Are the data rates predicted by the analytic analysis of receivers with low resolution ADCs achievable?

Kilian Roth *,‡, Hessam Pirzadeh †, A. Lee Swindlehurst ‡§, Josef A. Nossek ‡¶
* Next Generation and Standards, Intel Deutschland GmbH, Neubiberg 85579, Germany.
e-mail: {kilian.roth}@intel.com.
† Department of Electrical and Computer Engineering, Technical University Munich, München 80333, Germany.
e-mail: {kilian.roth, josef.a.nossek}@tum.de.
‡ Center for Pervasive Communications and Computing, University of California, Irvine, CA 92697 USA.
e-mail: @uci.edu; {hpirezade, swindle}@uci.edu.
§ Institute for Advanced Study, Technical University of Munich, München 80333, Germany.
¶ Department of Teleinformatics Engineering, Federal University of Ceara, Fortaleza 60020-181, Brazil.

Abstract—Leveraging the available millimeter wave spectrum will be important for 5G. In this work, we compare an information theory based analytic result for the achievable rate of digital beamforming systems using low resolution Analog-to-Digital-Converters (ADCs), to a link-level simulation of the same system. For both evaluation methods we include imperfect channel knowledge at the receiver and the effects of the ADC in a wideband multi-user uplink scenario. For the system we choose a configuration defined by 3GPP New Radio (5G). Our results show a performance gap between the two systems, which is comparable to other evaluations of this kind for systems like LTE (4G). In the low-to-medium SNR regime from -20 to 0 dB there is a constant difference of about 4 dB between the two evaluation methods. However, due to a number of effects not modeled in the achievable rate analysis the maximum spectral efficiency saturates at a significantly lower value.

Index Terms—MIMO equalization, low resolution ADC, millimeter wave, wireless communication.

I. INTRODUCTION

Due to the propagation conditions at millimeter Wave (mmWave) frequencies, wireless communication in this band is especially attractive for high data rate, shorter range applications. In recent years, the availability of spectrum and consumer grade systems at mmWave frequencies has led to a huge increase in academic and industrial research. However, to fully leverage the spectrum in a power-efficient way, the BaseBand (BB) and Radio Front-End (RFE) capabilities must be drastically changed from current state-of-the-art cellular devices.

Most mmWave systems being developed in the industry concentrate on analog or sub-array hybrid beamforming. As the beams can only be adapted to one user, not all degrees of freedom are available to each user in a Multi User - Multiple Input, Multiple Output (MU-MIMO) setup. This lead to the interesting result that a digital beamforming system using low resolution Analog-to-Digital-Converters (ADCs) is more energy efficient in a wide range of scenarios [1].

Evaluation of digital beamforming systems with low resolution ADCs has thus mainly concentrated on either signal processing aspects, achievable rate or Bit Error Rate (BER) analyses. There have been analyses considering uplink wideband systems [2], multiple users in the uplink [3] or the effects of imperfect Channel State Information (CSI) at the receiver [4]. Different signal processing issues such as channel estimation [5] and Multiple Input, Multiple Output (MIMO) equalization [6], [2] have been analyzed individually without considering all aspects of a receiver combined.

In this work we want to bridge the gap between these two evaluation methods. The main target is to show that indeed we can approach the attractive data rates promised by analytic methods using standard signal processing techniques. In [7] the authors show a similar comparison for a point-to-point Long Term Evolution (LTE) system. Since we concentrate on a system setup that is almost identical with a third Generation Partnership Project (3GPP) New Radio (NR) system and a subcarrier spacing scaled to 480 kHz, we believe that these results show that such a receiver might be attractive for a practical system.

Due to the limited space not all details are given. In this work we only describe the overview and the differences with our theoretical evaluation in [1] and the link-level evaluation in [8]. The remaining paper is organized as follows: In the first section we present the chosen channel estimation method as well as the model for the theoretical evaluation. Afterwards, the approach used for the information theoretic rate evaluation as well as MIMO detection for the link-level evaluation are introduced. To highlight that it is possible to achieve the theoretically predicted rates in a link-level based evaluation, we show the comparison between these two methods for the same scenario. At the end of the paper we summarize our findings.

Throughout the paper we use boldface lower and upper case letters to represent column vectors and matrices, respectively. The term \(a_{m,l}\) is the element on row \(m\) and column \(l\) of matrix \(A\) and \(a_m\) is the \(m\)th element of vector \(a\). The expressions \(A^*, A^T, A^H\), and \(A^{-1}\) respectively represent the complex
conjugate, the transpose, the Hermitian, and the inverse of the matrix $A$. The symbol $R_{ab}$ is the correlation matrix of zero-mean vectors $a$ and $b$ defined as $E[ab^H]$.

II. CHANNEL ESTIMATION

For this evaluation we use the 3GPP NR Orthogonal Frequency Division Multiplexing (OFDM) type 1 DeModulation Reference Signals (DMRS) consisting of a pseudo noise sequence defined by parts of a length-31 Gold sequence described in [9]. These reference signals use three techniques to orthogonalize the signals from different users:

- Time domain Cyclic Shift (CS)
- Frequency Division Multiplexing (FDM)
- Code Division Multiplexing (CDM) for the case of more than four users

In this work we assume only four users, thus only time domain CS and FDM are used. These techniques in conjunction with the use of the Gold sequences provides sufficient orthogonality. Therefore, for the following theoretical calculation of the channel estimation Mean Square Error (MSE) as well as the channel estimation algorithm for the link-level simulation, we assume that the signals from different users are completely orthogonal.

For both the link-level simulation and calculation of the theoretical channel estimation MSE we use separate interpolation in the time and frequency dimensions. The work in [10] shows that this leads to only a minor performance degradation compared to joint time-frequency interpolation at a greatly reduced computational complexity. To further reduce the complexity we also limit the number of Sub-Carriers (SCs) that are used for the interpolation in the frequency direction. Again, we use the same setting for the theoretical and link-level based evaluation. This is different from the evaluation in [1] which did not place any restriction on the length of each interpolation filter and also included spatial smoothing. In this work we do not assume the algorithm to have knowledge about the antenna array geometry. Another important aspect is that for these channel estimation and equalization techniques to work properly, it must be ensured that the power received from different users is not very different. In a practical system this is ensured via power control.

Assuming perfect synchronization of the timing and carrier frequency, the received OFDM signal $Y_{k,\ell}$ for SC $k$ and OFDM symbol $\ell$ can be written as

$$ Y_{k,\ell} = H_{k,\ell}X_{k,\ell} + \eta_{k,\ell}, $$

(1)

where we assume that the channel impulse response is shorter than the Cyclic Prefix (CP), and $H_{k,\ell}, X_{k,\ell}$ and $\eta_{k,\ell}$ represent the channel, transmit signal and white Gaussian noise, respectively. Since the channel estimation procedure is executed separately for each antenna and user, no user or antenna index is used. It is important to note that this technique assumes knowledge of the following statistical channel parameters:

- Doppler spread
- Delay spread
- Received Signal-to-Noise-Ratio (SNR) of each user

Assuming a reference symbol is present on SC $q$ and OFDM symbol $p$, we multiply the signal with the known reference signal to obtain the corresponding channel estimate for this resource element

$$ \hat{H}_{p,q} = Y_{p,q}X_{p,q}^* = H_{p,q} + \eta_{p,q}, $$

(2)

where we assume that $|X_{p,q}^*| = 1$. By combining the channel estimates for all resource elements on $K$ SCs and $L$ symbols we get

$$ \hat{h}_r = \left[ \hat{H}_{1,1}, \hat{H}_{1,2}, \ldots, \hat{H}_{K-1,L}, \hat{H}_{K,L} \right]^T. $$

(3)

For all positions where no reference signals were sent, the corresponding element of $\hat{h}_r$ is set to zero. The set $P$ contains the indices of the reference symbols in $\hat{h}_r$. Applying the matrices for interpolation and smoothing in the time $A_t$ and frequency $A_f$ domain, we get the overall estimate of the channel at each position

$$ \hat{h} = A_t A_f \hat{h}_r = (A_t \otimes A_f) \hat{h}_r. $$

(4)

In many cases the covariance of the channel is unknown, and it is necessary to generate the interpolation matrices based on a model for the covariance matrix. The parameters of this model also need to be estimated. It is important to mention that the same interpolation matrix $A_f$ is used for all OFDM symbols containing reference symbols. This means that if the reference signal pattern in these symbols is not the same, we need to apply different matrices for each symbol. In the case of 3GPP NR OFDM type 1 DMRS the pattern on each OFDM symbol carrying reference signals is the same. In this work we assume the Doppler spread to be small enough that the channel can be assumed to remain static in one subframe of 14 OFDM symbols. Therefore, the time interpolation matrix $A_t$ consists only of an averaging among the OFDM symbols that contain reference symbols.

The frequency interpolation and smoothing matrix $A_f$ is based on the Minimum Mean Square Error (MMSE) solution described in [10]. To generate these matrices based on this method, we need to generate the auto- and cross-correlation of the channel $R_{h_a h_a}$ and $R_{h_b h_b}$. The vectors $h_a$ and $h_b$ represent the channel at the position of the reference signals and the channel for all SCs and for OFDM symbol $\ell$, respectively. In the case that we know all these matrices, the interpolation matrix is defined as

$$ A_f = R_{h_a h_a} (R_{h_a h_a} + R_{\eta \eta})^{-1}, $$

(5)

where $R_{\eta \eta}$ is the noise covariance matrix. In a practical system we could easily have hundreds of SCs, thus the complexity of the matrix inversion is extreme. Even if the values of the inversion are precomputed, the subsequent matrix-vector multiplication has high complexity. We therefore limit the length of the reference symbols considered for the interpolation to a certain number $K_C$. Since the channels for widely separated SCs is nearly uncorrelated, this has only a minor impact on the performance but significantly decreases the complexity.
In addition, for a practical system it is often not possible to directly observe and estimate the covariance matrix of the channel, especially with no reference signals present. Consequently, we use a model to generate the covariance matrices. For many real-world scenarios, multipath signals that arrive later at the receiver propagate over a longer distance, thus leading to lower energy at the receiver. Since this can be well approximated by an exponential Power Delay Profile (PDP), we use this as a model for the interpolation matrix.

Since the elements of $R_{hh^t}$ and $R_{hh^r}$ represent the cross correlation of different elements of the channel, all elements of these matrices are defined by the cross correlation between the channels on SC $i$ and $j$

$$E[h_i h_j^*] = \frac{1}{1 - j^2 \pi \tau_{\text{RMS}} d(i,j)},$$

(6)

where $\tau_{\text{RMS}}$ is the Root Mean Square (RMS) delay spread as defined in [11] and $d(i,j)$ the frequency separation of the $i$th and $j$th SCs. Since the RMS delay spread usually remains stable over a long period it can be estimated and we assume that it is perfectly known for the link-level simulation as well as the theoretical evaluation.

If we exclude the spatial interpolation from the theoretical channel estimation error derived in [1] we get the following expression for the average per-SC channel estimation error

$$\frac{1}{KL} \sum_{i=1}^K \sum_{j=1}^J \| h_i - h_j \|^2 = \frac{1}{KL} \left[ C1 - 2R(C2) + C3 \right],$$

(7)

with the components $C1$, $C2$ and $C3$ defined as:

$$C1 = \sum_{p \in P} \sum_{p2 \in P} \left( \sum \left[ (A_{i1})^H (A_{j1}) \right] \left( [A_{j}]_{k2}^H [A_{j}]_{k1} \right) \right) \left( R_{hh^t}\right)_{\ell1,\ell2} \left( R_{hh^r}\right)_{k1,k2} + \left( R_{\eta^t}\right)_{\ell1,\ell2} \left( R_{\eta^r}\right)_{k1,k2},$$

(8)

$$C2 = \sum_{p \in P} \left[ (A_{i1})^H R_{hh^t}\right]_{\ell1} \left[ (A_{j1})^H R_{hh^r}\right]_{k,k},$$

$$C3 = \text{tr} \left( R_{hh^t}\right) \text{tr} \left( R_{hh^r}\right),$$

where $k, k1, k2, \ell, f1$ and $f2$ are the frequency and time indices corresponding to the position of the reference symbols $p, p1$ and $p2$. The symbols $R_{hh^t}, R_{hh^r}, R_{\eta^t}$ and $R_{\eta^r}$ represent the temporal and frequency component of the channel $h$ and noise $\eta$ correlation.

For the simulations we use the channel model described in [12]. In Figure 1 we show the channel estimation error for different interpolation lengths. For low SNRs the additional benefits of averaging leads to the interpolation of length 16 having the best performance, and we note that the gap between the curves corresponds to the theoretically predicted 3 dB loss. In the high SNR regime the longest interpolation length overfits the exponential PDP. To achieve a good trade-off between complexity and performance over the whole SNR range we use a maximum interpolation length of 8 for the rest of this work.
B. Link-Level Simulation Based Rate Evaluation

For the link-level simulations we take the same effects into account as described in the preceding paragraph for the theoretical evaluation. There are some additional effects that must be accounted for, and we describe them in the following paragraph.

As mentioned above, since the processing is performed in the frequency domain, the quantization error can be modeled as Gaussian. Thus, in all the link-level processing steps we simply treat the quantization error as additional noise. Based on the channel estimate we also determine the signal power by averaging over the estimated power at each SC. In a similar fashion we estimate the noise power by regenerating the receive signal from the channel estimate and the known reference signal. Afterwards, we subtract this regenerated receive signal from each user to generate noise samples and then average over their power. This estimate is used to align the power of the noise at each antenna.

As in [8] we use an unbiased MMSE receiver for the MIMO equalization. We use the channel estimates to generate the MMSE MIMO equalizer. We also use the rate matching channel code and the Cyclic Redundancy Check (CRC) from a standard 3GPP LTE system defined in [15]. To adapt to different channel conditions we use QPSK, 16-QAM and 64-QAM modulation with different code rates. For each SNR we calculate the Block Error Rate (BLER) for each of the modulation formats and combine that with the bits transmitted in one block to calculate the transmitted data rate. Then we select the highest data rate separately for each SNR.

IV. Simulation Results

A. Simulation Parameters

All major simulation parameters are shown Table I. These describe a 3GPP NR system with a SC spacing of 480 kHz, but we used an LTE turbo code instead of the codes defined for NR. To enable simulations over a wide SNR range the modulation and code rate combinations in Table II are used. At the modulation format switching point we always choose a configuration for each of the two modulation formats to have the same spectral efficiency. This enables us to compare the effects of the different modulation formats. For the calculation of the spectral efficiency we included the CP and channel estimation reference signal overhead in both the theoretical and link-level evaluations. We also only used the occupied bandwidth of $1200 \times 32 \times 15 \text{ kHz} = 576 \text{ MHz}$ for the spectral efficiency calculation, thus neglecting the guardband required in a practical system. It is important to mention that the oversampling is also taken into account in the theoretical rate calculation, as it changes the quantization noise power for each frequency bin.

B. Simulation Results

The results for the comparison between the rate calculation and the link-level simulations are shown in Fig. 2, and we see a gap between the theoretical and link-level simulation results. There are multiple effects that make the results of the link-level simulations worse than the theoretical evaluation:

- Transmit symbols are not Gaussian
- Theoretical evaluation does not take the Inter Symbol Interference (ISI) of channels that are longer than the CP into account
- LTE turbo code performance is not ideal

Considering all these reasons for the performance gap, we believe that these results show that the interesting rates predicted in many theoretical evaluations could translate well to a practical implementation. The performance gap is consistent
with that observed in similar evaluations for a point-to-point LTE system in [7]. We note that the low-SNR regime is the more likely operating condition for such systems, and the observed performance difference is much smaller there. For the modulation and coding schemes that result in the same spectral efficiency, there is a performance gap of 5 dB between QPSK and 16-QAM modulation for 1 bit resolution, showing that all amplitude information is lost during the quantization of the time domain symbols. As the systems using ADCs with 2-3 bits of resolution do not remove the amplitude information for each time domain sample, the performance difference for the configuration with the same spectral efficiency is in the range of 0 to 1 dB. We also highlight that the SE of 10 bps/Hz results in a combined data rate of about 5.8 Gbit/s.

V. CONCLUSION

This comparison shows that indeed the data rates predicted for mmWave digital beamforming with low resolution ADCs translates well into link-level simulation results. At low per-antenna SNRs where such systems will likely operate, the performance of the link-level simulation can achieve the same data rate at about 4 dB lower SNR compared to the theoretical prediction. At high SNRs, the additional limitations of the constellation used in this work leads to a maximum data rate that is substantially lower compared to the theoretical prediction. The next steps going beyond this work could be to include more realistic aspects like beam alignment procedures, carrier frequency offset, timing offset and power offset of the different users in the link-level simulation.

ACKNOWLEDGMENT

This work has been performed in the framework of the Horizon 2020 project ONE5G (ICT-760809) receiving funds from the European Union. H. Pirzadeh and A. Swindlehurst were supported by the U.S. National Science Foundation under Grants ECCS-1547155 and CCF-1703635.

REFERENCES