algorithm, the trajectory for the max-min approach attempts in general to reduce the maximum distance between the UAV and any given user. Figs. 3 and 6 show the ergodic sum rate for the different scenarios. For each time step, the rate is calculated by averaging over 1000 independent channel realizations. Results for both $\Delta \delta = \frac{\pi}{6}$ and $\frac{\pi}{12}$ are plotted. In general, increasing the turning radius will provide better performance since it decreases the extra distance that must be flown to complete a loop maneuver and the amount of time that the aperture of the array is aligned with the angle-of-arrival of each user’s signal (where the ability of the array to suppress interference is minimized). The benefit of using SDMA is also apparent from Figs. 3 and 6, where we see that a gain of approximately 2.5x in rate is achieved over the TDMA scheme. We also note that the obtained sum rate is only about 13% less than what would be achieved assuming no interference, indicating the effectiveness of the beamforming algorithm.
V. Conclusion

In this paper, we investigated the UAV relay positioning problem for the ground-to-air uplink. An adaptive heading algorithm was proposed that uses predictions of the user terminal positions and beamforming at the UAV to maximize SINR at each time step. Two kinds of optimization problems were considered, one that maximizes a lower bound on the average uplink sum rate and one that maximizes a lower bound on the minimum average single-user rate. Simulation results indicate the effectiveness of the algorithms in automatically generating a suitable UAV heading for the uplink network, and demonstrate the benefit of using SDMA over TDMA in achieving the best throughput performance.

References


