Limited Rate Feedback for Two-User MISO Gaussian Interference Channel With and Without Secrecy

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Abstract—We study a two-user multiple-input single-output Gaussian interference channel with limited rate feedback and transmitter cooperation. Each receiver quantizes the channel state information of the direct and cross channels, and sends the codebook indices back to the transmitters through two sum-rate-limited feedback channels. The quantization errors reduce the beamforming gain from the direct transmitter, and cause interference leakage from the cross transmitter. Under an assumption of high signal-to-noise ratio, we first approximate the average transmission rate of each link, and use the sum rate to find the optimal transmit power and corresponding feedback bit allocation. We show that the maximum sum throughput is achieved using full transmit power, and the achievable sum rate under limited feedback is bounded above by a constant. We then extend the results to the case where secrecy is desired. In contrast to the first problem, increasing the transmit power beyond a certain point decreases the secrecy performance. Additionally, we analyze the asymptotic behavior of the solutions when the total feedback bandwidth and the number of transmit antennas grow large. We derive all results in closed form. Simulations validate the theoretical analysis and demonstrate the significant performance gains that result from the use of optimal transmit power control and intelligent feedback bit allocation.

Index Terms—Interference channel, limited rate feedback, multiple-input single-output (MISO), cooperative communication, physical layer secrecy, random vector quantization (RVQ).

I. INTRODUCTION

In an interference channel, multiple wireless communication links are simultaneously active in the same time and frequency resource, and hence potentially interfere with each other. A long history of studying the interference channel has provided various achievable rate regions [1]–[5]. Lately, extending the interference channel results to cases involving multiple antenna transceivers has drawn significant interest [6]–[9]. Employing multiple antennas increases the diversity gain and can help mitigate the interference in the system, provided that accurate channel state information (CSI) is available.

The most commonly assumed approach for obtaining CSI in a wireless network is to estimate the channel at the receivers based on pilot signals sent by the transmitters, and then feed back quantized CSI to the transmitters via a low-rate feedback channel. The impact of quantized CSI feedback on wireless communications performance has been studied by many researchers over the past two decades. Most of the initial work focused on performance in point-to-point multi-input multi-output (MIMO) links, but more recently studies have been undertaken that investigate quantized feedback in situations with multiple transmitters or multiple receivers (possibly all with multiple antennas). Quantized CSI for multi-antenna broadcast channels has been considered in [10]–[13], although these papers focused on situations where the number of feedback bits employed by each receiver was identical and not optimized. Optimized codebook design for CSI quantization in an OFDMA system was studied in [14].

More relevant to our paper is prior work on the impact of quantized CSI in multi-antenna interference channels (IC). The IC case is interesting because considerable gains can be obtained if the receivers feed back the quantized CSI not only to their desired transmitter, but also to the interfering transmitters as well in order to control the interference. These kinds of cooperative IC scenarios have been considered by a number of authors, but primarily for scenarios where, again, the number of bits fed back to the various transmitters is fixed and not optimized [15]–[25]. On the other hand, a few papers have considered optimizing the allocation of the feedback bits between the desired and interfering transmitters, under a constraint on the total feedback bandwidth [26]–[29]. However, the resulting optimization is quite complicated (e.g., requiring brute-force or greedy search techniques) and cannot be solved in closed form. An approximate closed-form solution is obtained in [27], but only for a soft handoff multicell scenario involving a linear array of cells with a single user per cell.

Interference is not only an issue for communication rates, but also for security since information can be extracted from the “interference” received by users who are not the intended recipients. In such scenarios it is desirable to minimize the leakage of information to those unintended receivers [30]. In our previous work [31], [32], we studied the impact of limited rate feedback on the wiretap channel with a helper, which can be considered as a special case of the two-user IC model. One receiver is a known eavesdropper, and a second transmitter is used to send a jamming signal to enhance secrecy. In these papers, we derived closed-form expressions for the optimal feedback bit allocation to the transmitter and the helper for MISO and MIMO cases.
respectively. Secrecy for the two-user IC has received considerable attention in recent years, and several approaches have been proposed that discuss how cooperation between the transmitters can be exploited to improve secrecy [33]–[42].

In this paper, we study strategies for transmit power control and feedback bit allocation for the two-user Gaussian MISO interference channel with limited rate feedback, and we separately consider the problems of maximizing the sum rate or secrecy sum rate of the IC. We assume random vector quantization (RVQ) of the CSI for the direct and cross channels, and assume that each receiver sends the indices of the corresponding codewords to both its own and the interfering transmitter through two sum-rate limited feedback channels. The contributions of this paper can be summarized as follows:

- For the case of maximizing the sum rate of the IC, we assume that the receivers only decode their own messages, and treat the interference as Gaussian noise. We derive an approximation for the average transmission rate of each link, and (unlike the prior work cited above) we find closed-form expressions for the transmit power and the feedback bit allocation that maximize the system sum throughput. Simulations demonstrate a significant gain in sum rate for the optimized system.

- We then consider the limited rate feedback IC with confidential messages, where each transmitter desires to send independent information to its intended receiver while ensuring mutual information-theoretic secrecy. In this case we assume the two transmitters are amenable to cooperation for improving the overall secrecy performance of the system. We derive an approximation for the average secrecy rate of each link, and we then obtain closed-form expressions for the transmit power and feedback bits allocated to the transmitters that maximize the sum secrecy rate. Again, simulations demonstrate that the optimized system achieves significantly higher secrecy rates.

- We study the asymptotic behavior of our solutions for the case where the number of antennas is large, and derive closed-form approximations for the sum rate, secrecy sum rate, and feedback bit allocation. Interestingly, we show that, for a fixed feedback bandwidth, the asymptotic sum rates converge to a constant value as the number of antennas increases, indicating that the gain of having more antennas is offset by the increased error in representing the larger channel with a fixed number of bits.

- We study the asymptotic behavior of our solutions for the case where the feedback bandwidth grows large, and again we derive closed-form approximations for the corresponding sum rate, secrecy sum rate, and feedback bit allocation. We show that, in the limit of a large feedback bandwidth, the optimal bit allocation is to assign half the bandwidth to each of the two transmitters.

The remainder of this paper is organized as follows. In Section II, we introduce the system model and assumptions, and provide preliminary background information. In Section III and IV, we discuss the power control and bit allocation algorithms for the two scenarios discussed above, and analyze the impact of limited rate feedback on the interference channel. In Section V, we investigate the asymptotic behavior of our solutions for large numbers of antennas and large feedback bandwidths. Finally, in Section VI, we present simulation results to validate our algorithms and analysis.

Throughout the paper we use standard lowercase letters to denote scalars, and lowercase boldface letters to denote vectors. The set of \( n \)-dimensional complex vectors is denoted by \( \mathbb{C}^n \). The Hermitian transpose is represented by \( (\cdot)^H \), the absolute value \( |\cdot| \), the Euclidean norm \( \|\cdot\| \), the expectation operator \( \mathbb{E}[\cdot] \), and the identity matrix \( I \). The symbol \( \{x\}^+ \) denotes \( \max\{x, 0\} \). The indices \( i, j \) will be used as subscripts to denote which of the two transmitters, receivers or channel links are being referred to, and since we are dealing exclusively with the two-user interference channel, \( i \) and \( j \) can only take on values in the set \( \{1, 2\} \). If \( i \) and \( j \) are used together in an expression or as a subscript, they represent different values \( i \neq j \).

II. ASSUMPTIONS AND PRELIMINARIES

A. Signal Modeling

We consider the two-user MISO interference channel in Fig. 1. We assume transmitter \( i \) possesses \( N_i \) antennas, while each receiver is equipped with a single antenna. We use \( h_{ii} \in \mathbb{C}^{N_i} \) to denote the direct channel from transmitter \( i \) to receiver \( i \); and \( h_{ij} \in \mathbb{C}^{N_i} \) to denote the cross channel from transmitter \( j \) to receiver \( i \) \( (\forall j \neq i; i, j \in \{1, 2\}) \). The elements of these channel vectors are assumed to be independent and identically distributed (i.i.d.), and have zero-mean complex Gaussian distributions with variance \( \sigma^2_{ii} \) and \( \sigma^2_{ij} \) respectively. All channels experience independent block fading.

The receivers are assumed to have perfect CSI for their own channels that they send to their respective transmitter through two error-free, insecure feedback channels. To reduce interference or enhance secrecy, the two links agree to cooperate and also provide CSI for the cross-links from each user to the interfering transmitter. The \( i \)-th receiver quantizes the CSI of its direct and cross links for each channel realization by selecting the closest codeword from two independent variable volume codebooks containing \( 2^{B_{ii}} \) and \( 2^{B_{ij}} \) entries, respectively. The
Specializing the results of Theorem 4 in [43] to our case, the distortion can be bounded below and above as follows:

\[
\left( \frac{N_j - 1}{N_j} \right) 2^{-\frac{N_j - 1}{2}} \leq D_{ij} \leq \Gamma \left( \frac{N_j}{N_j - 1} \right) 2^{-\frac{N_j - 1}{2}},
\]

where \( \Gamma(\cdot) \) represents the gamma function. We see that the upper and lower bounds converge to the same value as \( N_j \) increases and are thus tight. Consequently, in what follows we will use the upper bound to motivate the following approximations:

\[
E \left[ |\hat{h}_{ii}^H \hat{h}_{ii}|^2 \right] \approx 1 - \gamma_i 2^{-\frac{N_j - 1}{2}},
\]

\[
E \left[ |\hat{h}_{ij}^H \hat{h}_{ij}|^2 \right] \approx 1 - \gamma_j 2^{-\frac{N_j - 1}{2}},
\]

where \( \gamma_i \equiv \Gamma(\frac{N_j}{N_j - 1}) \) and \( \gamma_j \equiv \Gamma(\frac{N_j - 1}{N_j - 2}) \). We will see that the approximation is very accurate, even when the number of antennas is relatively small.

### C. Beamforming Design

We assume a relatively high signal-to-noise ratio (SNR) scenario such that the zero-forcing (ZF) transmit scheme is sum-rate optimal according to [44]. ZF beamforming is also known to be very close to optimal when secrecy rate is considered, as demonstrated in [45], [46]. Under the assumption of perfect feedback, transmitter \( i \) chooses a unit-norm beamforming vector \( w_i \), which is orthogonal to \( h_{ji} \) and maximizes \( |h_{ii}^H w_i|^2 \). This vector is defined as the ZF beamformer and is given by

\[
w_i^{ZF} = \frac{\left( \mathbf{1} - \hat{h}_{ji} \hat{h}_{ji}^H \right) h_{ii}}{\| \left( \mathbf{1} - \hat{h}_{ji} \hat{h}_{ji}^H \right) h_{ii} \|},
\]

where \( \mathbf{1} - \hat{h}_{ji} \hat{h}_{ji}^H \) denotes a projection onto the orthogonal complement of the column space of \( h_{ji} \). The vector \( w_i^{ZF} \) is constructed such that it nulls the interference at receiver \( j \), and the remaining degrees of freedom are used to maximize the transmission rate to receiver \( i \).

Under the limited rate feedback scenario, however, the transmitters only have access to the quantized version of their channels. Thus, the ZF beamforming vector at transmitter \( i \) becomes

\[
w_i = \frac{\left( \mathbf{1} - \hat{h}_{ji} \hat{h}_{ji}^H \right) h_{ii}}{\| \left( \mathbf{1} - \hat{h}_{ji} \hat{h}_{ji}^H \right) h_{ii} \|}.
\]

### D. Average Transmission Rate With Perfect CSI

The following lemmas provide useful preliminaries.

**Lemma 1:** Consider a \( t \)-dimensional vector \( g \in \mathbb{C}^t \) with zero-mean and independent complex Gaussian entries, i.e., \( g \sim \mathcal{CN}(0, \sigma_g^2 \mathbf{I}) \). The expected value of the logarithm of \( ||g||^2 \) is given by

\[
E \left[ \ln ||g||^2 \right] = \psi(t) + \ln \sigma_g^2,
\]

where \( \psi(\cdot) \) is the digamma function.

**Proof:** See Appendix A.
Lemma 2: For any two independent and isotropically distributed unit-norm vectors \( \mathbf{u}, \mathbf{v} \in \mathbb{C}^1 \), the following holds:
\[
\mathbb{E} \left[ \ln |\mathbf{u}^H \mathbf{v}|^2 \right] = \psi(1) - \psi(t) \tag{6}
\]
\[
\mathbb{E} \left[ \ln \left( 1 - |\mathbf{u}^H \mathbf{v}|^2 \right) \right] = \psi(t) - \psi(1). \tag{7}
\]
Proof: See Appendix B.

Lemma 3: Let \( \rho \) be a fixed constant and let \( Q \) be a random variable. The following approximation holds for large \( \rho \):
\[
\mathbb{E} \left[ \log_2 (1 + \rho Q) \right] \approx \log_2 \left( 1 + \rho e^{\mathbb{E} \ln Q} \right). \tag{8}
\]
Proof: See Appendix C.

Under perfect feedback, the transmitters can suppress all interference using the ZF beamforming vectors in (3). The average transmission rate of the \( i \)-th transmitter-receiver link with perfect CSI can be expressed as
\[
R_i^{ZF} = \mathbb{E} \left[ \log_2 \left( 1 + P_i |\mathbf{h}_{ii}^H \mathbf{w}_i^F|^2 \right) \right]. \tag{9}
\]

Proposition 1: The average transmission rate of the \( i \)-th transmitter-receiver link with perfect feedback satisfies the approximation:
\[
R_i^{ZF} \approx \log_2 \left( 1 + P_i \sigma_{ii}^2 e^{\psi(N_i-1)} \right). \tag{10}
\]
Proof: Plugging the ZF beamforming vector (3) into (9), we have
\[
R_i^{ZF} = \mathbb{E} \left[ \log_2 \left( 1 + P_i \frac{|\mathbf{h}_{ii}^H (\mathbf{I} - \tilde{\mathbf{h}}_{ji}, \tilde{\mathbf{h}}_{ji}^H) \mathbf{h}_{ii}|^2}{||\mathbf{I} - \tilde{\mathbf{h}}_{ji}, \tilde{\mathbf{h}}_{ji}^H|| \mathbf{h}_{ii}||^2} \right) \right]
= \mathbb{E} \left[ \log_2 \left( 1 + P_i \frac{||\mathbf{I} - \tilde{\mathbf{h}}_{ji}, \tilde{\mathbf{h}}_{ji}^H|| \mathbf{h}_{ii}||^2}{||\mathbf{I} - \tilde{\mathbf{h}}_{ji}, \tilde{\mathbf{h}}_{ji}^H|| \mathbf{h}_{ii}||^2} \right) \right]
= \mathbb{E} \left[ \log_2 \left( 1 + P_i \left( 1 - \frac{||\mathbf{h}_{ii}^H \mathbf{h}_{ii}|^2}{||\mathbf{h}_{ii}||^2} \right) \right) \right]
\approx \log_2 \left( 1 + P_i e^{\mathbb{E} \ln \left( \left|\frac{||\mathbf{h}_{ii}^H \mathbf{h}_{ii}|}{||\mathbf{h}_{ii}||} \right|^2 \right)} \right)
= \log_2 \left( 1 + P_i \sigma_{ii}^2 e^{\psi(N_i-1)} \right). \tag{11}
\]
Under the high SNR assumption, we use the approximation in (11) following from (8) in Lemma 3. Given that \( \mathbf{h}_{ii} \) is independent of \( \mathbf{h}_{ji} \), (12) is obtained from (5) in Lemma 1 and (7) in Lemma 2.

III. LIMITED RATE FEEDBACK ANALYSIS

Here we assume that each receiver only intends to decode the information signal from its paired transmitter, treating the message of the other transmitter as interference. There is no attempt to decode and subtract the interfering message. For the given beamforming vectors in (4), the average achievable transmission rate of the \( i \)-th transmitter-receiver link under limited rate feedback is:
\[
R_i = \mathbb{E} \left[ \log_2 \left( 1 + \frac{P_i |\mathbf{h}_{ii}^H \mathbf{w}_i|^2}{1 + P_j |\mathbf{h}_{jj}^H \mathbf{w}_j|^2} \right) \right]. \tag{13}
\]
In order to evaluate the performance metric, we use the following lemmas.

Lemma 4: The expected value of the signal power term can be approximated as
\[
\mathbb{E} \left[ \log_2 \left( 1 + P_i |\mathbf{h}_{ii}^H \mathbf{w}_i|^2 \right) \right]
\approx \log_2 \left( 1 + P_i \sigma_{ii}^2 e^{\psi(N_i-1)} \right) + \log_2 \left( 1 - \gamma_j 2^{-\frac{R_i}{N_i^e}} \right). \tag{14}
\]
Proof: See Appendix D.

Lemma 5: The expected value of the noise power term can be approximated as
\[
\mathbb{E} \left[ \log_2 \left( 1 + P_j |\mathbf{h}_{jj}^H \mathbf{w}_j|^2 \right) \right]
\approx \log_2 \left( 1 + P_j \sigma_{jj}^2 e^{\psi(N_j-1) + \psi(1)} \gamma_j 2^{-\frac{R_j}{N_j^e}} \right). \tag{15}
\]
Proof: See Appendix E.

To characterize the effect of limited feedback, we derive an approximation of the average transmission rate of the \( i \)-th transmitter-receiver link:

Proposition 2: The average transmission rate of the \( i \)-th transmitter-receiver link is approximately given by
\[
R_i \approx \log_2 \left( 1 + P_i \sigma_{ii}^2 e^{\psi(N_i-1)} \right) + \log_2 \left( 1 - \gamma_j 2^{-\frac{R_i}{N_i^e}} \right)
- \log_2 \left( 1 + P_j \sigma_{jj}^2 e^{\psi(N_j-1) + \psi(1)} \gamma_j 2^{-\frac{R_j}{N_j^e}} \right). \tag{16}
\]
Proof: The average transmission rate of the link from transmitter \( i \) to receiver \( i \) in (13) can be approximated as below:
\[
R_i \approx \mathbb{E} \left[ \log_2 \left( 1 + \frac{P_i |\mathbf{h}_{ii}^H \mathbf{w}_i|^2}{1 + P_j |\mathbf{h}_{jj}^H \mathbf{w}_j|^2} \right) \right]
= \mathbb{E} \left[ \log_2 \left( 1 + P_i \frac{|\mathbf{h}_{ii}^H \mathbf{w}_i|^2}{1 + P_j |\mathbf{h}_{jj}^H \mathbf{w}_j|^2} \right) \right]
\approx \log_2 \left( 1 + P_i \sigma_{ii}^2 e^{\psi(N_i-1)} \right) + \log_2 \left( 1 - \gamma_j 2^{-\frac{R_i}{N_i^e}} \right)
- \log_2 \left( 1 + P_j \sigma_{jj}^2 e^{\psi(N_j-1) + \psi(1)} \gamma_j 2^{-\frac{R_j}{N_j^e}} \right). \tag{17}
\]
The approximation in (17) holds by eliminating the interference term \( P_j |\mathbf{h}_{jj}^H \mathbf{w}_j|^2 \) in the numerator under the high SNR assumption. Equation (18) follows from (14) and (15) in Lemma 4 and Lemma 5.

We observe from (16) that the first term is the approximate average transmission rate under perfect feedback obtained from (10) in Proposition 1. Furthermore, the use of limited rate feedback produces two additional negative terms. The second term is due to the mismatch with the ideal beamformer for the direct transmitter, and the third term results from the existence of interference from the other transmitter that has not been canceled.
due to the quantized beamformer. Together, they constitute the throughput loss due to quantization errors.

Our optimization problem is to maximize the average sum transmission rate by adjusting the amount of transmit power and the feedback bit allocation for the direct and cross channels:

\[
\max_{P_i, B_{i,i}, B_{i,j}} R = \sum_{i=1}^{2} R_i \quad i \neq j; \quad i, j \in \{1, 2\} \tag{19}
\]

s.t. \[0 < P_i \leq P_{\text{max},i}, \]
\[B_{i+i} + B_{i,j} = b_i, \]
\[B_{i,i}, B_{i,j} \in \mathbb{Z}^+, \]

where \(\mathbb{Z}^+\) is the set of non-negative integers. Relaxing the integer constraint, the following closed-form expression for the optimal solution holds when the transmitters have the same number of antennas:

**Theorem 1:** Suppose the transmitters have the same number of antennas \(N\). Then the optimal solution to (19) without the integer constraint, and the corresponding maximum average sum transmission rate are shown in (20)–(22), shown at the bottom of this page, where \(\gamma = \Gamma(\frac{N}{N-1}), i \neq j\) and \(i, j \in \{1, 2\}\).

**Proof:** To solve the optimization problem, we first relax the integer constraint. Then, we use the sum of the average transmission rate in (16) as an approximation to the objective function. Substituting \(b_i - B_{i,j}\) for \(B_{i,j}\), the optimization problem in (19) can be rewritten in standard form as

\[
\max_{P_i, B_{i,i}} \sum_{i=1}^{2} \log_2 \left( \frac{1 + P_i \sigma_i^2 e^{\psi(N,-1)}}{1 + P_j \sigma_j^2 e^{\psi(N_j,-1) - \psi(N_j-1)} + (\psi(1) \gamma j) 2^{-\frac{\beta_j}{\gamma_j}}} \right)
\]

s.t. \[-P_i < 0,\]
\[-P_i, P_{\text{max},i} \leq 0,\]
\[-B_{i,i} \leq 0,\]
\[-B_{i,i} - b_i \leq 0.\]

This formulation leads to a convex optimization problem that can be solved by standard methods. We formulate the Lagrangian

\[
\mathcal{L} = \sum_{i=1}^{2} \log_2 \left( \frac{1 + P_i \sigma_i^2 e^{\psi(N,-1)}}{1 + P_j \sigma_j^2 e^{\psi(N_j,-1) - \psi(N_j-1)} + (\psi(1) \gamma j) 2^{-\frac{\beta_j}{\gamma_j}}} \right),
\]

where \(\mu_{ik}, k = 1, \ldots, 4\) is the Lagrange multiplier. By taking the derivative of (23) with respect to \(P_i\) and \(B_{i,i}\), and applying Karush-Kuhn-Tucker (KKT) conditions, the closed-form solution is given in (20)–(22) when the transmitters have the same number of antennas.

**Remark 1:** We see from Theorem 1 that transmitting with full power is the optimal transmission strategy. Note that the need for cooperation is evident since the cross channel variance \(\sigma_j^2\) and the maximum power of the other transmitter \(P_{\text{max},j}\) play a large role in determining the optimal feedback bit allocation \(B_{i,j}\).

In practice, \(P_{\text{max},j}\) will exceed the threshold condition specified in Equations (21)–(22) when \(b_i\) is sufficiently large. Consequently, the optimization often produces a non-integer bit allocation that is less than \(b_i\). For the actual feedback link, we search the integer values above and below \(B_{i,j}\) to determine the integer bit allocation \(B_{i,j}^*\) and \(B_{i,j}^* = b_i - B_{i,j}^*\) and the actual sum rate \(\tilde{R}^*\) according to the expression in (16):

\[
\tilde{R}^* \approx \sum_{i=1}^{2} \left[ \log_2 \left( 1 + P_i \sigma_i^2 e^{\psi(N,-1)} \right) + \log_2 \left( 1 - \gamma 2^{-\frac{\beta_j}{\gamma_j}} \right) \right. \]
\[\left. - \log_2 \left( 1 + P_j \sigma_j^2 e^{\psi(N_j,-1)} + (\psi(1) \gamma j) 2^{-\frac{\beta_j}{\gamma_j}} \right) \right].
\]

**Remark 2:** Due to the relaxation of the integer constraint, \(\tilde{R}^*\) is bounded above by \(R^*\). As the transmit power grows to infinity, the average sum transmission rate with a fixed number of antennas \(N\) approaches a power of \(N\), according to (23).
of feedback bits is bounded above by

$$\tilde{R}^i \leq \lim_{P_{\text{max},i} \to \infty} P^* \sum_{i=1}^{2} \log_2 \left( \frac{\sigma_i^2 2^{\frac{N_i}{N}}}{4 \gamma^2 \sigma_j^2 e^{\psi(N)} - 2e^{(N-1)} + \psi(1)} \right).$$

(24)

Unlike the perfect CSI scenario, where the average achievable transmission rate with the ZF beamformer can be made arbitrarily large by increasing the transmit power, the system performance of the limited rate feedback interference channel converges to a constant at high SNR.

IV. SECRECY RATE ANALYSIS

In this section, we consider the case where the transmitters wish to cooperate in order to enhance the secrecy of their messages. Here the cooperation is even more critical since for secrecy the goal is not just to reduce the interference of the transmitter’s signal at the other receiver, but to prevent the other receiver from obtaining any information about the signal itself. Cooperation can also be useful in determining if one of the users is feeding back erroneous CSI in order to thwart the secrecy measures. From the secrecy point of view, the metric of interest is the average achievable secrecy rate defined as in [32], [47]:

$$R_{\text{sec},i} = \left\{ \mathbb{E} \left[ \log_2 \frac{1 + P_i |h_{ji}^H w_i|^2}{1 + P_j |h_{ji}^H w_i|^2} \right] \right\}^+.$$ (25)

As mentioned above, maximizing the sum secrecy rate is a more stringent problem than that considered in the previous section, since the corresponding rate (and associated coding) prevents the non-intended receiver from receiving any information about the transmitted signal. We will see that the resulting solution for the case with secrecy is quite different from that obtained in the previous section; in particular, it will be shown that, in terms of maximizing the sum secrecy rate, it is no longer optimal for the base stations to transmit with full power.

We approximate the average secrecy rate of the i-th transmitter-receiver link, similarly to Proposition 2.

**Proposition 3:** The average secrecy rate of the i-th transmitter-receiver link is approximately given by

$$R_{\text{sec},i} \approx \left\{ \log_2 \left( 1 + P_i \sigma_i^2 e^{\psi(N-1)} \right) + \log_2 \left( 1 - \frac{2 B_{ji}}{N_i} \right) \right\}^+.$$ (26)

**Proof:** Similar to the proof of Proposition 2, the average secrecy rate of the i-th transmitter-receiver link in (25) can be approximated as

$$R_{\text{sec},i} = \left\{ R_i - \mathbb{E} \left[ \log_2 \left( 1 + P_j |h_{ji}^H w_i|^2 \right) \right] \right\}^+$$

$$\approx \left\{ \log_2 \left( 1 + P_i \sigma_i^2 e^{\psi(N-1)} \right) + \log_2 \left( 1 - \frac{2 B_{ji}}{N_i} \right) \right\}^+.$$ (28)

The last step is based on the approximation in Proposition 2 and Lemma 5.

Our goal is to maximize the average sum secrecy rate by adjusting the amount of transmit power and the feedback bit allocation:

$$\max_{P_i, B_{ii}, B_{ij}} R_{\text{sec}} = \sum_{i=1}^{2} R_{\text{sec},i} \quad i \neq j; \quad i, j \in \{1, 2\}$$

s.t. \(0 < P_i \leq P_{\text{max},i}\)

\(B_{ii} + B_{ij} = b_i\)

\(B_{ii}, B_{ij} \in \mathbb{Z}^+\).

**Theorem 2:** When the transmitters have the same number of antennas \(N\), the optimal solution to (27) without the integer constraint, and the corresponding maximum average sum secrecy rate are characterized by (28)–(30), shown at the bottom of this page, where \(i \neq j\) and \(i, j \in \{1, 2\}\).
Proof: After relaxing the integer constraint as in Theorem 1, we use the sum of the average secrecy rate in (26) as an approximation to the objective function. Substituting \( b_i - B_{ij} \) for \( B_{ij} \), the optimization problem in (27) becomes

\[
\max_{P_i, B_{ii}} \sum_{i=1}^{2} \log_2 \left( \frac{1 + P_i \sigma_{ii}^2 e^{\psi(N_i-1)}}{1 + P_i \sigma_{ij}^2 e^{\psi(N_j-1) - \psi(N_i-1) + \psi(1)} \gamma_j 2^{-\frac{b_i - B_{ij}}{N}}} \right)
\]

s.t.

\[
-P_i < 0, \\
P_i - P_{\max,i} \leq 0, \\
B_{ii} - b_i \leq 0.
\]

As before, we consider the Lagrangian

\[
L = \sum_{i=1}^{2} \log_2 \left( \frac{1 + P_i \sigma_{ii}^2 e^{\psi(N_i-1)}}{1 + P_i \sigma_{ij}^2 e^{\psi(N_j-1) - \psi(N_i-1) + \psi(1)} \gamma_j 2^{-\frac{b_i - B_{ij}}{N}}} \right)
\]

\[+ \mu_1 P_i - \mu_2 (P_i - P_{\max,i}) + \mu_3 B_{ii} - \mu_4 (B_{ii} - b_i).\]

(31)

By equating the derivative of (31) with respect to \( P_i \) and \( B_{ii} \), and applying KKT conditions, we present the closed-form results in (28)–(30) when the transmitters have the same number of antennas.

For the actual feedback link, we search the integer values above and below \( B_{ij}^* \) in (29) to determine the integer bit allocation \( \hat{B}_{ii}^* \) and \( \hat{B}_{ij}^* \). According to (26), the actual sum secrecy rate \( \hat{R}_{\sec}^* \) is:

\[
\hat{R}_{\sec}^* \approx \sum_{i=1}^{2} \left\{ \log_2 \left( 1 + P_i^* \sigma_{ii}^2 e^{\psi(N_i-1)} \right) + \log_2 \left( 1 - \gamma_i 2^{-\frac{b_i - B_{ij}}{N}} \right) \right\}
\]

\[+ \log_2 \left( 1 + P_i^* \sigma_{ij}^2 e^{\psi(N_j) - \psi(N_i-1) + \psi(1)} \gamma_j 2^{-\frac{b_i - B_{ij}}{N}} \right) + \log_2 \left( 1 + P_i^* \sigma_{ij}^2 e^{\psi(N_j) - \psi(N_i-1) + \psi(1)} \gamma_j 2^{-\frac{b_i - B_{ij}}{N}} \right).\]

Remark 3: The need for cooperation is again clear since the optimal \( B_{ij}^* \) requires knowledge of the power employed by the other transmitter \( P_j^* \) and the cross channel statistics \( \sigma_{ij}^2 \), but with a minor difference compared to (21) in Theorem 1. However, the transmission strategy behaves differently when secrecy is taken into consideration. Beyond a certain threshold, increasing transmit power hurts the secrecy performance due to the limited feedback.

V. ASYMPTOTIC RESULTS

In this section, we study the asymptotic behavior of the solutions obtained in the previous sections for the total feedback bandwidth and the number of transmit antennas.

A. Large Feedback Bandwidth

If \( B_{ii}, B_{ij} \rightarrow \infty \), from (16) and (26) it is clear that \( R_i \) and \( R_{\sec,i} \) converge to the rates that would be obtained in the case with perfect CSI at the transmitters, and thus the rate loss in both cases is 0.

For sufficiently large \( b_i \), the optimal bit allocation for the transmission rate in (21) has the following approximation:

\[
B_{ii}^* \approx \frac{b_i}{2} - \frac{N - 1}{2} \log_2 \left( \frac{P_{\max,j} \sigma_{ij}^2 e^{\psi(N_i-1) + \psi(1)}}{P_{\max,i} \sigma_{ii}^2 e^{\psi(N_i-1) + \psi(1)}} \right),
\]

(32)

The second term in (32) is negligible as \( b_i \rightarrow \infty \), and thus we see that the feedback bandwidth is asymptotically divided into two and allocated equally to each transmitter.

For sufficiently large \( b_j \), the optimal transmit power for secrecy rate in (28) can be approximated as

\[
P_i \approx \min \left\{ \frac{2^{-\frac{N-1}{2}}}{2^\gamma \sigma_{ii}^2 e^{\psi(N_i-1) + \psi(1)}}, P_{\max,i} \right\}.
\]

We investigate two cases regarding the transmit power constraint \( P_{\max,i} \). If the optimal transmit power is limited by \( P_{\max,i} \), i.e., \( P_i = P_{\max,i} \), the optimal bit allocation in (29) that maximizes the secrecy rate is:

\[
B_{ii}^* \approx \frac{b_i}{2} - \frac{N - 1}{2} \log_2 \left( \frac{P_{\max,j} \sigma_{ij}^2 e^{\psi(N_i-1) + \psi(1)}}{P_{\max,i} \sigma_{ii}^2 e^{\psi(N_i-1) + \psi(1)}} \right),
\]

(33)

which also approaches \( b_i/2 \) as \( b_i \rightarrow \infty \). Comparing (33) to (32), optimizing the secrecy rate requires an additional \( N - 1 \) bits to be allocated to the cross channel.

On the other hand, if the transmit power is not constrained, the optimal power is:

\[
P_i \approx \frac{2^{-\frac{N-1}{2}}}{2^\gamma \sigma_{ii}^2 e^{\psi(N_i-1) + \psi(1)}}.
\]

(34)

Applying (34) to Equation (29), the optimal bit allocation as \( b_i \rightarrow \infty \) is:

\[
B_{ii}^* \approx (N - 1) \log_2 (2\gamma).
\]

In this case, only a relatively small number of bits is allocated to the direct channel.

B. Large Number of Transmit Antennas

As \( N \) goes to infinity, we obtain that \( B_{ii}^* = b_i \) and \( P_i = P_{\max,j} \) after solving the optimization problem in (19). This is because channel between the desired and interfering transmitter becomes asymptotically orthogonal. The sum transmission rate \( R^* \) has the following asymptotic approximation:

\[
R^* \approx \sum_{i=1}^{2} \left[ \log_2 \left( \frac{P_{\max,i} \sigma_{ii}^2 \ln 2}{1 + P_{\max,j} \sigma_{ij}^2 e^{\psi(1)}} \right) + \log_2 b_i \right],
\]

(35)

which approaches a constant, and increases with the total feedback bandwidth \( b_i \). This is an interesting result, indicating that
the gain of having more antennas is offset by the increased error in representing the larger channel with a fixed number of bits.

The analysis for the secrecy rate is analogous. As \( N \to \infty \), the optimal transmit power in (28) is approximately

\[
P^*_i \approx \min \left\{ \frac{2^{b_i}}{2\gamma^2 \sigma^2_{ji} e^{(N-1) + \psi(1)}}, P_{\text{max},i} \right\}
\]

\[
= \frac{2^{b_i}}{2\gamma^2 \sigma^2_{ji} e^{(N-1) + \psi(1)}} \sim \frac{1}{2\gamma^2 \sigma^2_{ji} e^{(1)}}, \tag{36}
\]

where the asymptotic result in (36) leads to a small constant that is assumed to be always less than the transmit power constraint \( P_{\text{max},i} \). Then, we obtain the asymptotic result for the optimal bit allocation as

\[
B_{ii}^* \approx \min \{(N - 1) \log_2 (2\gamma), b_i\} \sim b_i.
\]

The sum secrecy rate \( R^*_s \), therefore

\[
R^*_s \sim \left\{ \sum_{i=1}^{2} \left[ \log_2 \left( \frac{2\sigma^2_{ji} \ln 2}{9\sigma^2_{ji} e^{(1)}} \right) + \log_2 b_i \right] \right\}^+ \tag{37}
\]

which also approaches to constant for large \( N \), and increases with the total feedback bandwidth \( b_i \).

If we wish to enhance the rate for the \( i \)-th transmitter-receiver link by \( \Delta r_i \) for both scenarios as \( N \to \infty \), we have the following property:

\[
\Delta r_i = \log_2 (b_i + \Delta b_i) - \log_2 b_i.
\]

Therefore, it requires \( \Delta b_i = (2^{\Delta r_i} - 1)b_i \) extra bits to enhance the rate for each link by \( \Delta r_i \).

VI. SIMULATION RESULTS

In this section, we validate our analytical results through Monte Carlo simulations. For a given channel realization, we use the numerical results in [48] to randomly generate the associated quantized feedback. Utilizing the statistics of RVQ codebooks, this method simulates the quantization procedure without generating an actual codebook, and reduces the computational complexity as the number of feedback bits grows. For simplicity and demonstration purposes, we assume the system is symmetric. Transmitters have the same number of antennas and power constraints, i.e., \( N_1 = N_2 = 4 \) and \( P_{\text{max},1} = P_{\text{max},2} = P_{\text{max}} \). Unless stated otherwise, the limited feedback bandwidth for both receivers is set to 20 bits. The i.i.d. entries of the direct and cross channels are distributed as \( \mathcal{CN}(0, \sigma_i^2) \) and \( \mathcal{CN}(0, \sigma_j^2) \), i.e., \( \sigma_{11}^2 = \sigma_{22}^2 = \sigma_i^2 \) and \( \sigma_{12}^2 = \sigma_{21}^2 = \sigma_j^2 \). In each figure, the values of \( \sigma_i^2 \) and \( \sigma_j^2 \) will be depicted. All results are based on averages obtained over 1000 independent channel realizations.

Fig. 2 compares the numerical evaluation of the average sum transmission rate according to (13) with the approximate average sum transmission rate based on (16) as a function of the number of bits allocated to the direct channel (\( B_{ii} \)). The figure shows the results when the transmit power is fixed at \( P_i = 10 \text{ dB} \) and \( 20 \text{ dB} \) and for various channel conditions. The approximate average sum transmission rate is especially accurate at the maximum, where \( B_{ii} \) and \( B_{ij} \) are properly allocated. Most importantly, the peaks of the two curves coincide exactly.

We show the average sum transmission rate as a function of transmit power in Fig. 3. The dashed lines indicate the numerical simulations with perfect CSI and with quantization under the optimal feedback allocation. The solid lines represent the respective approximate sum rates based on (10) and (16). The figure further verifies the accuracy of the approximate sum rates in terms of the transmit power. In addition, it indicates that the system throughput using ideal ZF beamforming grows without bound as SNR increases. However, the limited feedback system is interference-limited and the sum rate converges to an upper limit.

Fig. 4 compares the optimal transmit power and feedback bit allocation results (solid lines) in (20) and (21) to the optimal results obtained from a grid search method (dashed lines). Fig. 5 demonstrates the accuracy of the average sum transmission rate with respect to the transmit power constraint \( P_{\text{max}} \). The actual average sum transmission rate based on the optimal results (\( P^*_i \) and \( B_{ii}^* \)) in Fig. 4 is essentially identical to the best possible

![Fig. 2. Accuracy of the approximate average sum transmission rate for the fixed transmit power \( P_i = 10 \text{ dB} \) and \( 20 \text{ dB} \).](image)

![Fig. 3. Approximate average sum transmission rate versus transmit power.](image)
average sum transmission rate obtained from the simulations. Fig. 4 and 5 illustrate that transmitting with full power achieves the maximum average sum transmission rate. Additionally, we depict the sum rate achieved using infinite transmit power (dash-dot line) given by (24). We see that the approximate upper bound in (24) roughly predicts the limiting throughput.

Figure 6 and 7 plot the numerical evaluation of the average sum secrecy rate according to (25) along with the approximate average sum secrecy rate based on (26) as a function of $B_i$ and $P_i$ respectively. Both figures verify the accuracy of the approximate average sum secrecy rate, which is used to identify the optimal results. The peak values of the approximate rate coincide with the peak values of the actual average sum secrecy rate. By using the approximation, we can find the optimal system parameters without knowing the exact rate. Furthermore, Fig. 7 illustrates the trade-off associated with increasing transmit power in a scenario with limited feedback and hence inaccurate CSI; beyond a certain point increasing transmit power decreases the secrecy performance.

Similarly, Fig. 8 and 9 present the accuracy of the optimal parameters obtained from Theorem 2 with respect to $P_{\text{max}}$. The figures indicate that the transmit power of $P_i = 16$ dB and 19 dB (vertical lines) derived in (28) attains the maximum
Fig. 9. Accuracy of the actual average sum secrecy rate versus transmit power constraint.

Fig. 10. Asymptotic behavior of the total feedback bandwidth.

of the average sum secrecy rate for the corresponding channel conditions. Increasing the power constraint beyond this threshold does not improve the secrecy performance of the system. In Fig. 9, we also illustrate the approximate upper bound given by (30).

Figure 10, 11 and 12 demonstrate the asymptotic behavior of the optimal solutions when $\sigma_d^2 = \sigma_c^2 = 1$. Fig. 10 plots the bit allocation ratio of the direct channel versus the total feedback bandwidth. When the maximum transmit power is 10 dB, the feedback bits allocated to the direct channel approach approximately half of the total feedback bandwidth as $b_i$ increases. A few extra bits are sent to the interfering transmitter when secrecy is considered. However, if the transmit power is not constrained, $P^*_i$ grows very large, and most of the feedback bandwidth is allocated to the cross channel for mitigating interference. Fig. 11 plots the sum rates as a function of the number of transmit antennas, and we see that for given $b_i$, the sum transmission rate and the sum secrecy rate converge to the respective constants in (35) and (37) as $N$ grows large. Fig. 12 shows that increasing the feedback bandwidth can further improve the sum rates.

VII. CONCLUSION

This paper has considered the allocation of transmit power and feedback bits in a cooperative two-user MISO interference channel with limited rate feedback. We analyzed two scenarios regarding the system throughput and secrecy performance. For each case, we derived an approximate rate of the transmission link and maximized the sum rate to find closed-form expressions for the optimal transmit power and feedback bit allocation. Moreover, we showed that the system throughput is interference-limited at high SNR under the assumption of limited feedback. There exists a critical value of the power constraint above which increasing transmit power reduces the sum secrecy rate. We also studied the behavior of the optimal bit and power allocations for the cases where the number of antennas or the feedback bandwidth grows large. For a large number of antennas and a fixed total feedback bandwidth, the asymptotic sum rate converges to a constant, indicating that the increased array gain is offset by the increased feedback quantization error. For a large feedback bandwidth and a fixed number of antennas, the optimal solution is to assign each one
half of the bandwidth to each of the feedback channels. Simulation results justify the approximations used in the paper, and demonstrate how the proper choice of the transmit power and feedback bit allocation can dramatically enhance the system performance.

APPENDIX A
PROOF OF LEMMA 1
Let \( z = \| \mathbf{g} \|^2 \). The pdf of \( z \) is given by [49]:

\[
f(z) = \begin{cases} 
\frac{t^{t-1/2} e^{-z/s_g^2}}{\Gamma(t) s_g^2} & z > 0 \\
0 & \text{otherwise}
\end{cases}
\]

The expected value of the logarithm of \( z \) is:

\[
E[\ln z] = \int_0^\infty \ln z \frac{t^{-1} e^{-z/s_g^2}}{\Gamma(t) s_g^2} \, dz \\
= \frac{1}{\Gamma(t) s_g^2} \int_0^\infty z^{t-1} e^{-z/s_g^2} \ln z \, dz \\
= \frac{1}{\Gamma(t) s_g^2} \sigma_g^2 t \psi(t) + \ln \sigma_g^2 \\
= \psi(t) + \ln \sigma_g^2 ,
\]

where [50] applies (38). Therefore,

\[
E[\ln \| \mathbf{g} \|^2] = \psi(t) + \ln \sigma_g^2.
\]

APPENDIX B
PROOF OF LEMMA 2
The squared inner product of any two \( t \)-dimensional independent and isotropically distributed unit-norm vectors is Beta-distributed [11]:

\[
|\mathbf{u}^H \mathbf{v}|^2 \sim \beta(1, t-1) .
\]

Due to the properties of the beta distribution, we have

\[
1 - |\mathbf{u}^H \mathbf{v}|^2 \sim \beta(t - 1, 1) ,
\]

and the expected values of the logarithm of the beta random variables are

\[
E[\ln |\mathbf{u}^H \mathbf{v}|^2] = \psi(1) - \psi(t) \\
E[\ln (1 - |\mathbf{u}^H \mathbf{v}|^2)] = \psi(t - 1) - \psi(t).
\]

APPENDIX C
PROOF OF LEMMA 3
Let \( q_i \) be a realization of the random variable \( Q \). We have

\[
\frac{1}{n} \sum_{i=1}^n \log_2 (1 + \rho q_i) - \log_2 \left( 1 + \rho e^{\frac{1}{n} \sum_{i=1}^n \ln q_i} \right) \\
= \frac{1}{n} \sum_{i=1}^n \log_2 (1 + \rho q_i) - \log_2 \left( 1 + \rho e^{\frac{1}{n} \sum_{i=1}^n \ln q_i} \right) \\
= \frac{1}{n} \left[ \log_2 \prod_{i=1}^n (1 + \rho q_i) - \log_2 \left( 1 + \rho \left( \prod_{i=1}^n q_i \right)^\frac{1}{n} \right) \right] \\
= \frac{1}{n} \log_2 \prod_{i=1}^n \frac{1 + \rho q_i}{1 + \rho \left( \prod_{i=1}^n q_i \right)^\frac{1}{n}} \\
\approx \frac{1}{n} \log_2 \prod_{i=1}^n \frac{q_i}{\left( \prod_{i=1}^n q_i \right)^\frac{1}{n}} \\
= \frac{1}{n} \log_2 \prod_{i=1}^n q_i = 0 .
\]

The approximation in (39) is based on the assumption that \( \rho \) is sufficiently large. Therefore,

\[
\frac{1}{n} \sum_{i=1}^n \log_2 (1 + \rho q_i) \approx \log_2 \left( 1 + \rho e^{\frac{1}{n} \sum_{i=1}^n \ln q_i} \right) .
\]

As \( n \to \infty \), by the law of large numbers,

\[
\frac{1}{n} \sum_{i=1}^n \log_2 (1 + \rho q_i) \to E[\log_2 (1 + \rho Q)] \\
\frac{1}{n} \sum_{i=1}^n \ln q_i \to E[\ln Q] .
\]

Hence, the following approximation holds:

\[
E[\log_2 (1 + \rho Q)] \approx \log_2 \left( 1 + \rho e^{E[\ln Q]} \right) .
\]

APPENDIX D
PROOF OF LEMMA 4
Using the beamforming vectors in (4), the expected value of the received signal power term under the limited rate feedback
scheme can be derived as follows:
\[
E \left[ \log_2 \left( 1 + P_i \frac{\| h_i^H h_i \|^2}{\| (1 - \hat{h}_i \hat{h}_i^H) h_i \|^2} \right) \right]
\]
\[
= E \left[ \log_2 \left( 1 + P_i \frac{\| h_i^H (I - \hat{h}_i \hat{h}_i^H) \hat{h}_i \|^2}{\| (I - \hat{h}_i \hat{h}_i^H) \hat{h}_i \|^2} \right) \right]
\]
\[
\approx E \left[ \log_2 \left( 1 + \frac{P_i e^{2\psi(N-1)-2\psi(N_i)}}{1 - \| \hat{h}_i \|^2} \left( 1 - \gamma_i 2^{\frac{\| h_i \|^2}{2}} \right) \| \hat{h}_i \|^2 \right) \right]
\]
\[
= E \left[ \log_2 \left( 1 + \frac{P_i e^{2\psi(N-1)-2\psi(N_i)}}{1 - \| \hat{h}_i \|^2} \left( 1 - \gamma_i 2^{\frac{\| h_i \|^2}{2}} \right) \| \hat{h}_i \|^2 \right) \right]
\]
\[
= E \left[ \log_2 \left( 1 + \frac{P_i e^{2\psi(N-1)-2\psi(N_i)}}{1 - \| \hat{h}_i \|^2} \left( 1 - \gamma_i 2^{\frac{\| h_i \|^2}{2}} \right) \| \hat{h}_i \|^2 \right) \right]
\]
\[
= E \left[ \log_2 \left( 1 + \frac{P_i e^{2\psi(N-1)-2\psi(N_i)}}{1 - \| \hat{h}_i \|^2} \left( 1 - \gamma_i 2^{\frac{\| h_i \|^2}{2}} \right) \| \hat{h}_i \|^2 \right) \right]
\]
Equation (43) is obtained from (1). The approximation in (44) is due to the high SNR assumption, (45) is based on (8) in Lemma 3, and (46) follows from (5) in Lemma 1 and (7) in Lemma 2.

APPENDIX E
PROOF OF LEMMA 5

Using the beamforming vectors in (4) and defining \( \tilde{h}_j^H \triangleq \frac{h_j^H (1 - \hat{h}_i \hat{h}_i^H)}{\| h_j^H (1 - \hat{h}_i \hat{h}_i^H) \|^2} \), the average received noise power under the limited rate feedback scheme can be derived as follows:

\[
E \left[ \log_2 \left( 1 + P_j \| h_j^H w_j \|^2 \right) \right]
\]
\[
= E \left[ \log_2 \left( 1 + \frac{P_j \| h_j^H (I - \hat{h}_j \hat{h}_j^H) \hat{h}_j \|^2}{\| (I - \hat{h}_j \hat{h}_j^H) \hat{h}_j \|^2} \right) \right]
\]
\[
= E \left[ \log_2 \left( 1 + \frac{P_j \| h_j^H (I - \hat{h}_j \hat{h}_j^H) \hat{h}_j \|^2}{\| \hat{h}_j \|^2 \| (I - \hat{h}_j \hat{h}_j^H) \hat{h}_j \|^2} \right) \right]
\]
\[
\approx E \left[ \log_2 \left( 1 + \frac{P_j \| \hat{h}_j \|^2 \| (I - \hat{h}_j \hat{h}_j^H) \hat{h}_j \|^2}{\| \hat{h}_j \|^2 \| (I - \hat{h}_j \hat{h}_j^H) \hat{h}_j \|^2} \right) \right]
\]
\[
= E \left[ \log_2 \left( 1 + \frac{P_j \| \hat{h}_j \|^2 \| (I - \hat{h}_j \hat{h}_j^H) \hat{h}_j \|^2}{\| \hat{h}_j \|^2 \| (I - \hat{h}_j \hat{h}_j^H) \hat{h}_j \|^2} \right) \right]
\]
We replace \( \| h_i \|^2 \) with its expected value in (47). Equation (48) is obtained from (2). Equation (49) is based on (8) in Lemma 3. Equation (50) follows from (5) in Lemma 1 and (6)-(7) in Lemma 2.
APPENDIX F
PROOF OF EQUATION (41)

The expected value of matrix term \((I - \hat{h}_j \hat{h}_j^H)\) can be approximated as

\[
E \left[ I - \hat{h}_j \hat{h}_j^H \right] = I - E \left[ \hat{h}_j \hat{h}_j^H \right] = \left[ 1 - \frac{1}{N} \right] I \quad (51)
\]

\[
= \left( 1 + \frac{1}{N-1} \right)^{-1} I \quad (52)
\]

\[
= \left( e^{\psi(N-1) - \psi(N)} \right)^{-1} I \quad (53)
\]

where (51) holds since \(E[\hat{h}_j \hat{h}_j^H] = \frac{1}{N} I\). Equation (52) is from the first order Taylor approximation, and (53) is due to the recurrence relation of digamma function.

REFERENCES


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