Multiple-Antenna Capacity in Correlated Rayleigh Fading with Channel Covariance Information

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Abstract — **For a block fading multiple input multiple output (MIMO) wireless link comprising M transmit and N receive antennas operating in a spatially correlated Rayleigh flat fading environment with coherence time T and only knowledge of channel correlations at the transmitter and receiver, we show that the channel capacity is independent of the smallest $M-T$ eigenvalues of the transmit fade covariance matrix. We characterize the structure of the input signal that achieves capacity. In contrast to the previously reported results for the spatially white fading model where adding more transmit antennas beyond the coherence interval length ($M > T$) does not increase capacity, we find that additional transmit antennas always increase capacity as long as their channel fading coefficients are spatially correlated, making the case for smaller spacing between transmit antennas. For fast fading channels the capacity gained from multiple transmit antennas is bounded by the array gain $10 \log M$.**

I. Introduction

While enormous capacity gains have been predicted for multiple antenna wireless systems with perfect channel knowledge at the receiver, reliable channel estimation may not be possible for a mobile receiver that experiences rapid fluctuations of the channel coefficients. For a fast fading channel in the absence of instantaneous channel knowledge at the transmitter or receiver it was shown by Marzetta and Hochwald in [1] that increasing the number of transmit antennas does not increase channel capacity. Specifically, they showed that with uncorrelated flat Rayleigh fading, increasing the number of transmit antennas (M) beyond the channel coherence interval length (T) does not increase capacity. Since the absence of instantaneous channel knowledge is a realistic assumption for users moving at high speeds where the coherence interval is small and the channel fluctuates rapidly, the capacity advantage of using multiple antennas for such systems seem questionable.

However, note that the results of [1] assume spatially uncorrelated fading. While uncorrelated fading is a valid assumption for systems with widely spaced antennas, in general the channel coefficients are correlated in space. This general case forms the focus of this paper. Specifically, we analyze MIMO capacity under correlated fading when only the channel fade covariance information is available at both the transmitter and the receiver.

II. System Model

Following notation and system model consistent with [1], we have

$$X = \sqrt{\frac{\rho}{M}} SH + W \tag{1}$$

where X is the $T \times N$ received signal matrix, S is the $T \times M$ transmitted signal matrix, H is the $M \times N$ channel matrix, and W is the $T \times N$ additive noise vector. The noise components are assumed to be i.i.d. zero mean unit variance circularly symmetric complex Gaussian. The channel coefficients remain fixed for the coherence time T after which they change to an independent set of values generated according to the spatial correlation model

$$H = (R^t)^{1/2} H_w (R^r)^{1/2}. \tag{2}$$

where H_w is a spatially white $M \times N$ MIMO channel with i.i.d. zero mean unit variance circularly symmetric complex Gaussian components, and R^t and R^r are called the transmit and receive fade covariance matrices respectively.

III. Results

We derive the conditional probability density of the received signal X conditioned on the transmitted signal S as

$$p(X|S) = \frac{\exp\left\{-\text{tr}\left[\left(I_{NT} + \frac{\rho}{M} R^r \otimes S R^t S^\dagger\right)^{-1} \text{vec}(X)\text{vec}(X)^\dagger\right]\right\}}{\pi^{TN} \det\left[I_{NT} + \frac{\rho}{M} R^r \otimes S R^t S^\dagger\right]}$$

We use the special properties of the conditional pdf $p(X|S)$ to obtain various results in [2]. The main results of [2] are summarized in the following theorems.

Theorem 1 *(Structure of signal that achieves capacity) The signal matrix that achieves capacity can be written as $S = \Phi V U$ where Φ is a $T \times T$ isotropically distributed unitary matrix, and V is an independent $T \times M$ real, nonnegative, diagonal matrix. U is a unitary matrix with the eigenvectors of R^t as its columns.*

Theorem 2 *For any coherence interval T and any number of receiver antennas, the capacity obtained with $M > T$ transmitter antennas in spatially correlated fading depends on the $M \times M$ transmit fade correlation matrix R^t through only the T largest eigenvalues of R^t and is not a function of the eigenvectors of R^t or R^r.*

Theorem 3 *Capacity increases almost surely (with probability 1) with the number of transmit antennas when the transmit antenna fades are spatially correlated.*

As a corollary of Theorem 2 we prove that for a system with coherence time $T = 1$ symbol period, capacity is a Schur-concave function of the eigenvalues of the transmit fade covariance matrix. In other words, for fast fading or fast frequency hopping systems spatial correlations are beneficial. The additional capacity due to spatial correlations is bounded by the array gain $10 \log M$.

IV. Conclusion

We conclude that in the absence of channel knowledge at the transmitter or receiver multiple transmit antennas are beneficial only if the fades associated with different transmit antennas are correlated. For fast fading channels, the additional capacity gained by using multiple transmit antennas is bounded by the array gain $10 \log M$.

References

[1] T. Marzetta, B. Hochwald, "Capacity of a Mobile Multiple-Antenna Communication Link in Rayleigh Flat Fading", IEEE Transactions on Information Theory, Volume:45, Issue: 1, Jan. 1999, Page(s): 139-157.

[2] S. Jafar, A. Goldsmith, "Multiple-Antenna Capacity in Correlated Rayleigh Fading with Channel Covariance Information", full paper in preparation.

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