On the Vector Broadcast Channel with Alternating CSIT: A Topological Perspective

Jinyuan Chen
Stanford University, CA
jinyuanc@stanford.edu

Petros Elia
EURECOM, France
elia@eurecom.fr

Syed Ali Jafar
University of California, Irvine
syed@uci.edu

***Abstract*—In many wireless networks, link strengths are affected by many topological factors such as different distances, shadowing and inter-cell interference, thus resulting in some links being generally stronger than other links. From an information theoretic point of view, accounting for such topological aspects has remained largely unexplored, despite strong indications that such aspects can crucially affect transceiver and feedback design, as well as the overall performance.**

The work here takes a step in exploring this interplay between topology, feedback and performance. This is done for the two user broadcast channel with random fading, in the presence of a simple two-state topological setting of statistically strong vs. weaker links, and in the presence of a practical ternary feedback setting of *alternating channel state information at the transmitter* (alternating CSIT) where for each channel realization, this CSIT can be perfect, delayed, or not available.

In this setting, the work derives generalized degrees-of-freedom bounds and exact expressions, that capture performance as a function of feedback statistics and topology statistics. The results are based on novel *topological signal management* (TSM) schemes that account for topology in order to fully utilize feedback. This is achieved for different classes of feedback mechanisms of practical importance, from which we identify specific feedback mechanisms that are best suited for different topologies. This approach offers further insight on how to split the effort — of channel learning and feeding back CSIT — for the strong versus for the weaker link. Further intuition is provided on the possible gains from topological spatio-temporal diversity, where topology changes in time and across users[1].

I. Introduction

A vector Gaussian broadcast channel, also known as the Gaussian MISO BC (multiple-input single-output broadcast channel) is comprised of a transmitter with multiple antennas that wishes to send independent messages to different receivers, each equipped with a single antenna. In addition to its direct relevance to cellular downlink communications, the MISO BC has attracted much attention for the critical role played in this setting by the feedback mechanism through which channel state information at the transmitter (CSIT) is typically acquired. Interesting insights into the dependence of the capacity limits of the MISO BC on the timeliness and quality of feedback have been found through degrees of freedom (DoF) characterizations under perfect CSIT [1], no CSIT [2], [3], compound CSIT [4], delayed CSIT [5], CSIT comprised of channel coherence patterns [6], mixed CSIT [7]–[10], and alternating CSIT [11]. Other related work can be found in [12]–[15].

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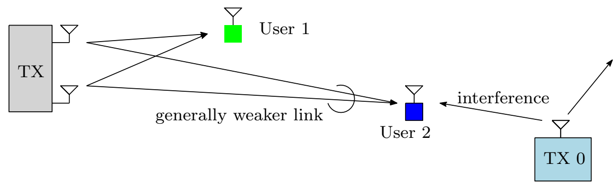

Fig. 1. Topology where link 2 is weaker due to distance and interference.

As highlighted recently in [16], while the insights obtained from DoF studies are quite profound, they are implicitly limited to settings where all users experience comparable signal strengths. This is due to the fundamental limitation of the DoF metric which treats each user with a non-zero channel coefficient, as capable of carrying exactly 1 DoF by itself, regardless of the statistical strength of the channel coefficients. Thus, the DoF metric ignores the diversity of link strengths, which is perhaps the most essential aspect of wireless communications from the perspective of interference management. Indeed, in wireless communication settings, the link strengths are affected by many topological factors, such as propagation path loss, shadow fading and inter-cell interference [17], which lead to statistically unequal channel gains, with some links being much weaker or stronger than others. Accounting for these topological aspects, by going beyond the DoF framework into the *generalized* degrees of freedom (GDoF) framework, is the focus of the topological perspective that we seek here.

The work here combines considerations of topology with considerations of feedback timeliness and quality, and addresses questions on performance bounds, on encoding designs that account for topology and feedback, on feedback and channel learning mechanisms that adapt to topology, and on handling and even exploiting fluctuations in topology.

II. System model for the topological BC

A. Channel, topology, and feedback models

We consider the broadcast channel, with a two-antenna transmitter sending information to two single-antenna receivers. The corresponding received signals at the first and second receiver at time t, can be modeled as

$$y_t = \sqrt{\rho}\boldsymbol{h}_t^{'\mathsf{T}}\boldsymbol{x}_t + u_t^{'} \quad (1)$$

$$z_t = \sqrt{\rho}\boldsymbol{g}_t^{'\mathsf{T}}\boldsymbol{x}_t + v_t^{'} \quad (2)$$

where ρ is defined by a power constraint, where $\boldsymbol{x}_t$ is the normalized transmitted vector at time t — normalized here to satisfy $||\boldsymbol{x}_t||^2 \leq 1$ — where $\boldsymbol{h}_t^{'}, \boldsymbol{g}_t^{'}$ represent the vector fading channels to the first and second receiver respectively, and where $u_t^{'}, v_t^{'}$ represent equivalent receiver noise.

1) Topological diversity: In the general topological broadcast channel setting, the variance of the above fading and equivalent noise, may be uneven across users, and may indeed fluctuate in time and frequency. These fluctuations may be a result of movement, but perhaps more importantly, topological changes in the time scales of interest, can be attributed to fluctuating inter-cell interference. Such fluctuations are in turn due to different allocations of carriers in different cells or — similarly — due to the fact that one carrier can experience more interference from adjacent cells than another.

The above considerations can be concisely captured by the following simple model

$$y_t = \rho^{A_{1,t}/2} \boldsymbol{h}_t^{\intercal} \boldsymbol{x}_t + u_t \tag{3}$$

$$z_t = \rho^{A_{2,t}/2} \boldsymbol{g}_t^{\intercal} \boldsymbol{x}_t + v_t \tag{4}$$

where now $\boldsymbol{h}_t, \boldsymbol{g}_t$ and u_t, v_t are assumed to be spatially and temporally i.i.d[2] Gaussian with zero mean and *unit variance*. With $||\boldsymbol{x}_t||^2 \leq 1$, the parameter ρ and the *link power exponents* $A_{1,t}, A_{2,t}$ reflect — for each link, at time t — an *average* received signal-to-noise ratio (SNR)

$$\mathbb{E}_{\boldsymbol{h}_t,\boldsymbol{x}_t} |\rho^{A_{1,t}/2} \boldsymbol{h}_t^{\intercal} \boldsymbol{x}_t|^2 = \rho^{A_{1,t}} \tag{5}$$

$$\mathbb{E}_{\boldsymbol{g}_t,\boldsymbol{x}_t} |\rho^{A_{2,t}/2} \boldsymbol{g}_t^{\intercal} \boldsymbol{x}_t|^2 = \rho^{A_{2,t}}. \tag{6}$$

In this setting we adopt a simple two-state topological model where the link exponents can each take, at a given time t, one of two values

$$A_{k,t} \in \{1, \alpha\} \quad \text{for} \quad 0 \leq \alpha \leq 1, \quad k = 1, 2$$

reflecting the possibility of either a strong link ($A_{k,t} = 1$), or a weaker link ($A_{k,t} = \alpha$). The adopted small number of topological states, as opposed to a continuous range of $A_{k,t}$ values, is motivated by static multi-carrier settings with adjacent cell interference, where the number of topological states can be proportional to the number of carriers.

Remark 1: We clarify that the rate of change of the topology — despite the use of a common time index for $A_{k,t}$ and $\boldsymbol{h}_t, \boldsymbol{g}_t$ — need not match in any way, the rate of change of fading. We also clarify that our use of the term 'link' carries a statistical connotation, so for example when we say that at time t the first link is stronger than the second link, we refer to a statistical comparison where $A_{1,t} > A_{2,t}$.

2) Alternating CSIT formulation: Finally in terms of feedback, we draw from the alternating CSIT formulation by Tandon et al. [11], which can nicely capture simple feedback policies. In this setting, the CSIT for each channel realization can be immediately available and perfect (P), or it can be delayed (D), or not available (N). In our notation, $I_{k,t} \in \{P, D, N\}$ will represent the CSIT about the fading channel of user k at time t.

B. Problem statement: generalized degrees-of-freedom, feedback and topology statistics

1) Generalized Degrees-of-Freedom: In a setting where (R_1, R_2) denotes an achievable rate pair for the first and second user respectively, we focus on the high-SNR regime and seek to characterize sum generalized degrees-of-freedom

$$d_\Sigma = \lim_{\rho\to\infty} \max_{(R_1,R_2)} \frac{R_1 + R_2}{\log \rho}$$

performance bounds.

It is easy to see that in the current two-state topological setting, a strong link by itself has capacity that scales as $\log \rho + o(\log \rho)$, while[3] a weak link has a capacity that scales as $\alpha \log \rho + o(\log \rho)$. Setting $\alpha = 1$ removes topology considerations, while setting $\alpha = 0$ almost entirely removes the weak link, as its capacity does not scale with SNR.

Example 1: One can see that, in the current setting of the two-user MISO BC, having always perfect feedback (P) for both users' channels, and having a static topology where the first link is stronger ($A_{1,t} = 1, \forall t$) than the second throughout the communication process ($A_{2,t} = \alpha, \forall t$), the sum GDoF is $d_\Sigma = 1 + \alpha$, and it is achieved by zero forcing.

Example 2: Furthermore a quick back-of-the-envelope calculation, can show that in the same fixed topology $A_{1,t} = 1, A_{2,t} = \alpha, \forall t$, the original MAT scheme — originally designed in [5] without topology considerations for the $\alpha = 1$ case — after a small modification that regulates the rate of the private information to the weaker user, achieves a sum GDoF of $d_\Sigma = \frac{2}{3}(1 + \alpha)$. This performance will be surpassed by a more involved topological signal management (TSM) scheme, to be described later on.

2) Motivation of the GDoF setting: Often, taking a strict interpretation of the limiting nature of GDoF, leads to confusion because, strictly speaking, any reasonable channel model would force a limiting α to be 1, since all powers would go to infinity the same way. Towards convincing the skeptical reader of the usefulness of our approach, we offer the following thoughts which can help clarify any misconceptions.

Our GDoF approach here is based on two crucial premises. i) Network links generally have different capacities, and in the perfectly conceivable case where a link has a capacity that is a fraction α of another link's capacity, a good approximation is that the weaker link has average power that is close to the α^{th} power of the aforementioned power of the strong link. ii) Albeit depending on the *limiting* behavior of random variables, our result here can also be interpreted in the *large* SNR regime, where you pick α based on the aforementioned premise, and once this α is picked and fixed, the high-SNR approximation can yield expressions which, for sufficiently large SNR, have a gap from reality that is expected to be substantially smaller than the derived expression — thus allowing for the derived expression to offer a good qualitative estimate of the overall behavior. Deviating from the strict and literal interpretation of GDoF, while still mathematically rigorous, the current approach allows us to consider topological settings that are motivated by reasonable scenarios that include distance variations and interference fluctuations, and does not constrain us to 'limiting' awkward scenarios where variable geometries have distances that scale in different specific ways.

3) Feedback and topology statistics: Naturally performance is a function of the feedback and topology statistics. Towards

[2]This suggests the simplifying formulation of unit coherence time.

[3]$o(\bullet)$ comes from the standard Landau notation, where $f(x) = o(g(x))$ implies $\lim_{x\to\infty} f(x)/g(x) = 0$. Logarithms are of base 2.

capturing these statistics, we draw from the formulation in [11] and consider

$$\lambda_{I_1,I_2}$$

to denote the fraction of the time during which the CSIT state is described by a pair $(I_1, I_2) \in (P, D, N) \times (P, D, N)$, as well as consider

$$\lambda_{A_1,A_2}$$

to denote the fraction of the time during which the gain exponents of the two links are some pair $(A_1, A_2) \in (1, \alpha) \times (1, \alpha)$, where naturally $\lambda_{1,\alpha} + \lambda_{\alpha,1} + \lambda_{1,1} + \lambda_{\alpha,\alpha} = 1$.

Example 3: $\lambda_{P,P} = 1$ (resp. $\lambda_{D,D} = 1$, $\lambda_{N,N} = 1$) implies perfect CSIT (resp. delayed CSIT, no CSIT) for both users' channels, throughout the communication process. Similarly $\lambda_{P,N} + \lambda_{N,P} = 1$ restricts to a family of feedback schemes where only one user sends CSIT at a time (more precisely, per channel realization), and does so perfectly. From this family, $\lambda_{P,N} = \lambda_{N,P} = 1/2$ is the symmetric option. Similarly, in terms of topology, $\lambda_{1,\alpha} = 1, \ \alpha < 1$ implies that the first link is stronger than the second throughout the communication process, while $\lambda_{1,\alpha} = \lambda_{\alpha,1} = 1/2$ implies that half of the time, the first user is statistically stronger, and vice versa.

In addition to the feedback and topology statistics, the formulation here can allow for description of feedback mechanisms. Towards this we use

$$\lambda_{I_1,I_2}^{A_1,A_2}$$

to denote the fraction of the time during which the CSIT state is (I_1, I_2) and the topology state is (A_1, A_2).

Example 4: Having $\lambda_{P,D}^{1,\alpha} + \lambda_{D,P}^{\alpha,1} = 1$ implies a mechanism that asks — for any channel realization — the statistically stronger user to send perfect feedback, and the statistically weaker user to send delayed feedback.

C. Conventions and structure

Throughout this paper, we adhere to the common convention and assume perfect and global knowledge of channel state information at the receivers (perfect and global CSIR).

We proceed with the main results. We first present sum GDoF outer bounds as a function of the CSIT and topology statistics, and then proceed to derive achievable and often optimal sum GDoF expressions for pertinent cases of practical significance. Due to lack of space, we will present no formal proofs, which will instead appear in the journal version [18] of this work. For the achievability part, we instead sketch the description of an encoding scheme for a specific setting of practical interest, which serves as an indication of how the inner bounds are achieved.

III. OUTER BOUNDS

We first proceed with a simpler version of the outer bound, which encompasses all cases of alternating CSIT, and all *fixed* topologies ($\lambda_{1,\alpha} = 1$, or $\lambda_{\alpha,1} = 1, \ \alpha \in [0, 1]$).

Lemma 1: The sum GDoF of the two-user MISO BC with alternating CSIT and a fixed topology, is upper bounded as $d_\Sigma \leq \min\{d_\Sigma^{(1)}, d_\Sigma^{(2)}\}$, where

$$\begin{aligned} d_\Sigma^{(1)} \triangleq & (1+\alpha)\lambda_{P,P} + \frac{3+2\alpha}{3}(\lambda_{P,D} + \lambda_{D,P} + \lambda_{P,N} + \lambda_{N,P}) \\ & + \frac{3+\alpha}{3}(\lambda_{D,D} + \lambda_{D,N} + \lambda_{N,D} + \lambda_{N,N}) \\ d_\Sigma^{(2)} \triangleq & (1+\alpha)(\lambda_{P,P} + \lambda_{P,D} + \lambda_{D,P} + \lambda_{D,D}) \\ & + \frac{2+\alpha}{2}(\lambda_{P,N} + \lambda_{N,P} + \lambda_{D,N} + \lambda_{N,D}) + \lambda_{N,N}. \end{aligned}$$

We now proceed with the general outer bound, for any alternating CSIT mechanism, and any topology, i.e., for any $\lambda_{I_1,I_2}^{A_1,A_2}$. In order to achieve a concise description of the bound, we provide the following notation.

$$\begin{aligned} \lambda_{P\leftrightarrow N}^{A_1,A_2} &\triangleq \lambda_{P,N}^{A_1,A_2} + \lambda_{N,P}^{A_1,A_2} \\ \lambda_{D\leftrightarrow N}^{A_1,A_2} &\triangleq \lambda_{D,N}^{A_1,A_2} + \lambda_{N,D}^{A_1,A_2} \\ \lambda_{P\leftrightarrow D}^{A_1,A_2} &\triangleq \lambda_{P,D}^{A_1,A_2} + \lambda_{D,P}^{A_1,A_2}. \end{aligned}$$

As a clarifying example, $\lambda_{P\leftrightarrow D}^{1,\alpha}$ simply denotes the fraction of the communication time during which the first link is stronger than the second, and during which, *any one* of the users feeds back perfect CSIT while the other feeds back delayed CSIT.

Lemma 2: The sum GDoF of the topological two-user MISO BC with alternating CSIT, is upper bounded as

$$d_\Sigma \leq \min\{d_\Sigma^{(3)}, d_\Sigma^{(4)}\} \tag{7}$$

where

$$\begin{aligned} d_\Sigma^{(3)} \triangleq & (1+\alpha)(\lambda_{P,P}^{\alpha,1} + \lambda_{P,P}^{1,\alpha}) + \frac{3+2\alpha}{3}(\lambda_{P\leftrightarrow D}^{\alpha,1} + \lambda_{P\leftrightarrow D}^{1,\alpha}) \\ & + \frac{3+2\alpha}{3}(\lambda_{P\leftrightarrow N}^{\alpha,1} + \lambda_{P\leftrightarrow N}^{1,\alpha}) + \frac{3+\alpha}{3}(\lambda_{D,D}^{\alpha,1} + \lambda_{D,D}^{1,\alpha}) \\ & + \frac{3+\alpha}{3}(\lambda_{D\leftrightarrow N}^{\alpha,1} + \lambda_{D\leftrightarrow N}^{1,\alpha}) + \frac{3+\alpha}{3}(\lambda_{N,N}^{\alpha,1} + \lambda_{N,N}^{1,\alpha}) \\ & + 2\lambda_{P,P}^{1,1} + \frac{5}{3}\lambda_{P\leftrightarrow D}^{1,1} + \frac{5}{3}\lambda_{P\leftrightarrow N}^{1,1} + \frac{4}{3}\lambda_{D,D}^{1,1} \\ & + \frac{4}{3}\lambda_{D\leftrightarrow N}^{1,1} + \frac{4}{3}\lambda_{N,N}^{1,1} + 2\alpha\lambda_{P,P}^{\alpha,\alpha} + \frac{5\alpha}{3}\lambda_{P\leftrightarrow D}^{\alpha,\alpha} \\ & + \frac{5\alpha}{3}\lambda_{P\leftrightarrow N}^{\alpha,\alpha} + \frac{4\alpha}{3}\lambda_{D,D}^{\alpha,\alpha} + \frac{4\alpha}{3}\lambda_{D\leftrightarrow N}^{\alpha,\alpha} + \frac{4\alpha}{3}\lambda_{N,N}^{\alpha,\alpha} \end{aligned}$$

$$\begin{aligned} d_\Sigma^{(4)} \triangleq & (1+\alpha)(\lambda_{P,P}^{1,\alpha} + \lambda_{P,P}^{\alpha,1}) + (1+\alpha)(\lambda_{P\leftrightarrow D}^{1,\alpha} + \lambda_{P\leftrightarrow D}^{\alpha,1}) \\ & + (1+\alpha)(\lambda_{D,D}^{1,\alpha} + \lambda_{D,D}^{\alpha,1}) + \frac{2+\alpha}{2}(\lambda_{P\leftrightarrow N}^{1,\alpha} + \lambda_{P\leftrightarrow N}^{\alpha,1}) \\ & + \frac{2+\alpha}{2}(\lambda_{D\leftrightarrow N}^{1,\alpha} + \lambda_{D\leftrightarrow N}^{\alpha,1}) + \lambda_{N,N}^{1,\alpha} + \lambda_{N,N}^{\alpha,1} \\ & + 2\lambda_{P,P}^{1,1} + 2\alpha\lambda_{P,P}^{\alpha,\alpha} + 2\lambda_{P\leftrightarrow D}^{1,1} + 2\alpha\lambda_{P\leftrightarrow D}^{\alpha,\alpha} \\ & + 2\lambda_{D,D}^{1,1} + 2\alpha\lambda_{D,D}^{\alpha,\alpha} + \frac{3}{2}\lambda_{P\leftrightarrow N}^{1,1} + \frac{3\alpha}{2}\lambda_{P\leftrightarrow N}^{\alpha,\alpha} \\ & + \frac{3}{2}\lambda_{D\leftrightarrow N}^{1,1} + \frac{3\alpha}{2}\lambda_{D\leftrightarrow N}^{\alpha,\alpha} + \lambda_{N,N}^{1,1} + \alpha\lambda_{N,N}^{\alpha,\alpha}. \end{aligned}$$

The above bounds will be used to establish the optimality of different encoding schemes and practical feedback mechanisms.

IV. Practical feedback schemes over a fixed topology

We proceed to derive different results for the case of any fixed topology. Here, without loss of generality, we will consider the case where $\lambda_{1,\alpha} = 1$, while the case of $\lambda_{\alpha,1} = 1$ is handled simply by interchanging the role of the two users. In the presence of a fixed topology, we initially focus on different practical feedback schemes for which we derive the exact sum GDoF expressions, and then proceed to explore the delayed CSIT case for which we derive a bound.

With emphasis on practicality, we first focus on three families of simple mechanisms which can be implemented so that, per coherence interval, only one user sends feedback[4].

Proposition 1: For the two-user MISO BC with a fixed topology and a feedback constraint $\lambda_{P,N} + \lambda_{N,P} = 1$ or $\lambda_{P,N} + \lambda_{N,P} = \lambda_{N,D} + \lambda_{D,N} = 1/2$ or $\lambda_{P,D} + \lambda_{D,P} = \lambda_{N,N} = 1/2$, the optimal sum GDoF is

$$d_\Sigma = 1 + \frac{\alpha}{2}. \tag{8}$$

In the first case, this is achieved by the symmetric mechanism $\lambda_{P,N} = \lambda_{N,P} = 1/2$, in the second case it is achieved by the symmetric mechanism $\lambda_{P,N} = \lambda_{N,D} = 1/2$ which associates delayed feedback with the weak user, and in the third case it is achieved by the mechanism $\lambda_{P,D} = \lambda_{N,N} = 1/2$, which again associates delayed feedback with the weak user.

Remark 2: The optimality of $\lambda_{P,N} = \lambda_{N,D} = 1/2$ (resp. $\lambda_{P,D} = \lambda_{N,N} = 1/2$) among all possible mechanisms $\lambda_{P,N} + \lambda_{N,P} = \lambda_{D,N} + \lambda_{N,D} = 1/2$ (resp. $\lambda_{P,D} + \lambda_{D,P} = \lambda_{N,N} = 1/2$), is due to the fact that delayed CSIT is associated to the weak link, which in turn allows for the unintended interference — resulting from communicating without current CSIT — to be naturally reduced in the direction of the weak link.

Remark 3: It is easy to see that the family $\lambda_{P,D} + \lambda_{D,P} = \lambda_{N,N} = 1/2$ is again a 'one-user-per-channel' family of feedback policies since it can be implemented by having half of the channel states not fed back, while having the other half fed back by any one user with no delay, and by the other user with delay.

A. Delayed CSIT and fixed topology

For the same setting of fixed topologies ($\lambda_{1,\alpha} = 1$ or $\lambda_{\alpha,1} = 1$, $\alpha \in [0, 1]$), we lower bound the sum GDoF performance for the well known delayed CSIT setting of Maddah-Ali and Tse [5], where feedback is always delayed ($\lambda_{D,D} = 1$). A brief description of the corresponding new encoding scheme will appear immediately afterwards.

Proposition 2: For the two-user MISO BC with a fixed topology and delayed CSIT, the sum GDoF is lower bounded as

$$d_\Sigma \geq 1 + \frac{\alpha^2}{2+\alpha}. \tag{9}$$

It is worth noting that the above sum GDoF surpasses the aforementioned performance of the original — and slightly modified MAT scheme [5] — over the same topology, which was mentioned in example 2 to be $d_\Sigma = \frac{2}{3}(1 + \alpha)$.

[4]In our formulation, which uses the simplifying assumption of having a unit coherence period, this simply refers to the case where only one user sends feedback at a time.

1) Topological signal management scheme: a sketch for the $\lambda^{1,\alpha}_{D,D} = 1$ case where $\alpha = 1/2$:
We now briefly sketch the description of the encoding scheme that achieves, in the presence of delayed CSIT, the sum GDoF $d_\Sigma = 1 + \frac{\alpha^2}{2+\alpha}$. For brevity we consider the simple setting where $\alpha = 1/2$, in which case the scheme has three phases with respective phase durations $T_1 = 2$, $T_2 = 1$, $T_3 = 2$ which, as we will see later on, are so chosen in order to balance the amount of side information that accumulates at the two users.

- During phase 1 ($t = 1, 2$) the transmitter sends $\boldsymbol{x}_1 = [a_1\ a_2]^\intercal$ and $\boldsymbol{x}_2 = [a_3\ a_4]^\intercal$ intended for user 1, where we recall that $\boldsymbol{x}_1, \boldsymbol{x}_2$ are normalized to have an average unit-power constraint. These are received by user 2 as interference, in the form of two linear combinations which we denote as $L_z(a_1, a_2)$ and $L_z(a_3, a_4)$. a_1 and a_3 each carry $\log\rho$ bits, while a_2 and a_4 each carry $\frac{1}{2}\log\rho$ bits.
- During phase 2 ($t = 3$), the transmitter sends — after normalization — $\boldsymbol{x}_3 = [b_1\ b_2]^\intercal$ intended for user 2, where again $\boldsymbol{x}_3$ is normalized to have an average unit-power constraint. This is received by user 1 as interference, in the form of a linear combination $L_y(b_1, b_2)$. b_1 carries $\log\rho$ bits, while b_2 carries $\frac{1}{2}\log\rho$ bits of information.
- Given delayed CSIT, at the beginning of the third phase ($t = 4, 5$), the transmitter can faithfully *reconstruct* the interference terms $L_z(a_1, a_2), L_z(a_3, a_4), L_y(b_1, b_2)$. As a result of the topology, $L_z(a_1, a_2), L_z(a_3, a_4)$ have power $\rho^{1/2}$, and can thus be reconstructed with $\frac{1}{2}\log\rho$ quantization bits each, with quantization error that is sufficiently small to not affect the DoF performance [19]. Similarly $L_y(b_1, b_2)$, which arrives with power ρ, is faithfully quantized with a total of $\log\rho$ quantization bits, which matches the number of quantization bits from the previous phase. Then these bits are mapped into common information symbols $\{c_1, c_2\}$ that are though represented by $\log\rho + o(\log\rho)$ bits, after the bits from the two phases are additively combined (vector XOR). Once this common information is eventually decoded, one user will be able to learn the other user's side information sufficiently well, by additively combining these bits with its own side information. As a result, during phase 3 ($t = 4, 5$), the transmitter sends — after normalization — $\boldsymbol{x}_4 = \begin{bmatrix} c_1 + a_5\rho^{-1/4} & 0 \end{bmatrix}^\intercal$ and $\boldsymbol{x}_5 = \begin{bmatrix} c_2 + a_6\rho^{-1/4} & 0 \end{bmatrix}^\intercal$, which means that it sends high-power common symbols $\{c_1, c_2\}$ to both users, and low-power private symbols $\{a_5, a_6\}$ for user 1, where this power is sufficiently lowered to account for the topology. Each c_1, c_2, a_5, a_6 carries $\frac{1}{2}\log\rho$ bits. As a result, summing up the bits, we have a total of $\frac{11}{2}\log\rho$ information bits, over 5 channel uses, which gives a sum GDoF of $\frac{11}{10}$, and which in turn matches the expression of the proposition for $\alpha = 1/2$.

V. Optimal sum GDoF of practical feedback schemes for the BC with topological diversity

We here explore a class of alternating topologies and reveal a gain — in certain instances — that is associated to topologies that vary in time and across users.

We first proceed, and for the delayed CSIT setting $\lambda_{D,D} = 1$, derive the optimal sum GDoF in the presence of the symmetrically *alternating topology* where $\lambda_{1,\alpha} = \lambda_{\alpha,1} = 1/2$.

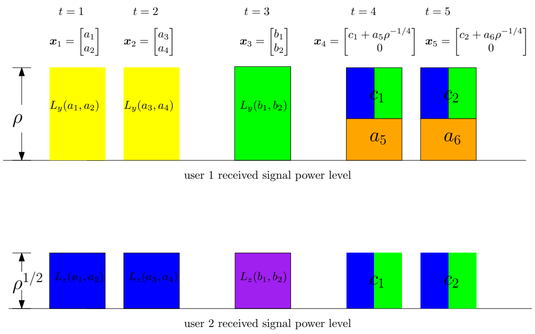

Fig. 2. Received signal power level illustration for the proposed TSM scheme: The case with $\lambda_{D,D}^{1,\alpha} = 1$ and $\alpha = 1/2$.

Proposition 3: For the two-user MISO BC with delayed CSIT $\lambda_{D,D} = 1$ and topological spatio-temporal diversity such that $\lambda_{1,\alpha} = \lambda_{\alpha,1} = 1/2$, the optimal sum GDoF is

$$d_\Sigma = 1 + \frac{\alpha}{3} \tag{10}$$

which exceeds the optimal sum GDoF $d'_\Sigma = \frac{2}{3}(1+\alpha)$ of the same feedback scheme, over an equivalent[5] but spatially non-diverse topology $\lambda_{1,1} = \lambda_{\alpha,\alpha} = 1/2$.

We also briefly note that for the same feedback policy $\lambda_{D,D} = 1$, the optimal sum GDoF $d_\Sigma = 1 + \frac{\alpha}{3}$ corresponding to the topologically diverse setting $\lambda_{1,\alpha} = \lambda_{\alpha,1} = 1/2$, also exceeds the sum GDoF performance in Proposition 2 of the TSM scheme in the presence of any static topology (e.g. $\lambda_{1,\alpha} = 1$).

A similar observation to that of the above proposition, is derived below, now for the feedback mechanism $\lambda_{P,N} = \lambda_{N,P} = 1/2$.

Proposition 4: For the two-user MISO BC with $\lambda_{P,N} = \lambda_{N,P} = 1/2$ and topological diversity such that $\lambda_{1,\alpha} = \lambda_{\alpha,1} = 1/2$, the optimal sum GDoF is

$$d_\Sigma = 1 + \frac{\alpha}{2} \tag{11}$$

which exceeds the optimal sum GDoF $d'_\Sigma = \frac{3}{4}(1+\alpha)$ of the same feedback mechanism over the equivalent but spatially non-diverse topology $\lambda_{1,1} = \lambda_{\alpha,\alpha} = 1/2$.

Regarding this same feedback policy $\lambda_{P,N} = \lambda_{N,P} = 1/2$, it is worth to now note this policy's very broad applicability. This is shown in the following proposition.

Proposition 5: For the two-user MISO BC with any strictly uneven topology $\lambda_{1,\alpha} + \lambda_{\alpha,1} = 1$ and a feedback constraint $\lambda_{P,N} + \lambda_{N,P} = 1$, the optimal sum GDoF is

$$d_\Sigma = 1 + \frac{\alpha}{2} \tag{12}$$

and it is achieved by the symmetric feedback policy $\lambda_{P,N} = \lambda_{N,P} = 1/2$.

Remark 4: This broad applicability of mechanism $\lambda_{P,N} = \lambda_{N,P} = 1/2$, implies a simpler process of learning the channel and generating CSIT, which now need not consider the specific topology as long as this is strictly uneven ($\lambda_{1,1} = \lambda_{\alpha,\alpha} = 0$).

[5]The compared topologies are considered equivalent in the sense that the overall duration of weak links, is the same for the two topologies.

VI. Conclusions

The work explored the interplay between topology, feedback and performance, for the specific setting of the two-user MISO broadcast channel. Adopting a generalized degrees of freedom framework, and addressing feedback and topology jointly, the work revealed new aspects on encoding design that accounts for topology and feedback, as well as new aspects on how to handle and even exploit topologically diverse settings where the topology varies across users and across time.

In addition to the bounds and encoding schemes, the work offers insight on how to feed back — and naturally how to learn the channel — in the presence of uneven and possibly fluctuating topologies. This insight came in the form of simple feedback mechanisms that achieve optimality — under specific constraints — often without knowledge of topology and its fluctuations.

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