Transmitter Optimization for Multiple Antenna Cellular Systems

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Abstract — **We consider the problem of transmitter optimization to maximize the downlink sum rate of a multiple antenna cellular system with perfect data cooperation between the base stations and a separate power constraint per base station. To perform this non-convex downlink transmitter optimization we investigate incremental power allocation strategies combined with BC-MAC duality results.**

I. System Model and Problem Statement

Recent work by Shamai and Zaidel [1] pioneered the application of dirty paper coding to a multiple cell environment under a simplified system model. In this paper we explore this idea further under a generalized system model, taking into account the individual power constraints per base station and making use of the uplink-downlink duality results in [2]. We assume there are B base stations. All the base stations are connected through a high speed reliable wired network that allows the base stations to exchange all information and perform as one composite base station using a centralized optimal transmit policy. We denote the i^{th} user's channel path gains associated with base station b as $H_i^{[b]}$ and the transmitted vector for the i^{th} user from base station b as $X_i^{[b]}$. We define a composite channel matrix H_i and a composite transmit vector X_i for user i as $H_i = \left[H_i^{[1]} \quad \cdots \quad H_i^{[B]}\right]$, and $X_i = \left[X_i^{[1]T} \quad \cdots \quad X_i^{[B]T}\right]^T$. Then, with dirty paper coding at the transmitter and a transmit power constraint per base station

$$\sum_{i\in\mathcal{K}} \text{trace}\left(\text{E}\left[X_i^{[b]}X_i^{[b]\dagger}\right]\right) \leq P^{[b]}, \quad b = 1, 2, \cdots, B, \qquad (1)$$

the multicellular multiple antenna downlink can be characterized as $Y_i = H_i \sum_{j=1}^{K} X_j + N_i, \quad \forall i \in \mathcal{K}$, where Y_i, X_i, H_i, and N_i are, respectively, the output vector, the input vector, the channel matrix, and the AWGN vector for user i in the index set of users, $\mathcal{K} = \{1, 2, \cdots, K\}$. The transmitter optimization problem is to determine input covariance matrices $Q_i = \text{E}[X_i X_i^\dagger]$ and an encoding order that maximizes the sum rate.

II. Results and Discussion

For a single cell, the non-convex downlink transmitter optimization problem can be framed in terms of a convex dual-uplink transmitter optimization problem and efficient algorithms are known to solve the latter [3]. However, the dual-uplink for a downlink with power constraints given by (1) is not known. We propose a multistage incremental power allocation algorithm to perform this optimization. At each stage power is allocated in small increments until one of the base stations meets its transmit power constraint. The path gains corresponding to this base station are then set to zero in subsequent stages so that no further power is allocated to these antennas. The incremental algorithm itself works on the dual-uplink (ignoring the constraints (1)) and duality transformations are used to check if any of the constraints (1) is tight.

For simplicity we focus on the case where each user has a single receive antenna while the base stations can have multiple transmit antennas. On the dual uplink (without the constraints (1)), the algorithm determines the best direction to invest each incremental amount of power. Thus it builds up the overall optimal solution by proceeding through a series of solutions, each of which is optimal for the amount of power invested upto that point. The asymptotic optimality of this algorithm in the limit of infinitesimally small increments is easily established for the case when the users' channels are linearly independent.

Determining the optimal direction to invest each incremental amount of power involves inverting the Hessian of the sum rate function with respect to the users' transmit powers. The complexity of this operation motivates a simpler algorithm. This algorithm simply adds each incremental amount of power to the user that yields the highest first derivative of the sum rate function with respect to that user's transmit power. Unlike the previous algorithm, this algorithm can only be shown to be asymptotically optimal for two users. In fact, for more than two users it can be shown to be suboptimal even asymptotically. One feature of this algorithm is that with each successive increment the transmit powers of the users are non-decreasing. However, we provide numerical examples to show that with optimal power allocation the transmit power of a user may decrease as the overall transmit power increases. In spite of its suboptimality this algorithm is interesting because it is simpler, and numerical results indicate that the loss in average sum rates obtained with this algorithm compared to the sum rates under the optimal algorithm is negligible.

These algorithms form the basis for forthcoming numerical investigations to be presented at the conference that determine the impact of number of antennas at the base station, the number of users per cell, and the throughput gains of optimal power control relative to practical schemes in use today (e.g. HDR which transmits only to the best user) etc.

References

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