Selection Diversity for Interference Alignment Systems

Liangbin Li, Hamid Jafarkhani, Syed Ali Jafar

Center for Pervasive Communications & Computing, University of California, Irvine

***Abstract*—This paper explores the use of selection diversity for interference alignment systems. Multiple distinct alignment modes exist for certain types of interference network. By selecting the best alignment mode to support the signal-to-noise rate (SNR) of the worst user, a diversity gain improvement can be observed. This paper shows that in a 3-user double-antenna interference channel, a system switching between two alignment modes achieves a diversity gain of 2 in the fixed-rate regime and the maximum degree-of-freedom (DoF) gain of 3 in the variable-rate regime. Consequently, we disprove a previous feasibility condition for diversity in alignment systems, which requires a trade-off between the DoF gain and the diversity gain.**

I. Introduction

Interference alignment is a promising technique for interference management in multi-user networks. Through transmitter beamforming, multi-user interference is projected into overlapping subspaces at different receivers, thereby leaving the remaining signal space for desired signals. In [1], [2], interference alignment is shown to achieve the maximum degree-of-freedom (DoF) gain in different types of interference networks. Contrary to the DoF gain that is based on data rates, diversity gain measures system performance from the reliability perspective. For a system with minimum non-zero rate constraint, system outage cannot be avoided even if perfect channel state information at the transmitter (CSIT) is available. Because of the outage, the system is subject to finite diversity gains [3]. One natural question is how to improve the diversity gain for interference alignment systems.

The pursuit of these two ends, however, requires seemingly contradictory strategies. For given dimensions of signal space at each receiver, the DoF gain asks to accommodate as many messages as possible. Then, alignment constraints from different receivers limit the dimension of signal space occupied by each message. The diversity gain, on the other hand, asks each message to see as many dimensions as possible. Thus, to improve the diversity gain, the signal space for alignment is traded for diversity [4], [5]. Although each transmitter sends at a rate lower than the maximum possible rate, a higher diversity gain is achieved. In [4], a necessary and sufficient condition to achieve diversity gain more than 1 at rate one message per channel use per user was proposed for K-user interference channel with M antennas at each transmitter and N antennas at each receiver (Eqn. (21))

$$M + N \geq K + 2. \tag{1}$$

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Another idea to improve diversity for alignment systems is through space-time coding (STC). In 2-user double-antenna X channels, the alignment scheme with Alamouti structure achieves a diversity gain of 2 without losing the maximum DoF gain [6]. Yet, such a scheme requires redundant transmit dimensions after forming the alignment. Unfortunately, not all alignment systems provide such redundant dimensions.

This paper proposes a new approach to improve the diversity using selection. Antenna selection (or relay selection) is well investigated for diversity in point-to-point systems and cooperative networks (The interested readers are referred to [7] and references therein.). The key insight of selection is to confine the message in a good subspace. Thus, to achieve diversity gain, one message does not need to see as many dimensions as possible. Multiple distinct alignment modes exist for certain interference networks [8]. We propose to select the alignment mode by protecting the performance of the worst user. Particularly, we show a diversity gain of 2 is achievable for the 3-user double-antenna interference channel. For this setting, it can be verified that $M + N < K + 2$. Therefore, we disprove the diversity feasibility condition in (1), which assumes that seeing multiple dimensions is necessary for each message to achieve diversity more than 1.

The rest of the paper is organized as follows. Section II discusses previous results on feasibility condition for diversity. In Section III, we present our selection algorithm for alignment systems. Section IV provides diversity analysis. Simulations are demonstrated in Section V and conclusions are given in Section VI.

Notations: We use capitalized letter $\mathbf{A} \in \mathbb{C}^{m\times n}$ to denote a matrix drawn from the $m \times n$ matrix space defined on complex fields. We also use $\mathbf{A}^*$, $\mathbf{I}_n$, and $\mathbb{E}_x$ to denote Hermitian of matrix $\mathbf{A}$, an identity matrix of size $n \times n$, and expectation over random variable x, respectively. The notation $o(x)$ defines a function such that $\lim_{x\to 0} \frac{o(x)}{x} = 0$. $\mathcal{CN}(0, 1)$ denotes a circular symmetric complex Gaussian distribution with zero mean and variance 1.

II. System Model and Previous Results

Consider a K-user interference channel as illustrated in Fig. 1. Each transmitter is equipped with M antennas, and each receiver is equipped with N antennas. The channel matrix from Transmitter j to Receiver i is denoted as $\mathbf{H}^{[ji]} \in \mathbb{C}^{M\times N}, i, j \in \{1, 2, \ldots, K\}$. Throughout the paper, we use superscripts j, i as the indices for transmitter and receiver,

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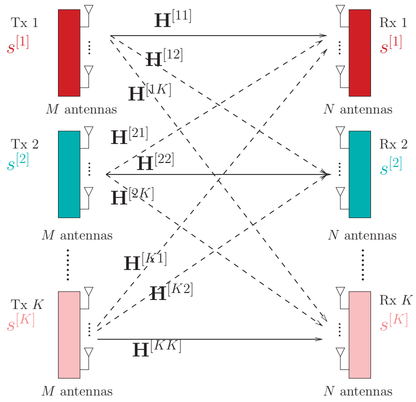

Fig. 1. K-user interference channel with M antennas at each transmitter and N antennas at each receiver.

respectively. We model each channel path as Rayleigh fading, i.e., each entry in $\mathbf{H}^{[ji]}$ is an i. i. d. $\mathcal{CN}(0,1)$ distributed random variable. During one transmission, we assume that channel matrices stay constant. Thus, channels are block fading. Let $\mathbf{x}^{[j]} \in \mathbb{C}^{M\times 1}$ be the precoded vector sent from Transmitter j in one channel use. The received vector at Receiver i can be written as

$$\mathbf{y}^{[i]} = \sum_{j\in\{1,2,\dots,K\}} \mathbf{H}^{[ji]}\mathbf{x}^{[j]} + \mathbf{w}^{[i]},\ i \in \{1,2,\dots,K\} \quad (2)$$

where $\mathbf{w}^{[j]} \in \mathbb{C}^{N\times 1}$ denotes the noise vector with i. i. d. $\mathcal{CN}(0,1)$ additive white Gaussian noise (AWGN) entries. We assume that perfect global channel information is accessible to all transmitters and receivers. For interference channel, Receiver i is only interested in decoding messages from Transmitter i. Since the network is fully connected, each receiver sees interference from $K-1$ users.

Assume that each user sends only one symbol $s^{[j]}$, drawn from a fixed-point constellation, in each channel use. The symbol $s^{[j]}$ is sent by linear beamformer $\mathbf{v}^{[j]} \in \mathbb{C}^{M\times 1}$ as

$$\mathbf{x}^{[j]} = \mathbf{v}^{[j]}s^{[j]}. \quad (3)$$

Each receiver uses a receive beamformer $\mathbf{u}^{[j]} \in \mathbb{C}^{1\times N}$ to extract its desired symbol. For normalization, we let $\|\mathbf{v}^{[j]}\| = 1$, $\|\mathbf{u}^{[j]}\| = 1$, and $\mathbb{E}\,|s^{[j]}|^2 = P$, where P is the transmitted power for each user.

A. Feasibility condition for diversity [4]

With a zero-forcing (ZF) design, the beamformers need to satisfy

$$\mathbf{u}^{[i]}\mathbf{H}^{[ji]}\mathbf{v}^{[j]} = 0, \forall i \neq j, \quad (4)$$

$$\text{Rank}\left\{\mathbf{u}^{[j]}\mathbf{H}^{[jj]}\mathbf{v}^{[j]}\right\} = 1, \forall j. \quad (5)$$

The above constraints can be represented by $K(K-1)$ equations. Note that the beamformers depend decisively on cross channels. The authors in [4] propose to use extra dimensions to protect the direct channels, corresponding to additional K equations. The total number of equations for their constraints are $K(K-1)+K = K^2$. Since each transmit beamformer has $M-1$ variables and each receive beamformer has $N-1$ variables, the total number of variables for design is $K(M-1+N-1) = K(M+N-2)$. To have a feasible design for diversity, the number of variables needs to be no less than the number of total equations,

$$K(M+N-2) \geq K^2,$$

which leads to (1). It can be checked that a setting with $K=3$ and $M=N=2$ is infeasible to achieve diversity more than 1. However, in this paper, we disprove this feasibility condition by showing that a diversity gain of 2 is achievable for this setting. In other words, the additional K constraints to protect direct channels are not necessary for diversity.

B. 3-user alignment by eigenvalue decomposition

We briefly review the eigenvector alignment for the setting with $K=3$ and $M=N=2$ [1]. In the sequel of this paper, we focus on $K=3$ and $M=N=2$. For this setting, the DoF gain outerbound, 3, is achievable through linear interference alignment. Each transmitter sends one message in one channel use. Since the receiver has two antennas, it observes a two-dimensional signal space. There are two interfering messages. With alignment, these two messages occupy only a one-dimensional subspace at each of the receivers, thus leaving one dimension for the desired message. To confine interfering messages, beamforming vectors are designed as

$$\text{R1}: \ \mathbf{H}^{[21]}\mathbf{v}^{[2]} = \lambda_1\mathbf{H}^{[31]}\mathbf{v}^{[3]}, \quad (6)$$

$$\text{R2}: \ \mathbf{H}^{[32]}\mathbf{v}^{[3]} = \lambda_2\mathbf{H}^{[12]}\mathbf{v}^{[1]}, \quad (7)$$

$$\text{R3}: \ \mathbf{H}^{[13]}\mathbf{v}^{[1]} = \lambda_3\mathbf{H}^{[23]}\mathbf{v}^{[2]}, \quad (8)$$

where $\lambda_i \in \mathbb{C}$. Since each channel matrix is square and almost surely full rank, we can obtain from (6) and (7) that $\mathbf{v}^{[2]} = \lambda_1\left(\mathbf{H}^{[21]}\right)^{-1}\mathbf{H}^{[31]}\mathbf{v}^{[3]}$ and $\mathbf{v}^{[3]} = \lambda_2\left(\mathbf{H}^{[32]}\right)^{-1}\mathbf{H}^{[12]}\mathbf{v}^{[1]}$, respectively. Thus, combing these two equations and canceling $\mathbf{v}^{[3]}$, we have $\mathbf{v}^{[2]} = \lambda_1\lambda_2\left(\mathbf{H}^{[21]}\right)^{-1}\mathbf{H}^{[31]}\left(\mathbf{H}^{[32]}\right)^{-1}\mathbf{H}^{[12]}\mathbf{v}^{[1]}$. Further replacing $\mathbf{v}^{[2]}$ in (8) with the above line, we obtain

$$\mathbf{H}^{[13]}\mathbf{v}^{[1]} = \lambda_1\lambda_2\lambda_3\mathbf{H}^{[23]}(\mathbf{H}^{[21]})^{-1}\mathbf{H}^{[31]}(\mathbf{H}^{[32]})^{-1}\mathbf{H}^{[12]}\mathbf{v}^{[1]},$$

which can be transformed into

$$\lambda\mathbf{v}^{[1]} = \mathcal{H}\mathbf{v}^{[1]}, \quad (9)$$

with $\mathcal{H} = (\mathbf{H}^{[13]})^{-1}\mathbf{H}^{[23]}(\mathbf{H}^{[21]})^{-1}\mathbf{H}^{[31]}(\mathbf{H}^{[32]})^{-1}\mathbf{H}^{[12]}$ and $\lambda = \frac{1}{\lambda_1\lambda_2\lambda_3}$. The matrix $\mathcal{H}$ is called the *alignment chain matrix*, and can be derived by tracing alignment path T1 $\rightarrow$ R2 $\rightarrow$ T3 $\rightarrow$ R1 $\rightarrow$ T2 $\rightarrow$ R3 $\rightarrow$ T1. Eqn. (9) follows the definition of eigenvalue decomposition, where λ and $\mathbf{v}^{[1]}$ are eigenvalue and normalized eigenvector of $\mathcal{H}$, respectively. Given $\mathcal{H}$, we can obtain the design of $\mathbf{v}^{[1]}$ by the eigenvalue decomposition of the alignment chain matrix. With $\mathbf{v}^{[1]}$, the other beamforming vectors $\mathbf{v}^{[2]}$ and $\mathbf{v}^{[3]}$ can be computed from (6) and (7). Since channels are generic, the condition of linear independency between the desired subspace and the interference subspace (shown in (5)) holds naturally.

Given transmit beamformers, the receive beamformer $\mathbf{u}^{[i]}$ can be computed from (4). Applying the receive beamformer to the received vector $\mathbf{y}^{[i]}$ to cancel the aligned interference, we have the following system equation

$$\mathbf{u}^{[i]}\mathbf{y}^{[i]} = \mathbf{u}^{[i]}\mathbf{H}^{[ii]}\mathbf{v}^{[i]}s^{[i]} + \mathbf{u}^{[i]}\mathbf{w}^{[i]}. \tag{10}$$

To decode the symbol, maximum-likelihood (ML) decoding is performed based on the resulting system equation,

$$\min_{s^{[i]}} \left| \mathbf{u}^{[i]}\mathbf{y}^{[i]} - \mathbf{u}^{[i]}\mathbf{H}^{[ii]}\mathbf{v}^{[i]}s^{[i]} \right|, \forall i. \tag{11}$$

Thus, each receiver performs one single-symbol decoding.

III. Selection of Alignment Modes

In this section, we propose to select alignment modes for diversity. We also discuss its analogy to a 3-user 2-frequency system without cross interference.

In a point-to-point system, there are two main approaches to obtain diversity. The first approach is to transmit each message through different independent fading paths, and constructively combining faded copies at the receiver. As a result, the diversity gain can be achieved without CSIT. One successful example of this approach is STC [9]. If the transmitter has CSIT, instead of blindly sending the message to all fading paths, a second approach is to choose the best path. One example of this second approach is antenna selection [9]. In fact, simply avoiding the worst path can also result in a diversity more than 1. The fundamental difference between these two approaches is the dimension of the signal space occupied by one message. STC requires each message to see the entire signal space, whereas selection requires each message to see only a good subspace. Recall that in alignment systems, because of the multiple alignment constraints, it is hard for each message to see the entire signal space. To apply the STC idea, a redundant dimension is needed at each transmitter[1]. However, a selection-based alignment system is less restrictive compared to STC. From the above discussion, if none of the transmitters touches the worst subspace, a diversity gain more than 1 can be achieved. This intuitively explains the motivation behind using selection.

A. Distinct alignment mode

For a 3-user double-antenna interference channel, an alignment solution can be found by eigenvalue decomposition of the alignment chain matrix. Since the alignment chain matrix is generated by a generic channel matrix and is 2×2, there exists two distinct eigenvectors, each corresponds to one *alignment mode*. In total, the system has two alignment modes. For convenience, we add a subscript $n \in \{1, 2\}$ to denote the alignment mode.

What determines the resulting system performance is the signal-to-noise ratio (SNR) at the output of each receive beamforming vector. Denote the interference subspace at Receiver i as $\mathbf{I}_n^{[i]}$. With a ZF design, the receive beamforming vector $\mathbf{u}_n^{[i]}$ is any normalized row vector in the row span of

$$\mathbf{\Phi}_n^{[i]} = \mathbf{I}_2 - \frac{\mathbf{I}_n^{[i]}\mathbf{I}_n^{[i]*}}{\mathbf{I}_n^{[i]*}\mathbf{I}_n^{[i]}}, \; i \in \{1, 2, 3\}, n \in \{1, 2\}, \tag{12}$$

[1]A redundant transmit dimension naturally exists in a 2-user X channel [6]; or it can be created by sacrificing the DoF gain [4], [5].

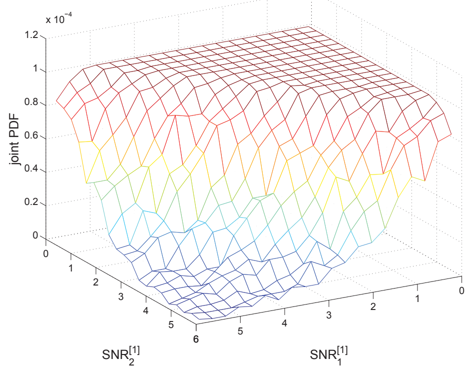

Fig. 2. Simulation of joint PDF of $\text{SNR}_1^{[1]}$ and $\text{SNR}_2^{[1]}$.

which is the projection matrix to the null space of $\mathbf{I}_n^{[i]}$. Then, the SNR after cancelling the aligned interference can be computed as

$$\text{SNR}_n^{[j]} = \mathbf{v}_n^{[j]*}\mathbf{H}^{[jj]*}\mathbf{\Phi}_n^{[j]}\mathbf{H}^{[jj]}\mathbf{v}_n^{[j]}. \tag{13}$$

We define two alignment modes to be *distinct* if $\text{SNR}_1^{[j]}$ is not exactly the same as $\text{SNR}_2^{[j]}$ at all receivers. Note that $\text{SNR}_1^{[j]}$ can be correlated with $\text{SNR}_2^{[j]}$ due to the involved eigenvalue decomposition. In Fig. 2, we show the joint probability density function (PDF) of $\text{SNR}_1^{[1]}$ and $\text{SNR}_2^{[1]}$ through simulation. It can be observed that the joint PDF is symmetrical with respect to $\text{SNR}_1^{[1]}$ and $\text{SNR}_2^{[1]}$, because the two alignment modes are statistically the same. In addition, the correlation is not very high as the joint PDF is relatively flat around zeros.

The system has two distinct alignment modes. All users need to be in the same alignment mode simultaneously for the purpose of alignment. In other words, given User 1 is in alignment mode 1, Users 2 and 3 cannot be in alignment mode 2. To protect the performance of the worst user, we select the mode that maximizes the minimum of SNRs among all users. In other words, we formulate a max-min optimization to choose the alignment mode n by

$$\max_{n \in \{1,2\}} \min_{j \in \{1,2,3\}} \text{SNR}_n^{[j]}. \tag{14}$$

Thus, the SNR of the worst user is enhanced by switching between the two alignment modes.

The selection algorithm needs to compute six SNRs and compare among them to choose the best alignment mode out of two possible alignment modes. A centralized controller can be used to collect global channel information and compute the best alignment mode. The set of transmit and receive beamforming vectors in the best alignment mode can be sent from the centralized controller to the corresponding node to release the computation complexity. All nodes perform transmit beamforming, receive beamforming, and single-symbol decoding, as expressed in (3), (10), and (11), respectively.

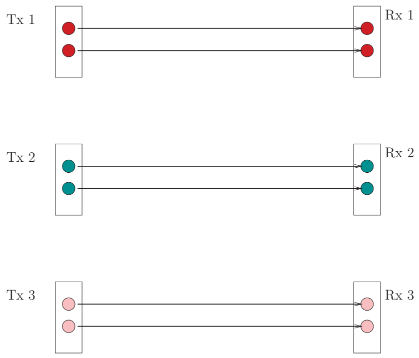

Fig. 3. An analogous 3-user 2-frequency system. Only one frequency can be selected for each communication.

B. Analogy to a 3-user 2-frequency system

Consider a 3-user system without cross-interference as shown in Fig. 3. Two frequency tones are accessible for each node. The channel path between User pair j on Tone n is denoted as $h_n^{[j]}$, which is modeled by an i. i. d. $\mathcal{CN}(0,1)$ Gaussian random variable. The system can select only one frequency tone for communication. A max-min selection can also be formulated as

$$\max_{n\in\{1,2\}} \min_{j\in\{1,2,3\}} \left|h_n^{[j]}\right|^2. \tag{15}$$

We can compare the two systems as follows:

1) Since alignment cancels cross-interference, the selection of one alignment mode is analogous to the selection of one frequency tone.
2) For both systems, all users need to choose the same channel. In the interference channel, all users have to select the same alignment mode. In the 3-user 2-frequency system, the same frequency tone is used by all users. In other words, the selections of operating channels are correlated among all users.
3) For both systems, the SNRs are independent for all users. In the 3-user 2-frequency system, channels among different users are independent. Clearly, the output SNRs follow i. i. d. exponential distributions. In the interference channel, the output SNRs are computed by (13). If we condition on all cross channels and allow only direct channels to be random, $\mathbf{v}_n^{[j]}$ and $\mathbf{\Phi}_n^{[j]}$ are fixed. It can be verified that the resulting $\mathrm{SNR}_n^{[j]}$ also follows an i. i. d. exponential distribution for different j.
4) The difference between two systems hides in the correlation of two channels to the same user. In the 3-user 2-frequency system, channels are independent between two frequency tones to the same user. Then, $\left|h_1^{[j]}\right|^2$ is independent from $\left|h_2^{[j]}\right|^2$. On the other hand, in the interference channel with fixed cross channels, $\mathrm{SNR}_1^{[j]}$ is correlated to $\mathrm{SNR}_2^{[j]}$ due to the corresponding eigenvalue decomposition. $\mathrm{SNR}_1^{[j]}$ and $\mathrm{SNR}_2^{[j]}$ are correlated exponential random variables.

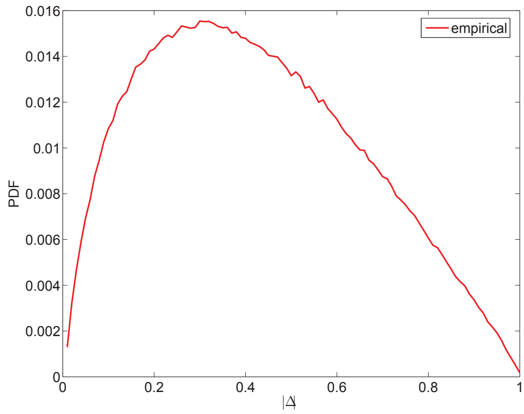

Fig. 4. The PDF of $|\Delta|$. Its near-one behavior is almost linear. Thus, we can expect $\mathbb{E}_{\mathbf{H}^{[ji]}} \frac{1}{1-|\Delta|^2} < \infty$.

In the following section, analysis is conducted to show the correlation between the two alignment modes does not decrease the diversity gain.

IV. Diversity Analysis

In this section, we analyze the achievable diversity for the proposed selection algorithm. Discussion on the connection between the number of distinct alignment modes and achievable diversity is also provided.

The proof is based on the outage probability of the instantaneous normalized receive SNR [10]. We show its near-zero behavior scales as

$$P\left(\max_n \min_j \mathrm{SNR}_n^{[j]} < \epsilon\right) = c\epsilon^2 + o(\epsilon^2), \tag{16}$$

where $c \in \mathbb{C}$ is an arbitrary constant. We need the following conjecture to prove the main theorem.

Conjecture 1. *Let $\Delta^{[j]} = \mathbf{v}_2^{[j]*}\mathbf{v}_1^{[j]}\mathbf{u}_1^{[j]}\mathbf{u}_2^{[j]*}$, which reflects the correlation of beamformers corresponding to the two modes. Since $\Delta^{[j]}$ is statistically equivalent for all users, we remove the superscript j for convenience. We conjecture the following expectation is upperbounded by a finite number*

$$\mathbb{E}_{\mathbf{H}^{[ji]}} \frac{1}{1-|\Delta|} < \infty. \tag{17}$$

Since Δ denotes the product of transmit correlation and receive correlation, its norm $|\Delta|$ is upperbounded by 1. Recall that Δ is derived from the eigenvectors of the alignment chain matrix. To conduct the analysis, upper bounds on inner-product of eigenvectors of non-Hermitian random matrices are needed. Although the probability of two parallel eigenvectors, i.e., $|\Delta| = 1$, is zero, the near-one behavior of $|\Delta|$ determines whether $\mathbb{E}_{\mathbf{H}^{[ji]}} \frac{1}{1-|\Delta|}$ is integrable or not. We simulate the distribution of $|\Delta|$ in Fig. 4. It can be observed that $|\Delta|$ is almost linear in the near-one regime. In other words, the PDF $f(|\Delta| = \gamma| |\Delta| \to 1) = c(1-\gamma)$ for a constant c. Also, numerical integration says that $\mathbb{E}_{\mathbf{H}^{[ji]}} \frac{1}{1-|\Delta|} \approx 2.4168$. Thus, evidence shows that the conjecture holds.

Theorem 1. *With the max-min selection algorithm in* (14)*, a diversity gain of 2 is achievable for the 3-user double-antenna interference channel in the fixed-rate regime; a DoF gain of 3 is achievable in the variable-rate regime.*

Proof. The DoF gain claim holds naturally since both alignment modes achieve a DoF gain of 3. Switching between them cannot reduce the DoF gain.

For the proof of the diversity claim, first, we fix the cross-channels and analyze the direct channels. Then, we consider the influence of the cross-channels. We provide a lowerbound on the outage probability, which corresponds to the diversity upperbound. The max-min operations in (16) involves six SNRs. We can arbitrarily select two SNRs corresponding to different receivers and modes to lowerbound the outage as

$$P\left(\max_n \min_j \mathrm{SNR}_n^{[j]} < \epsilon\right) > P\left(\mathrm{SNR}_1^{[j_1]} < \epsilon, \mathrm{SNR}_2^{[j_2]} < \epsilon\right),$$

where $j_1 \neq j_2$. If we condition on the cross channels, as argued in Subsection III-B, $\mathrm{SNR}_n^{[j]}$ follows a conditional exponential distribution. In addition, $\mathrm{SNR}_1^{[j]}$ only depends on its direct channel $\mathbf{H}^{[jj]}$. Then, $\mathrm{SNR}_1^{[j_1]}$ is conditionally independent from $\mathrm{SNR}_2^{[j_2]}$. These arguments lead to

$$\begin{aligned} &P\left(\mathrm{SNR}_1^{[j_1]} < \epsilon, \mathrm{SNR}_2^{[j_2]} < \epsilon\right) \\ &= \mathop{\mathbb{E}}_{\mathbf{H}^{[ji]}, j\neq i} P\left(\mathrm{SNR}_1^{[j_1]} < \epsilon, \mathrm{SNR}_2^{[j_2]} < \epsilon | \mathbf{H}^{[ji]}\right) \\ &= \mathop{\mathbb{E}}_{\mathbf{H}^{[ji]}, j\neq i} P\left(\mathrm{SNR}_1^{[j_1]} < \epsilon | \mathbf{H}^{[ji]}\right) P\left(\mathrm{SNR}_2^{[j_2]} < \epsilon | \mathbf{H}^{[ji]}\right) \\ &= \mathop{\mathbb{E}}_{\mathbf{H}^{[ji]}, j\neq i} \epsilon^2 + o(\epsilon^2) = \epsilon^2 + o(\epsilon^2). \end{aligned} \tag{18}$$

The last step of (18) holds because $P\left(\mathrm{SNR}_n^{[j]} < \epsilon | \mathbf{H}^{[ji]}\right)$ is exponentially distributed with variance 1.

In what follows, we provide an upperbound on the outage probability, which corresponds to the diversity lowerbound. Let $E^{j_1 j_2}$ denote the event $\mathrm{SNR}_1^{[j_1]} < \epsilon$ and $\mathrm{SNR}_2^{[j_2]} < \epsilon$ with $j_1, j_2 \in \{1,2,3\}$. Then, the event $\max_n \min_j \mathrm{SNR}_n^{[j]} < \epsilon$ corresponds to $\bigcup_{j_1, j_2 \in \{1,2,3\}} E^{j_1 j_2}$. Its outage probability can be upperbounded by

$$P\left(\max_n \min_j \mathrm{SNR}_n^{[j]} < \epsilon\right) < \sum_{j_1, j_2 \in \{1,2,3\}} P(E^{j_1 j_2}). \tag{19}$$

We have shown that $P(E^{j_1 j_2})$ scales as $\epsilon^2 + o(\epsilon^2)$ for $j_1 \neq j_2$ in (18). It suffices to show that $P(E^{jj})$ also follows the same scaling.

Computing $P(E^{jj})$ needs the joint PDF of $f\left(\tilde{h}_1^{[j]}, \tilde{h}_2^{[j]}\right)$, where $\tilde{h}_n^{[j]} = \mathbf{u}_n^{[j]} \mathbf{H}^{[j]} \mathbf{v}_n^{[j]}$ denotes the equivalent channels incorporating the transmit and receive beamforming. From Subsection III-B, $\tilde{h}_1^{[j]}$ and $\tilde{h}_2^{[j]}$ are correlated Gaussian provided that cross interference channels are fixed. We can calculate the conditional covariance matrix of $[\tilde{h}_1^{[j]}, \tilde{h}_2^{[j]}]^{\mathrm{T}}$ as

$$\mathbf{\Sigma}^{[j]} = \mathop{\mathbb{E}}_{\mathbf{H}^{[jj]} | \mathbf{H}^{[ji]}} \begin{bmatrix} \tilde{h}_1^{[j]} \\ \tilde{h}_2^{[j]} \end{bmatrix} \begin{bmatrix} \tilde{h}_1^{[j]*} & \tilde{h}_2^{[j]*} \end{bmatrix} = \begin{bmatrix} 1 & \Delta \\ \Delta^* & 1 \end{bmatrix}.$$

The eigenvalues of $\mathbf{\Sigma}^{[j]}$ can be found as

$$\lambda_1^{[j]} = 1 - |\Delta|, \lambda_2^{[j]} = 1 + |\Delta|. \tag{20}$$

Note that $\mathrm{SNR}_1^{[j]} + \mathrm{SNR}_2^{[j]} = \left|\tilde{h}_1^{[j]}\right|^2 + \left|\tilde{h}_2^{[j]}\right|^2$, which follows a generalized Chi-square distribution with degree 4 by conditioning on cross-channels. Its conditional PDF is

$$\begin{aligned} &f(\mathrm{SNR}_1^{[j]} + \mathrm{SNR}_2^{[j]} = \gamma | \mathbf{H}^{[ji]}) \\ &= \frac{\exp\left(-\gamma/\lambda_1^{[j]}\right)}{\lambda_1^{[j]} - \lambda_2^{[j]}} + \frac{\exp\left(-\gamma/\lambda_2^{[j]}\right)}{\lambda_2^{[j]} - \lambda_1^{[j]}}. \end{aligned}$$

Therefore, we can upperbound $P(E^{jj} | \mathbf{H}^{[ji]})$ by

$$\begin{aligned} &P(E^{jj} | \mathbf{H}^{[ji]}) < P\left(\mathrm{SNR}_1^{[j]} + \mathrm{SNR}_2^{[j]} < 2\epsilon | \mathbf{H}^{[ji]}\right) \\ &= \int_{\gamma < 2\epsilon} \left(\frac{\exp\left(-\gamma/\lambda_1^{[j]}\right)}{\lambda_1^{[j]} - \lambda_2^{[j]}} + \frac{\exp\left(-\gamma/\lambda_2^{[j]}\right)}{\lambda_2^{[j]} - \lambda_1^{[j]}}\right) \mathrm{d}\gamma \\ &= \frac{2\epsilon^2}{\lambda_2^{[j]} \lambda_1^{[j]}} + o(\epsilon^2). \end{aligned}$$

Note that

$$P(E^{jj}) = \mathop{\mathbb{E}}_{\mathbf{H}^{[ji]}} P(E^{jj} | \mathbf{H}^{[ji]}) < \mathop{\mathbb{E}}_{\mathbf{H}^{[ji]}} \frac{2\epsilon^2}{\lambda_1^{[j]} \lambda_2^{[j]}} + o(\epsilon^2).$$

To complete the proof, we only need to show $0 < \mathop{\mathbb{E}}_{\mathbf{H}^{[ji]}} \frac{1}{\lambda_1^{[j]} \lambda_2^{[j]}} < \infty$, then each term in the RHS of (19) has an upperbound scaling as $c\epsilon^2 + o(\epsilon^2)$. From (20), we have $\frac{1}{\lambda_1^{[j]} \lambda_2^{[j]}} = \frac{1}{1 - |\Delta|^2} > 1$. It follows that $\mathop{\mathbb{E}}_{\mathbf{H}^{[ji]}} \frac{1}{\lambda_1^{[j]} \lambda_2^{[j]}} > 1$. For the upperbound, $\mathop{\mathbb{E}}_{\mathbf{H}^{[ji]}} \frac{1}{\lambda_1^{[j]} \lambda_2^{[j]}} = \mathop{\mathbb{E}}_{\mathbf{H}^{[ji]}} \frac{1}{1 - |\Delta|^2} = \frac{1}{2}\left(\mathop{\mathbb{E}}_{\mathbf{H}^{[ji]}} \frac{1}{1 + |\Delta|} + \mathop{\mathbb{E}}_{\mathbf{H}^{[ji]}} \frac{1}{1 - |\Delta|}\right)$. Clearly, the first term has a finite value. Based on Conjecture 1, the second term also has a finite value. Therefore, $\mathop{\mathbb{E}}_{\mathbf{H}^{[ji]}} \frac{1}{\lambda_1^{[j]} \lambda_2^{[j]}}$ has a finite value. This concludes the proof. □

The theorem gives an impression that the number of distinct alignment modes is equal to the achievable diversity. In what follows, we discuss its extension to other network settings. Let us consider the 2-user X channels, where each of the two transmitters sends one message to each of the two receivers. One alignment mode depends on the aligned interference subspace at each of the two receivers. Since we can arbitrarily choose the aligned interference subspace, there are infinite alignment modes. Clearly, the achievable diversity is upperbounded by the diversity of the corresponding point-to-point channel, thus cannot be infinite. Identifying distinct alignment modes in terms of diversity is an interesting problem for future works.

V. Simulation

In this section, we simulate the bit error rate (BER) performance of the proposed selection algorithm, and compare it with the scenario without selection and the analogous 3-user 2-frequency system. We simulate an uncoded system with fixed-rate modulation for diversity. BPSK modulation

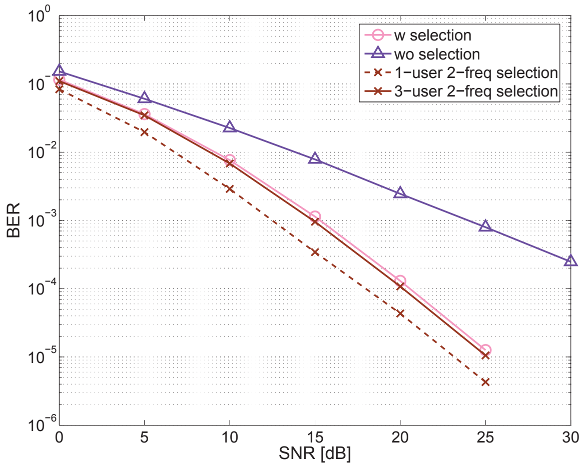

Fig. 5. BER comparison for 3-user double-antenna interference channel using linear alignment. BPSK modulation is used to carry each symbol. Each user achieves an uncoded rate of one bit per channel use.

is used for symbol $s^{[j]}$, because we only need to illustrate the achievable diversity gain. Then, the uncoded rate is one bit per channel use per user. Each user is assigned with equal transmit power P. Since the noise variance is normalized, the average receive SNR is equal to the transmit power P. Note that the design is statistically equivalent for all users. The BERs are equal for all receivers. Without loss of generality, we only illustrate the BER at Receiver 1.

Fig. 5 shows the simulation results. The horizontal and vertical axes represent the average receive SNR and BER, respectively. It can be observed that without selection, the eigenvector alignment [1] achieves only a diversity gain of 1 (labelled as ’wo selection’). The diversity gain can be improved to 2 using the proposed max-min selection given in (14) (labelled as ’w selection’). Compared to the 3-user 2-frequency system, our proposed system achieves the same diversity gain (a diversity gain of 2) but has a small SNR offset (less than 0.5 dB). This is due to the correlation between two alignment modes. Further, we compare our proposed system to a single-user 2-frequency selection system, where the user selects the stronger frequency channel for communication. It can be observed that the SNR offset increases to approximately 2 dB. This implies that the correlation of selections among users (all users are required to choose the same alignment mode) outweighs the correlation between the two alignment modes.

VI. Conclusions

This paper presents a selection algorithm for the 3-user double-antenna interference channel to improve the diversity gain. For the considered network setting, there are two optimal alignment modes. These two modes are equivalent in terms of the DoF gain. The fact that they are distinct provides an opportunity to improve reliability. We propose an algorithm switching between the two modes to maximize the minimum of SNRs among all users. Consequently, we show that the proposed system achieves a diversity gain of 2 in the fixed-rate regime and the maximum DoF gain of 3 in the variable-rate regime. The idea can be generalized to other interference networks, e.g., X channels, that have many optimal alignment modes. Diversity gain can be increased without losing the optimal rate.

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