Performance of MIMO spatial multiplexing algorithms using indoor channel measurements and models

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Summary

Several algorithms have recently been proposed for multiplexing multiple users in multiple input multiple output (MIMO) downlinks. The ability of a transmitter to accomplish this using spatial methods is generally dependent on whether the users’ channels are correlated. Up to this point, most of the multiplexing algorithms have been tested on uncorrelated Gaussian channels, a best-case scenario. In this paper, we examine the performance of multiplexing multiple users under more realistic channel conditions by using indoor channel measurements and a statistical model for test cases. We use a block zero-forcing algorithm to test performance at various user separation distances, optimizing for both maximum throughput under a power constraint and minimum power under a rate constraint. The results show that for the measured indoor environment (rich scattering, non-line-of-sight), a separation of five wavelengths is enough to achieve close to the maximum available performance for two users. Since many spatial multiplexing algorithms require channel state information (CSI) at the transmitter, we also examine the performance loss due to CSI error. The results show that a user can move up to one-half wavelength before the original channel measurement becomes unusable. Copyright © 2004 John Wiley & Sons, Ltd.

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1. Introduction

Multiple input multiple output (MIMO) wireless systems offer the possibility of spatially multiplexing multiple users, creating an additional dimension for multiple access beyond time-, frequency- and code-division approaches. The uplink of a multi-user MIMO system follows a model for which numerous algorithms already exist; although non-trivial, this problem has been studied (either directly or indirectly) by numerous researchers. Less understood is the MIMO downlink or ‘multicast’ channel, where a multiple antenna transmitter (e.g. a basestation) attempts to send different messages to multiple users, each of which may be equipped with a multiple antenna receiver. If one assumes that all users have only one antenna, the problem is somewhat simplified, and optimal transmitter beamforming solutions exist [1,2], as well as sub-optimal, but simpler solutions [3–5]. When the users have multiple antennas, achieving

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space-division multiplexing in the downlink in a way that optimizes use of resources is considerably more complex. The theoretical limits of this channel have been considered in Reference [6], and several algorithms have been suggested recently for designing transmit vectors [7–12]. This work has shown that the availability of channel state information (CSI) at the transmitter can significantly improve the capacity of the system by allowing simultaneous transmission to multiple users in a way that minimizes inter-user interference. CSI is much more critical in the multi-user scenario than it is in single-user point-to-point MIMO systems, since without it interference mitigation is not possible.

The performance of the algorithms cited above has usually been evaluated for the idealized case of a Gaussian distributed channel with gains that are uncorrelated from antenna-to-antenna and from user-to-user. Such an assumption provides a best-case scenario since the algorithms rely on users with uncorrelated channels. Of course, it is clear that if two users are located too closely together, or if there is insufficient multipath scattering, this assumption will be violated. Thus, an important question to address is what propagation characteristics allow one to achieve effective spatial multiplexing. In this paper, we study aspects of this question for realistic indoor environments using data from two sources. The first is measurement data from an indoor MIMO channel sounding experiment [13], and the second is a statistical channel model that has been designed to simulate typical indoor channel conditions [14]. Channel sounding measurements have been used to study the multipath richness of the single-user MIMO channel [15–21], but to date there has not been a similar study for multi-user MIMO channels.

We address two important issues in this work. The first deals with how closely two users can be located in space before a significant reduction in spatial multiplexing performance (measured in terms of either capacity, total transmitted power or signal to interference and noise ratio [SINR]) is observed. The second is relevant when users in the network are mobile, and addresses the question of how far a receiver terminal can move before updated CSI is required. For the type of environment we studied, we find that, although the absolute performance of the algorithms tested is somewhat different depending on whether actual measurement data or synthetic models are used, the answers to these questions are not.

In the next section, we briefly outline the representative spatial multiplexing algorithms that were studied in this paper. Section 3 describes the experimental system that was used to collect the channel measurements, and Section 4 presents a corresponding statistical channel model for purposes of comparison. The results of our study are then presented in Section 5, which addresses the issue of inter-user channel correlation, and Section 6 investigates the effects of user mobility.

2. Spatial Multiplexing for Multi-User MIMO

The spatial multiplexing methods we tested more or less attempt to decompose the channel into independent sub-channels. There are two general approaches to doing this: (1) obtain sub-channels that are perfectly orthogonal, or (2) relax the orthogonality constraint to achieve sub-channels that balance inter-user interference with the effects of noise. The results presented later are based on simulations using three different algorithms, which we describe briefly in this section.

For a narrowband system with K users, let the matrix $H_j$ represent the matrix transfer function from the antennas of the base station with $n_T$ antennas to the $n_R$ antennas of user $j$. The $m_j$-dimensional vector $d_j$ represents the data transmitted to user $j$ at a specific time; each of the elements of this vector represents a different sub-channel for user $j$. The ‘modulation’ matrix $M_j$ is the $n_T \times m_j$ matrix of transmit vectors designed for user $j$, and the resulting signal received by user $j$ is:

$$x_j = \sum_{i=1}^{K} H_j M_i d_i + n_j$$  \hspace{1cm} (1)

$$= H_j M_j d_j + n_j$$  \hspace{1cm} (2)

$$= H_j M_j d_j + H_j \bar{M}_j \bar{d}_j + n_j$$  \hspace{1cm} (3)

where $n_j$ is additive noise, $M_j$ and $d_j$ are respectively defined as the combined modulation matrix and transmitted data vectors for all users:

$$M_j = [M_1 \ldots M_K]$$  \hspace{1cm} (4)

$$d^T = [d_1^T \ldots d_K^T]$$  \hspace{1cm} (5)

and $\bar{M}_j$ and $\bar{d}_j$ contain the same quantities for all users other than user $j$:

$$\bar{M}_j = [M_1 \ldots M_{j-1} M_{j+1} \ldots M_K]$$  \hspace{1cm} (6)

$$\bar{d}_j = [d_1^T \ldots d_{j-1}^T d_{j+1}^T \ldots d_K^T]$$  \hspace{1cm} (7)
The second term in Equation (3) represents all of the inter-user interference seen by user $j$. The spatial multiplexing algorithms that have been proposed recently in [5,7,8,11,22,23] try to reduce or eliminate this interference by choosing an $M_j$ that optimizes some appropriate criteria.

### 2.1. Block-Diagonalization

The first method we describe is an algorithm for generating orthogonal sub-channels reported independently in References [7,8,24–26], and referred to in References [8,25] as ‘Block-Diagonalization’ (BD). This algorithm attempts to completely cancel all inter-user interference, and is close to optimal at high SNR. Define

$$
\bar{H}_j = \begin{bmatrix} H_1^T & \ldots & H_{j-1}^T & H_{j+1}^T & \ldots & H_K^T \end{bmatrix}^T \tag{8}
$$

For channels where $n_T$ is large enough, specifically $n_T > \max_j \text{rank}(\bar{H}_j)$, it is possible to completely cancel out the interfering signals for each user, so that $H_j M_j = 0$ for all $i \neq j$. This can be done by choosing $M_j$ so that it lies in the null space of $\bar{H}_j$, as follows. Let $V_j^{(0)}$ be an orthogonal basis for the null space of $\bar{H}_j$, which can be derived from computing the singular value decomposition (SVD) of $\bar{H}_j$. We want to select an optimal combination of these null space vectors that maximizes the energy directed to user $j$ and avoids transmitting into the null space of $H_j$. To do this, we define the SVD of $H_j V_j^{(0)}$ as

$$
H_j V_j^{(0)} = U_j \begin{bmatrix} \Sigma_j & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_j^{(1)} \\ V_j^{(0)} \end{bmatrix}^H \tag{9}
$$

where $\Sigma_j$ represents the non-zero singular values, and $V_j^{(1)}$ the corresponding right singular vectors. The system modulation matrix $M_5$ is then chosen as

$$
M_5 = \begin{bmatrix} V_1^{(0)} V_1^{(1)} & V_2^{(0)} V_2^{(1)} & \ldots & V_K^{(0)} V_K^{(1)} \end{bmatrix} P^{1/2} \tag{10}
$$

where the values of the diagonal matrix $P$ are used for power distribution among the various data channels that have been created by the decomposition. The product of the system channel matrix

$$
H_S = \begin{bmatrix} H_1^T & H_2^T & \ldots & H_K^T \end{bmatrix}^T \tag{11}
$$

and modulation matrix $M_5$ now has a block-diagonal structure, which motivates the use of the term ‘Block-Diagonalization’ to refer to the algorithm.

The choice of the diagonal elements of $P^{1/2}$ is determined by the singular values in the various $\Sigma_j$ matrices, with the power allocation strategy being dependent on the desired optimization criterion. To maximize the total system throughput (sum capacity) given a constraint on total transmitted power, the singular values of all the $\Sigma_j$ matrices are sorted collectively, and power allocation determined by the water-filling solution [27]. The sum capacity is useful as an analysis tool, but it is often achieved at the expense of weaker users whose channels are less than ideal. Consequently, we also consider the dual, or ‘power control’ problem. Suppose each user in an spatial multiplexing system has requested an arbitrary quality of service (QoS). In this case, the requested transmission rate can be used as a constraint, and the power distribution can be calculated to minimize total transmitted power, which has the benefit of helping to reduce interference in a network with multiple base stations. To accomplish this, the values of $\Sigma_j$ for each user are considered separately, and power is added using the water-filling solution until the QoS is met for that user. The BD algorithm and its adaptations to both of these problems is explained in greater detail in Reference [8], where some additional extensions are also discussed.

### 2.2. Coordinated Zero-Forcing

Coordinated zero-forcing (CZF) [10,12] is a generalization of BD algorithm that allows for arbitrary channel dimensions, provided $K \leq n_T$. The basic concept is that the transmitter predicts the optimal receiver beamformers, and uses that information to design optimal transmit vectors. This allows additional degrees of freedom because some interference can be transmitted to a user if it falls in the nulls of that user’s beamformer. The CZF algorithm and others like it [23] are iterative because changing the transmitter’s beamformers changes the optimal solution for the receivers’ beamformers, and vice versa. Thus, it is necessary to alternatively update each set of beamformers until convergence. CZF allows some flexibility in determining the number of data streams, $m_j$, to be assigned to each user. The simulation results section provides some insight into this. The following summary of the algorithm assumes that $m_j$ has already been determined:

**Coordinated Zero-Forcing Algorithm**

1. For each user, initialize $W_j$ as the $m_j$ dominant left singular vectors of $H_j$, and define $\bar{H}_j = W_j^H H_j$. 

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(2) For each user, define $\tilde{H}_j = [H_{j1}^T \ldots H_{jLj}^T]^T$, and compute the SVD

$$H_j \tilde{V}_j^{(0)} = \left[ U_j^{(1)} \ U_j^{(0)} \right] \Sigma_j \left[ V_j^{(1)} \ V_j^{(0)} \right]^H$$

where $U_j^{(1)}$ and $V_j^{(1)}$ represent the first $m_j$ left- and right-singular vectors. Update the transmitter and receiver beamformers: $W_j = U_j^{(1)}$ and $M_j = V_j^{(1)}$, and define

$$S = \left[ W_1^T H_1 \right] \ldots \left[ W_K^T H_K \right]$$

(3) Repeat step 2 until

$$\min_{i=1,...,K} \sum_{j \neq i} |S_{i,i,j}| < \epsilon$$

for some value of $\epsilon$.

(4) Use water-filling to determine power allocation given the diagonal values of the $\Sigma_j$ matrices as the channel gains.

Note that the reason power allocation is not necessary until step 4 (rather than at step 2) is that since zero-forcing is used, the sub-channels do not interfere with each other. If $m_j = n_{R,j}$, this algorithm is equivalent to the BD algorithm and the solution is achieved after only one iteration. In a single-user MIMO channel, if the channel has rank $L$, the solution that maximizes capacity will use all $L$ available sub-channels. In a multi-user channel, if $H_j$ has rank $L_j$, transmitting to user $j$ with $L_j$ sub-channels would maximize capacity for user $j$, but does not necessarily maximize system capacity, since it is possible to benefit other users by choosing $m_j < L_j$. We investigate the question of system performance as a function of $m_j$ in the simulation results section.

2.3. Hybrid Spatial Multiplexing Algorithm

In addition to the orthogonal BD and CZF algorithms, a non-orthogonal spatial multiplexing method is also tested in Section 6. The method studied is a hybrid algorithm proposed in References [9,10]. The algorithm consists of two steps: computing the CZF solution followed by a coordinated beamforming algorithm. The number of data streams for each user is assumed to be assigned in advance, using an algorithm such as the one in Reference [12]. The effects of this choice are illustrated later in the simulation results. The optimal sub-channel power allocation is assumed to be determined via the Hughes–Hartogs bit-loading algorithm [28,29]. The hybrid algorithm is summarized below:

Hybrid Spatial Multiplexing Algorithm

(1) Given $R_j$, the required transmission rate, and $m_j$, the number of data streams, for each user, compute the CZF solution and use bit-loading to determine the sub-channel power distribution that meets the rate constraints for all users. Let $\gamma_{j,1} \ldots \gamma_{j,m_j}$ represent the resulting minimum sub-channel SINRs for user $j$ and let $w_{j,l}$ be the $l$th column of $W_j$.

(2) Coordinated beamforming:

(i) Define $R_{j,k} = H_j^H w_{j,k} u_{j,k}^* H_j$. Find the unit vectors $u_{j,k}$ and gain coefficients $\sqrt{\zeta_{j,k}}$ such that $\sum_l \zeta_{j,k}$ is minimized and such that the SINR constraints are met:

$$\zeta_{j,k} u_{j,k}^* R_{j,k} u_{j,k} - \sum_{j \neq l, k \neq m} \gamma_{j,k} \zeta_{l,m} u_{l,m}^* R_{j,k} u_{l,m} \geq \gamma_{j,k}$$

This expression assumes that the channel matrices have been normalized so that the noise has unit variance.

(ii) Recalculate the predicted receiver weights $w_{j,l}$ according to the algorithm used at the receiver (e.g. receiver beamformers based on minimum mean-squared error or maximal ratio combining criteria), and normalize so that $w_{j,l}^* w_{j,l} = 1$.

(iii) Repeat steps (i) and (ii) until convergence.

The problem in step 2 can be solved by the optimal downlink beamforming algorithm of Reference [1]. If the optimal number of sub-channels per user and associated SINR requirements could be known in advance, step 2 would result in the optimal solution for the given set of users and rate requirements. However, it is difficult to know this in advance, so the zero-forcing step (step 1) serves the purpose of providing an intelligent estimate of these values, resulting in a solution that is not guaranteed to be optimal, but will be better than equal power distribution. The zero-forcing step also provides a good initialization point that has been shown to reduce the number of iterations required for step 2 to
converge. Although it requires more computation, the hybrid algorithm will find a solution that requires less power than the corresponding zero-forcing result, since it allows for a larger set of possible solutions. More details on the algorithm can be found in References [9,10].

3. Experimental Channel Measurements

A narrowband custom-made MIMO channel probing system designed and built at Brigham Young University (BYU) was used to collect channel measurements. Some details of the measurement system are discussed here, but a more complete description of the system, including diagrams, can be found in Reference [16]. For the measurements used here, the system was equipped with ten monopoles forming a uniform circular array at both transmit and receive. The system was operated at a carrier frequency of 2.43 GHz, and the elements were positioned in a circle with a radius of 0.86 wavelengths so that the separation between adjacent elements was approximately one-half wavelength. The resulting $10 \times 10$ MIMO channel was sampled every 2.5 ms with a measurement bandwidth of 25 kHz.

The measurements used in this study were collected on the fourth floor of the Clyde Engineering Building on the BYU campus, which is constructed with steel-reinforced concrete structural walls and cinder-block partition walls. The measurements were taken with the transmitter in a fixed location and the receiver moving along a long corridor as illustrated in Figure 1. All channels in this scenario are non-line-of-sight (NLOS), which generally allows for better multipath diversity but with reduced gain compared to the line-of-sight (LOS) case. A total of 10 000 samples of the channel were taken over a length of 42.6 m, which corresponds to 29 samples per wavelength. For a more detailed description of the measurement scenario and the equipment, see Reference [13]. Multi-user scenarios were created by using multiple points along the path as channels for different users. To test the effects of a particular separation distance $d$, channel measurements at points separated by distance $d$ were compared along the entire length of the measurement set to calculate average and worst-case performance values.

Most of the test cases considered scenarios with fewer antennas than the original $10 \times 10$ data set. Appropriate antenna subsets were selected as follows. On the transmit side, antennas with maximal separation were chosen to mimic a fixed basestation that uses the entire 0.86 wavelength array aperture. For example, the solid circles in Figure 2 indicate how a subset of four antennas would be chosen from among the ten possible antennas for the transmitter. A mobile receiver, on the other hand, would be expected to have limited size, and thus only adjacent antennas were used in forming the subarrays for the end users.

An important issue that arises in MIMO channel data sets is how the various channels are normalized prior to processing. There are two common approaches. The first is to normalize over the entire data set, so that the ensemble of all of the measured channels has a given average size (usually measured

![Fig. 1. Map of the location of the measurement data.](image-url)
in terms of the Frobenius norm). This approach preserves relative power relationships between different channel samples, but is subject to large fluctuations due to multipath fading and variations in the number of walls between transmitter and receiver. We refer to this normalization approach as global channel normalization (GCN). The second approach is to scale each individual channel sample to have the same Frobenius norm, and is referred to as local channel normalization (LCN). This approach allows for more consistent comparisons with simulated data and makes the results less dependent on the specific physical environment. For the sake of comparison, both methods were used in the results that follow.

4. Statistical Model

Channel measurements are of great value in accurately predicting algorithm performance, but obtaining large quantities of measurement data can be prohibitive. When simulating communication systems, it is useful to be able to test them over a very broad range of channels, rather than on a data set from one specific location or group of locations. For this reason, statistical models are useful. Assuming that a model accurately reflects the channel conditions likely to be encountered, it is a relatively simple matter to generate large quantities of channels for simulation purposes. To complement the measured channels used in testing various spatial multiplexing algorithms, we also employ randomly generated data from a realistic statistical channel model.

The model used in this paper is the ‘double bounce’ indoor channel model proposed in Reference [14], which is a modified version of the models originally developed in References [30,31]. When applied to MIMO channels, a good statistical match was observed with measured capacities in Reference [32] and for a similar model in Reference [33]. The model groups arrivals in to clusters, where the time of each cluster is Poisson with parameter $\lambda$ and the time within each cluster is also Poisson with parameter $\lambda$. The amplitudes have a Rayleigh distribution, where the mean is characterized by two decay parameters $\Gamma$ and $\gamma$, due to exponential decay in time of the cluster amplitudes and the ray amplitudes within each cluster. The clusters are scattered uniformly in angle, and the members of each cluster are randomly distributed in angle with respect to the cluster mean, characterized by a standard deviation $\sigma$.

For MIMO channel simulations, it is also necessary to characterize the angle-of-departure at the transmitter. While this quantity was not explicitly measured in Reference [31], it is reasonable to assume that for the indoor environment studied here, the departure angle statistics are similar to and statistically independent of the arrival angle statistics [14]. As shown in Reference [32], this model matches measured MIMO channels well, but it lacks a mechanism for characterizing channel changes when either the transmitter or receiver moves an arbitrary distance. As discussed below, one approach to modeling motion is to base the multipath arrivals on the physical locations of scatterers in the channel, rather than simply time and angle-of-arrival [34,35].

To incorporate motion in our channel simulations, the double-bounce model of Reference [14] takes a modified version of the statistical model described above and uses it to generate scatterer locations. To do this, a random set of clusters are generated with their associated times, amplitudes, and angles as seen by the transmitter and receiver. It then assumes that each of the waves detected by the receiver are due to two bounces, one local to the transmitter and one local to the receiver, and calculates the location of the scatterers that produce the desired time and angle-of-arrival. This is illustrated in Figure 3. Given the location of each scatterer, it is then possible to allow a limited amount of mobility on the part of either the transmitter or receiver or both, which is adequate for
the distances considered here. The clusters are distributed uniformly in angle, and the angles of arrival within a cluster have a Gaussian distribution with respect to the angular center of the cluster. In order to simplify the model, the specific arrival times of each ray within a cluster are ignored, and are determined by the propagation distance specified by the angles-of-departure and arrival. This eliminates the need for the $C_21$ parameter. Likewise, all members of a cluster are assumed to have equal mean amplitude, eliminating the $C_{13}$ parameter. The original model in Reference [31] assumes that the arrivals and clusters continue to be generated until some noise floor is reached. Here, we simplify things further by fixing the number of clusters and arrivals per cluster in advance.

### 4.1. Model Parameters

In this section, we briefly discuss the choice of the model parameters that were used in the simulation. Since the measurement data we are using as a reference are narrowband, we conducted only narrowband tests on the simulated channels, so the time of arrival component was ignored in synthesizing the MIMO array responses from the path data. Thus, the main use of the time of arrival parameter $\lambda$ is in predicting the total delay spread, which influences the maximum distance at which a scatterer may be located from the transmitter or receiver. We now analyze the relationship between the amplitudes of each cluster with other clusters as a function of the model parameters.

The mean amplitude of the arrivals in a cluster decays exponentially over time, and the actual amplitudes are Rayleigh distributed with respect to the mean. Since the phase is uniformly distributed over the interval $[0,2\pi]$, each arrival is effectively a Gaussian random variable with zero mean and variance $\sigma_n^2$ for the $n$th cluster. Each $\sigma_{n+1}^2$ is related to the previous $\sigma_n^2$ by the same distribution. The relationship between the variance for arrivals $n$ and $n+1$ is

$$\sigma_{n+1}^2 = \sigma_n^2 e^{-\Delta T/\Gamma}$$  \hspace{1cm} (12)

where $\Delta T_n = T_{n+1} - T_n$ is the difference in time of arrival between clusters $n$ and $n+1$. The statistical model characterizes this time difference using an exponential distribution with parameter $\Lambda$:

$$f(\Delta T) = \Lambda e^{-\Lambda \Delta T}$$  \hspace{1cm} (13)

This results in the following distribution for $\sigma_{n+1}^2$ with respect to $\sigma_n^2$:

$$f(\sigma_{n+1}^2 \mid \sigma_n^2) = \Lambda \Gamma \left( \frac{\sigma_{n+1}^2}{\sigma_n^2} \right)^{\Lambda.1 - 1}$$  \hspace{1cm} (14)

It can be shown that the mean of this distribution is:

$$E(\sigma_{n+1}^2 \mid \sigma_n^2) = \frac{\Lambda \Gamma}{\Lambda \Gamma + 1} \sigma_n^2$$  \hspace{1cm} (15)

With this information, it is possible to predict the relationships between cluster amplitudes without time-of-arrival information. The original measurement data used to derive the model parameters for the Clyde Building at BYU yielded estimates of $1/\Lambda = 17$ ns and $\Gamma = 34$ ns, resulting in a product $\Lambda \Gamma \approx 2.0$ (note that this product is unitless). The expected amplitude of each cluster with respect to the previous one is thus 0.67.

In order to verify that reasonable values for the model parameters have been chosen, the singular values of the measured and simulated channel matrices were compared. In a channel with several clusters uniformly distributed in angle, there is likely...
to be some correlation between the amplitude spread of the clusters and the eigenvalue spread of the associated \( H \) matrix. To compare the properties of the measured and synthetic \( H \) matrices, we calculated the singular values of each measured \( H \) matrix, and a corresponding number of model-generated \( H \) matrices. Let \( \alpha_n \) represent the \( n \)th singular value of \( H \). We calculated \( \alpha_n/\alpha_{n+1} \) for \( n = 1 \ldots n_T - 1 \), and plotted the mean values of these ratios in Figure 4. The figure compares the case of \( \Gamma = 2.0 \), which was chosen based on previous measurements at higher frequencies, to a higher value of \( \Gamma = 4.0 \), which reduces the decay of secondary multipath components and more closely matches the singular values of the measured matrices. The larger discrepancy between the curves for the smaller singular values can be attributed to the fact that the statistical model assumed a limited number of clusters, and to the presence of some limited diffuse scattering in the measured data. This is not a significant issue since small subarrays are used in the simulation results, and only the first few singular values are important.

Table I lists all of the parameters that were used in the simulations presented in the next two sections.

### Table I. Model parameters used to generate synthetic channels.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2.43 GHz</td>
</tr>
<tr>
<td>Cluster rate of arrival (1/( \Lambda ))</td>
<td>17 ms</td>
</tr>
<tr>
<td>Cluster decay constant (( \Gamma ))</td>
<td>68 ms</td>
</tr>
<tr>
<td>Number of clusters</td>
<td>7</td>
</tr>
<tr>
<td>Scatterers per cluster</td>
<td>7</td>
</tr>
<tr>
<td>Angular Azimuth spread in clusters (( \sigma ))</td>
<td>25°</td>
</tr>
<tr>
<td>Tx – Rx separation distance (( R ))</td>
<td>10 m</td>
</tr>
<tr>
<td>Channel sample spacing</td>
<td>0.05 wavelengths</td>
</tr>
<tr>
<td>Total samples per channel realization</td>
<td>200</td>
</tr>
<tr>
<td>Number of channel samples</td>
<td>100</td>
</tr>
</tbody>
</table>

5. Effects of Inter-User Separation

In the following, the notation \( \{n_{R_1}, n_{R_2}\} \times n_T \) is used to indicate a scenario involving two receivers with \( n_{R_1} \) and \( n_{R_2} \) antennas respectively, and a transmitter with \( n_T \) antennas. In studying the effects of inter-user separation on spatial multiplexing performance, we tested two channel geometries: \( \{2, 2\} \times 4 \) and \( \{4, 4\} \times 10 \) in the measured and simulated environments. Throughout this section, various performance metrics have been computed as a function of the physical separation distance since this measure is easy to relate to the measured environment.

Figure 5 shows the results of implementing power minimization via BD for both channel geometries with two users separated by distances of 0.5, 1.5 and 5 wavelengths. The channels in the data set were scaled using GCN in this example. The power allocation was computed such that a total of 3 bits/symbol could be transmitted to each user at a symbol error rate of \( 10^{-5} \) while minimizing transmit power, and the data in the plot is the CCDF of the total transmitted power. The power fluctuations in the data set are evident in the irregular shape of the CCDFs. Due to the higher received power at certain locations along

Fig. 4. A comparison of the mean singular values of the \( H \) matrices generated from measurements and models.
the measurement track, the measurement-based channel occasionally outperforms an IID channel. Figure 6 shows results computed from the same data, this time using local normalization (LCN) and using the BD algorithm to maximize sum capacity given a constraint on total transmitted power of 10 dB relative to the noise level. In this case, the measured channels never outperform the IID channel. As expected, capacity increases with user separation. In the case of the \{2, 2\} × 4 channel, the capacities of the measured data approach the IID capacity, while in the case of the \{4, 4\} × 10 channel, they do not. This implies that the number of usable multipath components of the channel is less than four. This is not unexpected, since even in cases where the channel has full rank, the amplitude decay of the secondary multipath components will typically make the capacity somewhat less than that of IID channels.

Figure 7 illustrates the results for both sum capacity at a transmitted power of 10 dB above the noise level and the required transmitted power for 3 bits/symbol per user at a symbol error rate of \(10^{-5}\) as a function of separation distance from 0.5 to 10 wavelengths. There are two sets of capacity curves, one representing 10% outage, and the other labeled ‘minimum capacity’, is the worst case over the entire data set. The transmit
power results are based on GCN and represent the median of all realizations in the data set. While it is evident from Figure 6 that the \( \{4, 4\} \times 10 \) channel does not quite reach the IID channel capacity, it is clear that in both scenarios, five wavelengths of separation yields the maximum possible channel decorrelation. This five wavelength lower bound was also observed in other simulations not shown here involving a ten element transmitter with ten single-antenna users. At the carrier frequency of 2.43 GHz, this represents a distance of approximately 60 cm. This is a promising result since it is unlikely for two mobile users to operate wireless devices at distances much closer than this. It is important to note that this separation distance depends on the environment and the antenna arrangement. For instance, in an outdoor setting with LOS the channel, it is expected that larger separation distances might be required due to less multipath. However, most MIMO investigations have targeted environments with rich multipath such as the one studied here since the capacity of MIMO systems depends strongly on the multipath content.

Figures 8 and 9 are the same as Figures 6 and 7, except that they are derived from synthetic channels based on the statistical model described in the previous section. The model was used to generate scatterer locations, which were used to compute the matrix channel transfer functions of two users at various separation distances. Similar to the measurement data, the users were placed at a constant spacing

![Fig. 7. Sum capacity and power minimization performance as a function of separation distance for a two-user MU-MIMO system using measurement data.](image1)

![Fig. 8. CCDFs of sum capacity as a function of separation distance for a two-user MU-MIMO system using the statistical channel model. The data are normalized with local channel normalization (LCN).](image2)
and then moved several wavelengths. The direction of separation of the users was chosen randomly. In Figure 6, the capacity of the \( \{2,2\} \times 4 \) channels is higher than in the measurements, such that it is even higher than the IID case. This can happen when there is less total scattering, concentrating the energy in a few dominant multipath components. In Figure 7, it is clear that the relative values as a function of separation follow the same trend as the measurement data in Figure 9, with near-maximum performance at a user separation of about five wavelengths.

Figures 10 and 11 examine the performance of the CZF algorithm. In Figure 10, we examined the performance of three-user \( \{10,10,10\} \times 10 \) channels. Similarly to the two-user cases studied here, we generated test cases by moving the three users along the entire length of the data set at fixed separation distances. Since there are two separation parameters in this case, we tested all possible combinations of the two separation distances, resulting in the three-dimensional plot of Figure 10. In all cases, \( mj \) was fixed at three sub-channels for all users. The results show the same trend as the two-user channels, although there is a slightly larger increase in capacity when the minimum user separation is increased from five to ten wavelengths.

Figure 11 illustrates the capacity of a \( \{5,5\} \times 10 \) channel when CZF is used, as a function of both separation distance and the number of data streams (\( mj \)) allocated to the two users. The IID case is also
included as a reference. For these dimensions, choosing $m_j = 5$ results in CZF and BD solutions being equivalent. Interestingly, both the measurement-based and IID generated channels benefit from reducing the number of data streams per user to 3. This can be attributed to the interaction between the two users’ channels. For example, using only three sub-channels for one user allows more degrees of freedom for the other user to optimize its spatial channel allocation.

The performance at a separation of ten wavelengths in comparison to the IID channels suggests that there are two to three dominant paths that contribute most of the capacity of the channel. It is also interesting to note that for only one sub-channel per user, the performance varies very little from 0.5 to 10 wavelengths of separation, due to the fact that there is sufficient multipath richness to provide adequate performance for each user even when the two arrays are virtually superimposed. This consistent performance may make setting $m_j - 1$ for all users all the time an attractive design choice for some systems because this special case also provides reductions in computational complexity [10].

### 6. Effects of User Motion

The second application of the channel data was to measure the effects of channel latency due to user motion. The envisioned scenario in this case is that the transmitter is obtaining its estimates of the channel via feedback from the mobile users. By the time the transmitter has received and processed a channel estimate, the receiver may have moved slightly, causing a change in CSI. A similar scenario is considered in Reference [36]. The error due to user motion is in addition to estimation error that is already present due to additive noise in the signal received by the mobile from the base. Minimizing CSI error is critical in multi-user scenarios since inaccurate channel estimates reduce the effectiveness of interference reduction in spatial multiplexing algorithms.

The effect of CSI error due to user motion was quantified by measuring the mean SINR degradation of the sub-channels. Let $H_S$ represent the true channel matrix, and $\hat{H}_S$ be the information available to the transmitter that is corrupted by user motion and estimation errors. If $m_{ij}$ is the transmit beamformer for sub-channel $j$ of user $i$, and $w_{ij}$ is the corresponding receiver beamformer, then the SINR for the sub-channel at the output of the receiver is

$$\text{SINR}_{ij} = \frac{m_{ij}^* H_S^* w_{ij}^* m_{ij}}{\sum_{k,l \neq i,j} m_{ij}^* H_S^* w_{ij}^* m_{ij} + \sigma^2}$$

where $\sigma^2$ is the noise power. Since $m_{ij}$ is designed based on corrupted channel information, the true SINR will be different than what was intended. This SINR degradation is measured by comparing $\text{SINR}_{ij}$ computed using the true channel $H_S$ to the SINR resulting from the available channel information $\hat{H}_S$. The corrupted CSI was generated by taking a given measured or synthetic channel at one location, displacing it by a distance $d$ to obtain $H_{S,d}$, and then adding an estimation error term $N$ composed of uncorrelated Gaussian elements: $H_S = H_{S,d} + N$. The relative size of these quantities is specified using the ‘signal to
estimation error’ ratio $\|H_{S,D}\|^2_F/\|N\|^2_F$. A total of 1000 test cases were generated by randomly selecting locations for the two users in the data set. All channels were of dimension $\{2, 2\} \times 4$. Since the locations are selected randomly, it is very unlikely that the two users ever had a very small separation distance, so their channels can be considered to be statistically independent. GCN was used to include the effects of power fluctuations at different points in the measurement set.

For each realization, the channel was decomposed into orthogonal sub-channels using both BD and the hybrid algorithm. Each user was moved in a randomly chosen direction and the resulting SINR degradation was averaged over ten samples. Note that any direction of motion was possible with the statistical model, while motion was restricted to a straight-line path in the measured data. Figure 12 illustrates the SINR loss as a function of the estimation error and the distance between where the channel was measured and where it was used. Each point on the curves represents the median SINR loss over 1000 trials. The median is used rather than the mean in this case since it is not altered by the conversion into log space. Data for both orthogonal multiplexing (BD algorithm) and non-orthogonal multiplexing (hybrid algorithm) are included. The three curves for each case are for relative estimation error powers of 5, 10 and 15 dB below the power of $H_{S,D}$. Above a 15 dB, there is no noticeable improvement in performance, and below 5 dB, the performance degradation increases very quickly, and so is not included here.

Figure 13 shows a plot similar to Figure 12, but based on synthetic data where the two users’ channels
were generated independently of each other. The plot generally confirms the findings of the previous one, except that the performance reduction as the separation increases toward 1 wavelength is slightly greater, and the degradation is non-monotonic. One possible explanation of this behavior is correlation in the fading characteristics. Channels with a higher degree of spatial correlation will have more accurate channel estimates than channels with a lower degree of spatial correlation at the same measurement distance. Thus, a local maximum in the channel degradation would correspond to a local minimum in correlation. In Figure 13, the local maxima approximately correspond to the nulls that result from Jakes’ correlation model, which results from the structure imposed by the channel model.

Figures 12 and 13 also provide useful insights into the trade-offs of choosing orthogonal versus non-orthogonal multiplexing algorithms. In general, the non-orthogonal algorithms are capable of achieving better performance because they consider a larger set of possible solutions. Because we are considering only degradation rather than raw performance, this is not apparent from these plots, but it has been demonstrated elsewhere [10]. However, these plots demonstrate that non-orthogonal algorithms, in addition to having better overall performance, are more robust to channel estimation error. This is a consequence of the fact that they are already designed to tolerate a certain amount of inter-user interference, while the orthogonal methods are not.

Assuming one can tolerate an SINR loss of 3 dB due to channel mismatch, the above results show that motion in the order of one-half wavelength will invalidate the original channel estimate. At 2.43 GHz, this corresponds to a distance of only 6 cm. Assuming that the maximum speed of a mobile user indoors is around 5 m/s (walking speed), the required channel update rate would be in the order of 10 ms. Recent results [37] indicate that this rate could be significantly reduced through the use of MIMO channel prediction.

The synthetic and measured data considered in this paper have been strictly for an NLOS indoor environment, which has been observed to have somewhat more favorable characteristics for MIMO transmission than outdoor channels [38]. Users in outdoor environments will have higher mobility, but there is typically less multipath scattering and also the possibility of LOS propagation, so a user may be able to move a greater distance before a significant change in the channel occurs, and the orientation of the array and the users with respect to each other will have a greater bearing on performance. Thus, it is possible that the algorithms tested on the indoor data presented here may still be useful in outdoor channels, but further tests are necessary to validate such a claim.

7. Conclusion

Algorithms for spatial multiplexing in multi-user MIMO systems can substantially increase the capacity of a wireless network, assuming that accurate channel state information is available, and that the channels for different users are uncorrelated. In this paper, several multi-user MIMO downlink algorithms were tested against measured and synthetic indoor channels to determine their sensitivity to user motion and correlation among the users’ channels. Our analysis indicates that most of the potential throughput can be achieved at user separations as little as five wavelengths, and that even at shorter distances the performance can still be quite acceptable. This is a very promising result for future multi-user MIMO systems deployed in NLOS indoor environments. However, it is important to note that the separation distance depends on the environment under study and that further measurement studies are needed in order to find the required separation distances in other environments such as outdoor LOS environments. Measurement of the effects of channel estimation error reveals that motion in the order of 0.5 wavelengths between the channel measurement and the use of the measurement by the transmitter can provide acceptable performance if the expected losses are built into the design requirements.

We have also compared orthogonal multiplexing and a more computationally expensive non-orthogonal multiplexing algorithm in the presence of channel estimation error. The non-orthogonal algorithm, which has previously been shown to require less power to achieve the same data rates as the orthogonal algorithm, was observed here to be also more robust to errors in the channel information. This robustness comes from the fact that the transmit vectors are already designed to tolerate a certain amount of inter-user interference. Thus, the two algorithms provide a trade-off between cost and performance.

References

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Authors’ Biographies

Quentin H. Spencer received the B.S., M.S. and Ph.D. degrees in electrical engineering from Brigham Young University (BYU), Provo, Utah, U.S.A., in 1994, 1996 and 2004, respectively. From 1995 to 1996, he was a research assistant with the Department of Electrical Engineering, BYU, where he worked on propagation modeling for indoor wireless communication channels. From 1996 to 1997, he was an engineer with L-3 Communications, Salt Lake City, Utah, U.S.A. working on broadband military communication systems. From 1998 to 2003, he was a research assistant at BYU, where he worked on signal processing algorithms for MIMO wireless communication systems. During that time, he also worked concurrently as a control systems engineer at UniDyn Corporation, Orem, Utah, U.S.A. from 1999 to 2001, as a signal processing engineer at Inari Corporation, Draper, Utah in 2001. During 2002, he was a guest researcher at the Technical University of Ilmenau, Germany and Royal Institute of Technology, Stockholm, Sweden. Since 2003, he has been at Distribution Control Systems, Inc., Hazelwood, Missouri, U.S.A., where he leads research efforts in powerline communication systems. His research interests are array signal processing, and other applications of signal processing to communication systems. He is a member of the IEEE.

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