Abstract—Unmanned air vehicles (UAVs) can provide important communication advantages to ground-based wireless ad hoc networks. In this paper, the location and movement of UAVs are optimized such that the network connectivity can be improved. Two types of network connectivity are quantified: global message connectivity and worst-case connectivity. The problems of UAV deployment and movement are formulated for these applications. The optimization problems are NP hard and some heuristic adaptive schemes are proposed in order to yield simple solutions. From the simulation results, by deploying only a single UAV, the global message network connectivity and worst-case network connectivity can be improved by up to 109% and 60%, respectively.

I. INTRODUCTION

Unmanned air vehicles (UAVs) are playing increasingly prominent roles in the nation’s defense programs and strategy. While drones have been employed in military applications for many years, technological advances in microcontrollers, sensors, and batteries have dramatically increased their utility and versatility. Traditionally, emphasis has been placed on relatively large platforms such as Global Hawk and Predator, but increasing attention has recently been focused on small “mini-UAVs” (MUAVs) that offer advantages in flexibility and cost [1],[2],[3]. An example of an experimental MUAV built and tested at Brigham Young University is depicted in Figure 1. Because of their small size, they are difficult for others to detect and track, and they are able to more easily avoid threats in the environment they fly through. As a result, they can fly at much lower altitudes, on the order of tens or hundreds of feet, and collect much more precise, “localized” data. They are significantly cheaper and easier to fly, and can often be launched by an individual in any kind of terrain without a runway or special launching device.

Due to their mobility and elevation, UAVs equipped with communication capabilities can provide important advantages to ground-based ad hoc networks. Their use in routing, medium access control, and scheduling applications has been detailed in [4],[5],[6],[7]. These studies have been primarily heuristic, and have focused on simulations to qualitatively assess the benefits of UAV-assisted networks. In this paper, we take a mathematical approach to positioning and flying a UAV over a wireless ad hoc network in order to optimize the network’s connectivity for better QoS and coverage. We assume a single UAV flying over a connected network with knowledge of the positions and velocities of the network nodes. The UAV can position itself to the place where the land node cannot be located, such as within the enemy territory. The UAV itself acts as a node in the network, and can generate, receive or forward data packets to other users.

II. UAV-ASSISTED NETWORK MODEL

We assume a single UAV flying over a wireless mobile ad hoc network (MANET) that is able to obtain (e.g., through sharing of GPS data) the locations and velocities of the randomly distributed mobile users in the network. In particular, we assume the UAV to possess the following information:

- Locations of all users \((x_i, y_j)\), from which the distances between any two nodes is calculated to be \(D_{ij} = \sqrt{|x_i - x_j|^2 + |y_i - y_j|^2}\).
- Using the users’ locations at different times, the UAV can obtain the speeds and directions of mobile users:

\[
S_i = \frac{dx_i}{dt} + z \frac{dy_i}{dt}.
\]

From the simulation results, one UAV can improve the global message connectivity and worst-case connectivity by up to 109% and 60%, respectively.

This paper is organized as follows: In Section II, we describe the system model. In Section III, we quantify two matrices for network connectivity, and then formulate the problems for UAV deployment and movement to optimize the connectivity. In Section IV, we propose some engineering heuristic solutions. Simulation results are given in Section V and final conclusions are drawn in Section VI.
where $z = \sqrt{-1}$. Notice that the information has very low data rate and can be securely protected.

Suppose there are $K$ mobile users denoted as $N_1, \ldots, N_K$, plus one UAV denoted as $N_0$ in the MANET. The wireless channel response between any two nodes is $G_{ij}$, $i \neq j$. Suppose the transmitted power for each node is $P_i$ and the noise variance $\sigma^2$ is the same for all users. The received signal-to-noise ratio (SNR) $\Gamma_{ij}$ for the signal transmitted by the $i^{th}$ node and received by the $j^{th}$ node is

$$\Gamma_{ij} = \frac{P_i G_{ij}}{\sigma^2}. \quad (2)$$

Using a Raleigh statistical model, the channel gain can be expressed as

$$G_{ij} = \frac{C_{ij} |h_{ij}|^2}{(D_{ij})^\alpha}, \quad (3)$$

where $C_{ij}$ is a constant that takes into account the antenna gains and any propagation obstructions (shadowing), $|h_{ij}|^2$ is the squared magnitude of the channel fade and follows an exponential distribution with unit mean, $D_{ij}$ is the distance between user $i$ and user $j$, and $\alpha$ is the propagation loss factor. Here we assume the channels among different users are orthogonal. This assumption is valid for most military applications.

A sufficiently high SNR will guarantee that the receiver will have an acceptably small packet loss and successfully receive a transmitted packet, so that minimal link quality can be maintained. Suppose that the SNR threshold for successful packet reception is $\gamma$, so that using (2), (3), and the Raleigh statistical model, the probability of a successful transmission is given by:

$$P^{sj}_r (\Gamma_{ij} \geq \gamma) = \exp \left \{ -\frac{\sigma^2 \gamma (D_{ij})^\alpha}{G_{ij} P_i} \right \}. \quad (4)$$

Because transmission power is bounded, each user can only communicate with other users within a certain radius. In this paper, we will say that two nodes are connected if the probability defined in (4) is greater than or equal to some threshold $\delta$. Based on this, we define a graph $G(K, A)$ to describe the connectivity of the network, where the matrix $A$ has the following definition:

$$[A]_{ij} = \begin{cases} 1, & \text{if } P^{sj}_r \geq \delta; \\ 0, & \text{otherwise}. \end{cases} \quad (5)$$

We will assume that the network is connected, i.e. $\sum_{j=1}^{K} [A]_{ij} \geq 1, \forall i$, and we concentrate on how to improve the connectivity. If the network is not connected, then other methods such as those based on Steiner trees [8] must be employed. To quantify each link’s connectivity, we define the weight for each link as a function of the probability of successful transmission:

$$W_{ij} = -\log P^{sj}_r, \quad (6)$$

where the minus sign is added to make the weight positive. Suppose user $i$ wants to communicate with user $j$ via a relay with user $k$. Because of the log form, the sum of weights $W_{ik}$ and $W_{kj}$ will represent the probability of successful transmission between $i$ and $j$ as $P_{ik}P_{kj}$. The smaller the weight, the higher the connectivity.

III. CONNECTIVITY DEFINITION AND FORMULATION

In some applications such as military ad hoc networks, it is important to keep all the users connected. For example, in battlefield scenarios, it is essential to propagate commands to the distributed soldiers and vehicles. Given that the quality of each link can be represented by different weights as in (6), a natural question is how to select the links such that all the nodes are connected and the overall weights are minimized. The concept of a Minimal Spanning Tree (MST) from graph theory provides the solution to this question:

Definition 1: Given a graph, a spanning tree of that graph is a subgraph which is a tree and connects all the vertices together. A single graph can have many different spanning trees. We can also assign a weight to each edge, which is a number representing how unfavorable it is, and use this to assign a weight to a spanning tree by computing the sum of the weights of the edges in that spanning tree. A minimum spanning tree or minimum weight spanning tree is then a spanning tree with weight less than or equal to the weight of every other spanning tree. [8]

One example of such a MANET is shown in Figure 2. First, without considering the Steiner point which we will discuss later, there are total of 10 nodes. The possible connections between each node are marked with different costs. The bolded link shows the MST that connects all the nodes. To find the MST solution, Prim’s algorithm, Kruskal’s algorithm, and the Chazelle algorithm [8] can be utilized with polynomial time.

MSTs are widely used in wired networks to minimize the cost of transmission. Because of the broadcast nature of wireless communications, the transmissions of one user can be heard by many others. In [9], a pruning MST is proposed to yield energy efficient broadcast and multicast trees. In our work, however, we concentrate on how to improve the connectivity and not on how to construct the spanning tree. The approaches and discussions in the rest of this paper can be employed for any tree like those in [9]. Suppose the matrix $A'$ represents the MST, where $[A']_{ij} = 1$ if the link from user $i$ to user $j$ is in the MST, and $[A']_{ij} = 0$, otherwise. We have two different definitions of connectivity:

1) Global Message Connectivity

In some applications such as those in battle fields, a commander’s message should be transmitted globally to all users. If we define global message connectivity as
the probability that a message can be successfully transmitted to all users in the network under the unlimited energy but limited power condition, then the MST is the optimal solution for connectivity. Since the received SNR in (2) is a monotonically increasing function of power, to maximize the probability in (4), each user will transmit with maximal power. Because there is abundant energy, we do not have an energy efficiency problem. Suppose the MST has already been constructed with the weight defined in (6). The sum of the weights in the MST represents the overall probability that a message is successfully transmitted via this MST. Suppose there exists a tree that yields a higher probability to connect all users. We can convert the probability of successful transmission for each link in the tree to a corresponding weight for the link. Since the weights in (6) are monotonously decreasing with $P_{ij}$, the sum of the weights for the new tree will be smaller than that for the MST, which is a contradiction.

The probability that a command is transmitted to all the users, which can be computed as the product of successful transmission probabilities of the links on the MST. Maximizing this probability is equivalent to minimizing the weight defined in (6). The sum of the weights for the new tree will be smaller than that for the MST, which is a contradiction.

Since the UAV will be an additional node to the existing MST, another concept is defined to improve the MST as:

**Definition 2:** A node added to the network that minimizes the length of the spanning tree is called a Steiner point. The resulting tree is called a Steiner tree.

A Steiner point example is shown in Figure 2, where a Steiner point is added and the overall weight of the MST is reduced. In the sequel, we formulate the optimization problems for deployment and movement of the UAV. First, the UAV optimizes its location so that better network connectivity $U$ can be obtained. Second, the UAV needs to decide in which direction and at what speed it should move so that the connectivity can be better maintained. Recall that the UAV is denoted as node $U$.

The two problem formulations are given by the following:

1) **Formulation 1: UAV Deployment**

$$min \sum_{i=0}^{K} \sum_{j=0}^{K} [A']_{ij} W_{ij}. \quad (7)$$

2) **Worst-Case Connectivity**

If we define the worst-case connectivity as the lowest probability that part of the network can communicate the rest of the network, this connectivity measures how severe the network will be divided into two parts. Then under the unlimited energy but limited power condition, the largest weight in the MST can be used to measure this connectivity, i.e., maximizing this connectivity is equivalent to minimizing the weight of the worst-case MST edge:

$$U = \min_{[A']_{ij} = 1} \max W_{ij}. \quad (8)$$

**IV. UAV DEPLOYMENT AND MOVEMENT ALGORITHMS**

In this section, we analyze the two-user case first. Then we propose multi-user algorithms for the formulations in (9) and (10). The main approach is based on gradient methods and some engineering heuristics to reduce the complexity.

**A. Performance Analysis for Two-User Case**

Suppose two users are uniformly randomly located within a radius of $R$ as shown in Figure 3. Suppose the distance between two users is $d$. Obviously $0 \leq d \leq 2R$. We try to find the pdf of $d$. Suppose user 1 is located at the distance of $r$ to the center. There are three cases to discuss

1) $I(r, d) = 1$, if $r + d \leq R$.

Since the uniform distribution, we have the probability that the distance between two users is greater than $d$ as

$$P_r(d | I = 1, r) = \frac{\pi R^2 - \pi d^2}{\pi R^2} = 1 - \frac{d^2}{R^2}. \quad (12)$$

2) $I(r, d) = 2$, if $r + d > R$. $R - d > 0$.

Within the distance of $d$ to user 1, there are some areas that user 2 cannot be located, since it is outside the radius $R$. An example is shown in Figure 3. So the probability is proportional to the size of round dish with radius $R$ minus the round dish with radius $d$ that inside the dish with radius $R$. The shape looks like a waning crescent moon. We have the probability of user 2 located in that area as:

$$\Delta U = U(x_i(t+\Delta t), y_i(t+\Delta t)) - U(x_i(t), y_i(t)). \quad (11)$$
Initialize

1. Calculate gradient in (17).

If random initialization, select the best.

A linear search algorithm [10] can be utilized to reduce the complexity of the gradient method. The stopping criteria can be guaranteed to be globally optimal.

1) Random Initialization

This approach generates a number of seeds within the area of the MANET and lets the gradient method find the local optima. From the local optima, the global optimum is selected with the minimal overall weight. The advantage of this initialization method is that the global optimum can be obtained with high probability when the density of the initialization seeds is enough high. The disadvantage is that the computational complexity is high, especially when the number of users is large.

2) Heuristic Initialization

Suppose the MST without the UAV is constructed, and that the maximal link weight occurs between node \( i \) and node \( j \). The links between node \( i \) an node \( j \) are symmetric. The heuristic initialization for the UAV is at the middle of these two nodes, i.e.,

\[
x_i^0 = \frac{x_i + x_j}{2} \quad \text{and} \quad y_j^0 = \frac{y_i + y_j}{2}.
\]

The rationale is to improve the worst case link, so that the initial performance improvement can be good before applying the gradient method. This initialization cannot be guaranteed to be globally optimal.

Overall the algorithm to find the best location to deploy the UAV is shown in Table I. The complexity of the algorithm for each iteration is \( O(K^3 \log K) \). Since the MST cost at each iteration of the algorithm is non-increasing and the solution has a lower bound, the algorithm always converges.

### Table I: Algorithm to Find Best Deployment

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Calculate gradient in (17).</td>
</tr>
<tr>
<td>2.</td>
<td>( t = t + 1 ).</td>
</tr>
<tr>
<td>3.</td>
<td>line search ( \zeta &gt; 0 ) so that ( U ) is optimized with ( x_0^t + y_0^t = x_0^{t-1} + y_0^{t-1} - \zeta g_0 ).</td>
</tr>
<tr>
<td>Stopping Criteria</td>
<td>(</td>
</tr>
<tr>
<td>Return</td>
<td>((x_0, y_0)) If random initialization, select the best.</td>
</tr>
</tbody>
</table>

B. Deployment Formulation Solutions

In this subsection, we determine the UAV deployment, i.e., what is the optimal \((x_0, y_0)\). The problem is \( NP \) hard and we propose an adaptive algorithm to find a local optimum. Starting from any initialization point, we want to find out how to change the UAV’s location in some neighborhood around so that a better MST can be obtained. The gradient for such a search can be written as:

\[
g_0 = \frac{dU(x_0, y_0)}{dx_0} + \frac{dU(x_0, y_0)}{dy_0}.
\]

A linear search algorithm [10] can be utilized to reduce the complexity of the gradient method. The stopping criteria can be \( ||g_0||^2 \leq \varepsilon \) where \( \varepsilon \) is a small positive number, or where the KKT condition holds [10], i.e., at which the local optimum is achieved.

The problem in (9) has many local optima. This can be shown by the following extreme example. Suppose there are only 3 users and one UAV in the network. Three users are located in a line with locations \((0, 0)\), \((1, 0)\), and \((2, 0)\). With some simple calculations, we can see that there are two locally optimal locations for UAV at \((0.5, 0)\) and \((1.5, 0)\), respectively.

To overcome the local optimum problem, we propose the following two initialization methods.

1) Random Initialization

This approach generates a number of seeds within the area of the MANET and lets the gradient method find the local optima. From the local optima, the global optimum is selected with the minimal overall weight. The advantage of this initialization method is that the global optimum can be obtained with high probability when the density of the initialization seeds is enough high. The disadvantage is that the computational complexity is high, especially when the number of users is large.

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Overall the algorithm to find the best location to deploy the UAV is shown in Table I. The complexity of the algorithm for each iteration is \( O(K^3 \log K) \). Since the MST cost at each iteration of the algorithm is non-increasing and the solution has a lower bound, the algorithm always converges.

C. Movement Formulation Solutions

In this subsection, we assume that the initial UAV deployment has been done as described in the previous subsection; i.e., \((x_0, y_0)\) is known. We try to determine the movement of UAV so that the network connectivity can be improved in the future.

First we assume that within a short period of time \( dt \), the network and MST topologies do not change. The movement of the UAV will only affect the weights where the links are connected to the UAV. Define the set of nodes that are connected to the UAV as \( V \). The UAV only needs to monitor the nearby nodes in \( V \). The estimation, signaling, and overhead burdens can thus be greatly reduced.

From (3), (4), and (6), the gradient for the utility change can be written as:

\[
\frac{dU}{dt} = \frac{d}{dt} (U(D^*_i) - U(D_{ij}))
\]

where \( D^*_i = ||x_i + z y_i + S_i dt - x_j - z y_j - S_j dt||^2 \).
Since the UAV is airborne, we need to consider the issue of speed constraints. If the gradient in (19) is small, the connected nodes are hardly moving, and hence the UAV should not change position. Under this condition, the UAV must fly in a small circle. When the gradient is large enough, the UAV flies against the gradient direction with the speed proportional to the magnitude of the gradient. When the gradient is too large, the UAV can only fly in the direction of the gradient with its maximum speed $\mu_{\text{max}}$. The speed of the UAV $S_0$ can be calculated in Table II, where $\mu$ is a constant that can be determined experimentally.

The two algorithms in Table I and Table II can be utilized in turn. First, the deployment algorithm is used to find the best location the UAV should initially fly to. Then, the movement algorithm keeps track of the mobility of the distributed users. Occasionally, the network topology has been changed too much and the UAV falls to some local optimum. Under this condition, the deployment algorithm is reapplied to relocate the UAV into some better position. The frequency for employing the deployment algorithm depends on the mobility of the users.

### V. Simulations

To demonstrate the effectiveness of the proposed algorithms, we use the following simulation study: a total of $K$ users are randomly located within a square region of $1000$ m × $1000$ m. The transmission power is $300$ dbmW, the noise value $\sigma^2 = 10^{-7}$ dbmW, the SNR requirement $\gamma = 10$ dB, and the propagation loss factor is $\alpha = 3$. Without loss of generality, for the communication link between different mobile users, we assume $C_{ij} = C_1$, $\forall i, j \in \{1, 2, \ldots, K\}$; for the communication link between the UAV and the mobile users, $C_{ij} = C_0$, $i$ or $j = 0$. Here since UAV is on the sky, $C_0 > C_1$. For the simulations conducted here, we assumed that $C_0 = 2$ and $C_1 = 1$. 2500 initializations are realized for the random initialization methods.

In Figure 4, we show a snapshot of the global message connectivity as a function of the UAV location $(x_0, y_0)$. Here the number of users is $K = 10$. On the Z-axis, we show the connectivity of the network without the UAV as a star with a value of 0.2966. By deploying the UAV at the best location $(x_0, y_0) = (305, 951)$, the connectivity probability is improved to 0.4964 as shown by a diamond on the Z-axis. On the xy-plane, we show the MST with the users denoted by crosses and the UAV denoted by a circle. We can see that the UAV tries to improve the link from a faraway user to the rest of the network in this case. Moreover, from the curve, we can see that there are many local optima for $(x_0, y_0)$.

In Figure 5, we show one example of the UAV flying tracks with different types of initialization. Here the number of users is $K = 10$ and there are five different initial seeds for the random initialization. We can see that different initializations lead to different local optima. From the simulations, the heuristic initialization leads to global optimization most of the time. But there are cases where the random initialization leads to the global optimum, while the heuristic provides only a local optimum. On the other hand, there are also cases where the heuristic approach has a better solution, because the number of random initializations is not large enough. Moreover, the flying tracks are not smooth and the UAV may change directions. This is because the derivative of $U$ is not continuous, which can be easily observed from Figure 4.

In Figure 6, we show the network connectivity for different numbers of users. For both global message connectivity and worst-case connectivity, we show the performance of no UAV, random initialization, and heuristic initialization, respectively. We can see that the performance drops first when the number of users increases from a small number. This is because the users have to transmit over long distances for the message to propagate. When the number of users becomes large, the higher density of users makes the connectivity better. The addition of a single UAV can improve the connectivity of the network by 109% when the number of users is 4. This is because the size of V (the users connected to the UAV) is limited, while the rest of the links are kept the same. The improvement shrinks with larger number of users, since a higher node density means that most of links already have good connection probabilities and the addition of the UAV offers only a slight improvement. For worst-case connectivity, the UAV can improve performance by up to 60%. The heuristic initialization has a slightly worse performance compared to the random initialization because of the local optima, but the complexity of the heuristic approach is much lower.

### TABLE II: Algorithm to Find Best Movement

<table>
<thead>
<tr>
<th>Movement</th>
<th>Measure $(x_i, y_i)$ and $S_i$ for $i \in V$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monitor</td>
<td>Detect local optimum. If so, performance Table I.</td>
</tr>
<tr>
<td>Circle:</td>
<td>If $</td>
</tr>
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<td></td>
<td>If $</td>
</tr>
<tr>
<td></td>
<td>$S_i = \begin{cases} -\mu \frac{dU}{dt}, &amp; \text{if }</td>
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| Update   | Since the UAV is airborne, we need to consider the issue of speed constraints. If the gradient in (19) is small, the connected nodes are hardly moving, and hence the UAV should not change position. Under this condition, the UAV must fly in a small circle. When the gradient is large enough, the UAV flies against the gradient direction with the speed proportional to the magnitude of the gradient. When the gradient is too large, the UAV can only fly in the direction of the gradient with its maximum speed $v_{\text{max}}$. The speed of the UAV $S_0$ can be calculated in Table II, where $\mu$ is a constant that can be determined experimentally. The two algorithms in Table I and Table II can be utilized in turn. First, the deployment algorithm is used to find the best location the UAV should initially fly to. Then, the movement algorithm keeps track of the mobility of the distributed users. Occasionally, the network topology has been changed too much and the UAV falls to some local optimum. Under this condition, the deployment algorithm is reapplied to relocate the UAV into some better position. The frequency for employing the deployment algorithm depends on the mobility of the users. V. Simulations To demonstrate the effectiveness of the proposed algorithms, we use the following simulation study: a total of $K$ users are randomly located within a square region of $1000$ m × $1000$ m. The transmission power is $300$ dbmW, the noise value $\sigma^2 = 10^{-7}$ dbmW, the SNR requirement $\gamma = 10$ dB, and the propagation loss factor is $\alpha = 3$. Without loss of generality, for the communication link between different mobile users, we assume $C_{ij} = C_1$, $\forall i, j \in \{1, 2, \ldots, K\}$; for the communication link between the UAV and the mobile users, $C_{ij} = C_0$, $i$ or $j = 0$. Here since UAV is on the sky, $C_0 > C_1$. For the simulations conducted here, we assumed that $C_0 = 2$ and $C_1 = 1$. 2500 initializations are realized for the random initialization methods.

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We set up another simulation to test the analysis results. The users are uniformly randomly located within a cell with radius $R$. The rest of simulation settings are the same. In Figure 7, we show the global message connectivity for the different radius $R$. We can see that the analysis results match the numerical results well. The larger the cell size, the more improvement a UAV can provide. At the cell radius of 1000m,
the improvement is up to 240%.

In Figure 8, we show the average UAV speed and the probability that the UAV falls into a local optimum. Here $K = 5$. The mobile users move in arbitrary directions with the speeds uniformly distributed from zero to the value on the x-axis. The total time for each network situation is 300 seconds and the UAV updates its direction in every 10 seconds. $v_{\text{max}} = 30 \text{m/s}$. We average 500 different situations. We can see that the average speed of the UAV increases according to the users’ mobility. The probability that the UAV falls into a local optimum (the topology changes) during 300 seconds increases faster when the users’ speed is higher. According to the different users’ speeds, the frequency for applying the deployment algorithm in Table I should vary.

VI. CONCLUSIONS

In this paper, we study how to utilize UAVs to improve the network connectivity of a MANET. We define two types of connectivity, global message connectivity and worst case connectivity. Then formulate the deployment and movement problems for the UAV. Adaptive heuristic algorithms are proposed in order to provide a simple solution as well as good performance. From the simulation results, a UAV can improve the two types of connectivity by 109% and 60%, respectively.

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