SPACE-TIME POWER SCHEDULE FOR DISTRIBUTED MIMO LINKS WITHOUT CHANNEL STATE INFORMATION AT TRANSMITTING NODES

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ABSTRACT
A space-time optimal power schedule for multiple distributed MIMO links without the knowledge of channel state information at transmitting nodes is proposed. This new approach exploits both the spatial and temporal freedoms of distributed MIMO links. A readily computable expression for the ergodic sum capacity of the MIMO links is derived. Based on this expression, a projected gradient algorithm is developed to optimize the power allocation. For a symmetric set of MIMO links, it is observed that the space-time optimal power schedule reduces to a uniform isotropic power schedule when nominal interference is low, or to an orthogonal isotropic power schedule when nominal interference is high. Furthermore, the transition region between the latter two schedules is seen to be very small in terms of nominal interference-to-noise ratio.

Index Terms— MIMO systems, space-time power schedule, wireless mesh networks.

1. INTRODUCTION
In a large wireless mesh network of many MIMO nodes, multiple MIMO links must share a common frequency band concurrently to ensure a high spectral efficiency of the whole network [1]. Developing optimal power schedule for a set of co-channel, concurrent and neighboring MIMO links is therefore important.

Power schedule for multiple MIMO links has been studied in [2], [3], [4] and [5]. In [2], a space-only (i.e., time-invariant) power schedule is presented, and an iterative algorithm leading to the Nash equilibrium is developed. In [3], the same space-only criterion is used, but a projected gradient algorithm [6] is developed that yields a better result. In [4], the space-only approach is considered without channel state information (CSI) at transmitting nodes. In [5], a space-time power schedule is proposed that generalizes the approaches used in [2] and [3].

In this paper, we present a space-time optimal power schedule without CSI at transmitting nodes. This work goes beyond the work [5] that assumes CSI at transmitting nodes and also beyond the work [4] that assumes a time-invariant transmitting covariance matrix at each link.

In the absence of instantaneous CSI at transmitting nodes, the statistical CSI is necessary for designing power schedule. We assume that the MIMO channel between each transmitting node and its receiving node is a complex Gaussian matrix with independent and identically distributed (i.i.d.) entries. We derive a “closed form” expression for the ergodic sum capacity of multiple MIMO links. This expression consists of finite sums and a simple one-dimensional integral. It is readily computable. Another important result in this paper is the development of a projected gradient algorithm that allows one to maximize the ergodic sum capacity and hence to compute the corresponding optimal power schedule.

An important case study to be shown is based on a set of symmetric MIMO links. We will show that the space-time optimal power schedule reduces to a uniform isotropic power schedule when the interference level is low, or to an orthogonal isotropic power schedule when the interference level is high. Furthermore, the transition between the latter two schedules is very sharp along the interference-to-noise ratio (INR) axis. All space-only power schedules, including those in [4], are shown to be sub-optimal compared to ours. It is important to note that by INR, we refer to a nominal INR unless specified otherwise. The nominal INR can be kept constant while the actual INR changes, which will be further explained.

2. SYSTEM MODEL
We consider a network of $M$ nodes operating in a common time-frequency band. Each node has $N$ antennas. During every time slot, there are $L$ concurrent links. Each active transmitting node delivers information only to one active receiving node. And each active receiving node receives information only from one active transmitting node. We will assume that the channel matrices remain constant over $L$ consecutive time slots but change randomly over an interval of many multiples of $L$ time slots. We will design the power schedule to maximize an ergodic network capacity which is averaged over the statistical distribution of the channel matrices. This capacity is achievable (approximately) over the interval of many multiples of $L$ time slots.

The vector of the received signal $y_i$ at the $i$th receiving node can be written as

$$y_i = \sqrt{\frac{p_i}{N}} H_{i,i} x_i + \sum_{j=1, j \neq i}^{L} \sqrt{\frac{\beta_{i,j}}{N}} H_{i,j} x_j + v_i$$

\text{with $H_{i,j}$ is the $N \times N$ channel matrix between the $j$th transmitting node and the $i$th receiving node, $p_i$ denotes the signal-to-noise ratio (SNR) of the $i$th link, $\beta_{i,j}$ is the INR of the $j$th transmitting node to the $i$th receiving node, $x_i$ denotes the $N \times 1$ vector of the normalized transmitted signal from the $i$th transmitting node, and $v_i$ is the $N \times 1$ vector of the i.i.d. additive white Gaussian noise (AWGN) with zero mean and unit covariance matrix $C_{v_i} = I_N$. Here $I_N$ denotes an $N \times N$ identity matrix.}
The first term in (1) represents the signal-of-interest component at the $i$th receiving node, while the second term is the sum of interfering signals from all other $L - 1$ transmitting nodes. We assume that all the normalized transmitted signal is Gaussian distributed with zero mean vector and covariance matrix $P_i \triangleq \mathbb{E}\{x_i x_i^H\}$, where $\mathbb{E}\{\cdot\}$ stands for the statistical expectation, and $(\cdot)^H$ denotes the matrix Hermitian transpose. Without loss of generality, we assume that $tr\{P_i\} = N, i = 1, \ldots, L$, where $tr\{\cdot\}$ stands for the trace of a matrix. In the sequel, we make use of the following assumptions:

- There is no coding cooperation among different transmitting nodes and receiving nodes.
- The interfering signals are unknown to the receiving nodes, and single user receiver is used at each receiving node.
- The entries of $H_{i,j}$ are i.d. complex Gaussian distributed with zero mean and unit variance, that is, the channel is Rayleigh flat fading. The signal power loss is included in SNR $\rho_i$, and INR $\beta_{i,j}$.
- There is no CSI at any transmitting node, and the $i$th receiving node knows the CSI of the link-of-interest $H_{i,j}$.

3. ERGODIC SUM CAPACITY

For a given set of $P_i, i = 1, \ldots, L$, the overall ergodic sum capacity of the total $L$ links can be written as

$$I(P_1, \ldots, P_L) = \mathbb{E}_H \left\{ \sum_{i=1}^{L} \log_2 |I_N + \frac{P_i}{N} H_{i,i} P_i H_{i,i}^H R_i^{-1}| \right\}$$

(2)

where $|\cdot|$ denotes the determinant of a matrix, $\mathbb{E}_H\{\cdot\}$ stands for the statistical expectation with respect to all channel matrices $H \triangleq [H_{i,1}^T, \ldots, H_{i,L}^T]^T$, and $R_i$ is the interference-plus-noise covariance matrix at the $i$th receiving node

$$R_i = \sum_{j=1,j\neq i}^{L} \beta_{i,j} H_{i,j} P_j H_{j,j}^H + I_N.$$

Note that for a symmetric set of MIMO links, no link suffers a fairness problem under the ergodic sum capacity.

Let us denote $P_i = U_i D_i U_i^H$ as the eigenvalue decomposition of $P_i$, where $U_i$ is an $N \times N$ unitary eigenvectors matrix, and $D_i = \text{diag}\{d_{i,1}, d_{i,2}, \ldots, d_{i,N}\}$ is an $N \times N$ diagonal matrix of all eigenvalues. For convenience, we will use the $N \times 1$ column vectors $d_i \triangleq [d_{i,1}, d_{i,2}, \ldots, d_{i,N}]^T, i = 1, \ldots, L$. Since $H_{i,j}, i, j = 1, \ldots, L$, has i.i.d. entries, the statistics of $H_{i,j}$ is identical to that of $H_{i,j} U_j$ [7]. Hereafter, for simplicity, we write the ergodic capacity expression (2) as

$$I(d_1, \ldots, d_L) = \mathbb{E}_H \left\{ \sum_{i=1}^{L} \log_2 |I_N + \frac{H_{i,i} D_i H_{i,i}^H}{N} R_i^{-1}| \right\} - \mathbb{E}_R_i \{ \log_2 |I_N + H_{i,i} A_i H_{i,i}^H| \}$$

(3)

where

$$\mathbf{R}_i = \sum_{j=1,j\neq i}^{L} \beta_{i,j} H_{i,j} D_j H_{j,j}^H + I_N$$

$$H_i = [H_{i,1}, \ldots, H_{i,L}^H]$$

$$H_{i,j} = [H_{i,1}, \ldots, H_{i,j-1}, H_{i,j+1}, \ldots, H_{i,L}]$$

$$A_i = \text{diag}\{\lambda_{i,1}^2, \ldots, \lambda_{i,L}^2\}$$

$$\tilde{A}_i = \text{diag}\{\lambda_{i,1}^2, \ldots, \lambda_{i,i-1}^2, \lambda_{i,i+1}^2, \ldots, \lambda_{i,L}^2\}$$

$$\lambda_{i,j} = \left\{ \begin{array}{ll}
\frac{\beta_{i,j}}{\rho_i} d_{i,j} & j \neq i \\
\frac{\beta_{i,i}}{\rho_i} d_{i,j} & j = i.
\end{array} \right.$$
Similar to [5], in order to exploit both the spatial and temporal freedoms for power scheduling, we let \( \mathbf{P}_1, \ldots, \mathbf{P}_L \) be periodically time varying with the period equal to \( L \) time slots. We can write an averaged ergodic sum capacity over \( L \) time slots as

\[
I_a(\mathbf{P}) = \frac{1}{L} \sum_{l=1}^{L} \sum_{i=1}^{L} \log_2 \left( 1 + \frac{\rho_i}{N} \mathbf{H}_i \mathbf{P}_i(t) \mathbf{H}_i^H \mathbf{R}_i^{-1}(t) \right)
\]

where \( \mathbf{P} \) is a matrix stacking the source covariance matrices of all links:

\[
\mathbf{P} \triangleq \begin{bmatrix} \mathbf{P}_1^T, \ldots, \mathbf{P}_L^T \end{bmatrix}^T
\]

\[
\mathbf{P}_i \triangleq \begin{bmatrix} \mathbf{P}_i^T(1), \ldots, \mathbf{P}_i^T(L) \end{bmatrix}^T \quad i = 1 \ldots, L
\]

and \( \mathbf{P}_i \) is a matrix stacking the source covariance matrices of the \( i \)th link. Following the derivations in Section 3, we can write (7) into a closed form as

\[
I_a(d) = \frac{\log_2(e)}{L} \sum_{l=1}^{L} \sum_{i=1}^{L} \left[ \sum_{n=0}^{N-1} \sum_{k=1}^{N(L-1)} c_{i,k,n} Q(n, \lambda_{i,k,l}) - \sum_{n=0}^{N-1} \sum_{k=1}^{N(L-1)} d_{i,k,n} Q(n, \lambda_{i,k,l}) \right]
\]

where \( d \) is a vector stacking the power scheduling parameters of all links at all \( L \) time slots

\[
\mathbf{d} \triangleq \begin{bmatrix} d_1^T, \ldots, d_L^T \end{bmatrix}^T
\]

\[
d_i \triangleq \begin{bmatrix} d_i^T(1), \ldots, d_i^T(L) \end{bmatrix}^T \quad i = 1 \ldots, L
\]

and \( d_i \) is a vector stacking the power scheduling parameters of the \( i \)th link. In (8), the subscript \( i \) in the scalars \( c_{i,k,n}, d_{i,k,n}, \gamma_{i,k,l} \) and \( \gamma_{i,k,l} \) denotes the corresponding quantities for the \( i \)th time slot, and \( c_{i,k,n} \) and \( d_{i,k,n} \) have a structure similar to that of \( c_{i,k} \) in (5)-(6).

Taking the power constraint of each active link into account, our space-time power scheduling approach becomes the following optimization problem:

\[
\max_d \quad I_a(d) \quad \text{s.t.} \quad \|\mathbf{d}\|_1 = NL, \quad d_i \geq 0, \quad i = 1 \ldots, L \quad (10)
\]

where (10) is the set of the transmit power constraints at all transmitting nodes, and \( \| \cdot \|_1 \) denotes the sum norm (or \( L_1 \) norm) of a vector. For a vector \( x \), \( x \geq 0 \) means that each entry of \( x \) is nonnegative.

The results in [4] show that when the INR is sufficiently low, the ergodic sum capacity (4) is a concave function of the power allocation vectors \( d_1, \ldots, d_L \), but when the INR is sufficiently high, (4) becomes a convex function of the power allocation vectors. However, in general, it can be seen from (3) that due to the mutual interference among different links, the ergodic sum capacity is neither a convex function, nor a concave function, of the power allocation vectors \( d_1, \ldots, d_L \). Similarly, (8) is neither a convex nor a concave function of the power scheduling vector \( d \) in the general INR region. Thus, in general, (9)-(10) is a nonconcave optimization problem.

Since the constraints (10) are simple linear constraints, the projected gradient technique [6] can be applied to obtain a local optimal solution to the problem (9)-(10). Because of the space limitation, we omit the details, which can be found in the journal version of this paper.

The proposed space-time power scheduling requires only the knowledge of SNR and INR of each link at the transmitting nodes. This knowledge can be easily obtained by exploiting the topology of the wireless networks and the transmit power of each transmitting node. In practice, the optimization procedure can be run off-line for different combinations of SNR and INR. The resulting optimal parameters can be tabulated. Then, in real-time applications, we only need to look up this table to select the optimal power parameters.

Before finishing this section, we want to discuss a special case of the proposed space-time power schedule. When only one time slot is considered for power scheduling, we have the space-only power schedule, which can be written as the following constrained optimization problem

\[
\max_{d_1, \ldots, d_L} I(d_1, \ldots, d_L) \quad \text{s.t.} \quad \|d_i\|_1 = N, \quad d_i \geq 0, \quad i = 1 \ldots, L \quad (11)
\]

It has been shown in [4] that at a sufficiently low interference level, the space-only optimal power schedule is a uniform isotropic power schedule where all links use the samesource covariance matrix and the source covariance is the identity matrix. While at a sufficiently high interference level, it is shown in [4] that the space-only optimal power schedule becomes a low rank power schedule where each link uses a low rank source covariance matrix. However, the work [4] does not provide a good answer for the intermediate region of interference. Our optimization based on a single time slot yields the space-only optimal power schedule for any given interference level, which will be shown in Section 5.

5. NUMERICAL EXAMPLES

We now illustrate the performance of the space-time power scheduling scheme presented earlier. For comparison, we will consider the following schemes:

- **Scheme 1**: Space-time optimal power schedule based on (9) and (10).
- **Scheme 2**: Orthogonal isotropic power schedule, where during each time slot only one link has a non-zero source covariance matrix and the source covariance matrix is the identity matrix.
- **Scheme 3**: Uniform isotropic power schedule, where all links use the same source covariance matrix and the source covariance matrix is the identity matrix.
- **Scheme 4**: Space-only optimal power schedule based on (11) and (12).
- **Scheme 5**: Low rank power schedule [4], where each link uses a low rank source covariance matrix where the corresponding ranks for \( L \) links are denoted by the string of integers \( (r_1, r_2, \ldots, r_L) \). The \( i \)th link with the rank \( r_i \) uses a power vector \( d_i \) of \( r_i \) non-zero equal entries and \( N - r_i \) zero entries.

Schemes 1 and 2 are space-time based, and all other schemes are space-only based. Scheme 1 is space-time optimal while Scheme 2 is not. For Schemes 1 and 4, the power allocation vectors were
initialized randomly. For each simulation point, 30 initializations were tried and the best result was chosen.

For all examples, we consider a symmetric and circular network with \( L = 2 \) active and symmetric links. For each case to be considered, we set \( p_t = 20 \text{dB}, t = 1, \ldots, L, \) and \( \beta_{i,j} = \beta, i, j = 1, \ldots, L, j \neq i. \) The ergodic capacity shown in all figures is a per-link ergodic capacity.

Fig. 1 compares the ergodic capacities of the five schemes with \( N = 2. \) From Fig. 1, we can see that Scheme 2 is as optimal as Scheme 1 at high INR, and Scheme 3 is as optimal as Scheme 1 at low INR. More interestingly, the transition of the optimality from Scheme 2 to Scheme 3 along the INR axis is very sharp (within about 1.5 dB of INR). This optimality property of Schemes 2 and 3 is also observed in Fig. 2.

![Fig. 1. Comparison of ergodic capacities of five schemes. Scheme 5 uses \((r_1, r_2) = (1, 1)\). Here, \( N = 2, L = 2.\)](image)

Fig. 2 compares the ergodic capacities of the five schemes with \( N = 3. \) Here, Scheme 5 with \((r_1, r_2) = (1, 1)\) remains strongly suboptimal even compared to Scheme 4 over the whole range of INR. This is because under \( N = 3, \) there are effectively three independent streams. But Scheme 5 with \((r_1, r_2) = (1, 1)\) uses only two.

![Fig. 2. Comparison of ergodic capacities of five schemes. Scheme 5 uses \((r_1, r_2) = (1, 1)\). Here, \( N = 3, L = 2.\)](image)

Recall the optimality property of Schemes 2 and 3 that Scheme 2 is as optimal as Scheme 1 when INR is larger than a threshold and Scheme 3 is as optimal as Scheme 1 when INR is less than the threshold. We can determine the threshold INR value \( \beta^* \) by solving the following nonlinear equation

\[
I(\beta^*/N, \rho/N, N, L) = J(\rho/L/N, N, N)/L
\]

(13) where \( J(\rho/L/N, N, N)/L \) is the ergodic sum capacity using Scheme 2, and \( I(\beta^*/N, \rho/N, N, L) \) is the ergodic sum capacity of Scheme 3.

In practice, the threshold INR \( \beta^* \) can be tabulated for different network parameters such as the number of links and the number of antennas at each node. Once this table is available, it can be looked up in real time to determine whether each node should be scheduled under Scheme 2 or Scheme 3.

6. CONCLUSIONS

We have proposed a space-time power scheduling approach for multiple distributed MIMO links assuming no CSI at transmitting nodes. This approach leads to Scheme 1 which is a space-time optimal power schedule. With Scheme 1 as the optimal benchmark, we have observed that Scheme 2, an orthogonal isotropic power schedule (such as TDMA), is optimal when the INR is larger than a threshold, and Scheme 3, a uniform isotropic power schedule, is optimal when the INR is less than the threshold. The threshold INR value can be computed based on the network topology. This useful property has been observed from a symmetric and circular network. Whether or not such a simple property holds for asymmetric networks remains to be investigated.

7. REFERENCES