Optimum Power and Rate Allocation Strategies for Multiple Access Fading Channels

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Abstract

We obtain the (Shannon) optimal power and rate allocation strategies for the uplink on single cell systems. For an N user system, we show that successive decoding which is independent of channel state is optimal in the Shannon sense. Using this result, we frame a simple N dimensional convex optimization problem, which is solved to obtain optimum power and rate allocation as explicit functions of the channel state.

1. Introduction

In a typical uplink scenario, we have multiple users transmitting to a single basestation, where each user has a different data rate requirement (e.g. voice, data or video users). We wish to determine the Shannon capacity region for such a system, i.e. the set of rates that this system can achieve with each user guaranteed an arbitrarily small probability of error, using the best coding scheme possible and with no delay constraints. Assuming the transmitter and receiver have perfect and instantaneous knowledge of the channel, we are interested in resource allocation policies of the transmitter, as a function of channel state, that achieve a boundary point R^* of this Shannon capacity region.

The capacity region and optimal resource allocation policies for this scenario have been previously studied in [6]. In [6], the authors define a "marginal utility function" for each user at each channel state. This function represents the marginal increase in the objective function by allocating power to the user at an interference level z. This function is used to allocate power to the user with the highest marginal utility (called the greedy allocation problem). This approach, while powerful, is somewhat abstract, which makes it difficult to obtain intuition about the properties of the solution. In this paper we frame the resource allocation problem as a simple convex optimization, where both the objective function and the constraints are explicit differentiable functions of the power allocation policies of the users. Thus, differentiation and solving simultaneous equations are sufficient to obtain the optimal solution.

We present the system model in the next section. In Section 3 we show that it is optimal for the receiver to decode one user at a time in the *same* order at all channel states (termed the unique decoding order property). We frame the optimization problem and discuss the solution in Section 4. We provide an iterative algorithm to obtain the capacity region boundary in Section 5 and study the special case of rate-sum capacity in Section 6. The results and conclusions are in Section 7.

2. System Model

For notation, we use boldface to denote vectors (vector $\mathbf{v}$ denotes $\{v_1, v_2, \dots, v_N\}$) and $\mathbb{E}_f$ to denote expectation over the random variable f.

We consider a discrete time uplink channel with N users communicating with a single receiver. The received signal at time n is given by $y(n) = \sum_{i=1}^{N} \sqrt{h_i}(n)x_i(n) + z_i(n)$ where $x_i(n)$ and $h_i(n)$ are, respectively, the transmitted signal and channel gain for the ith user at time instant n, and $z_i(n)$ is additive white Gaussian noise (AWGN) with variance σ^2. Note that we can normalize the equations to set $\sigma^2 = 1$. We wish to determine the power and rate allocation policies, $P_i(\mathbf{h})$ and $R_i(\mathbf{h})$ respectively, for all N users, as a function of the channel state $\mathbf{h}$. We assume that all the transmitters and the receiver know all the channel states $\mathbf{h} = \{h_1(n), h_2(n) \dots h_N(n)\}$ at the time instant n (termed perfect transmitter and receiver side information). We also assume that $\mathbf{h}$ is a real and positive vector. If $h_i(n)$ is complex, we can replace it by $\sqrt{h_i h_i^*}$ without changing the final solution.

3. Uniqueness of Decoding Order

In this section, we shall prove a simple but important result about any resource allocation policy that achieves a point on the boundary of the n dimensional region described in Section 1.

Theorem 1 *For any point R^* on the boundary of the capacity region, the optimal decoding policy is successive decoding, with the same decoding order of users for all channel* *states. Thus every point on the boundary can be associated with a successive decoding order.*

Although this result was observed in [6] based on their optimization solution, it was neither emphasized nor used. We now obtain this result from first principles and then use it to greatly simplify the resource allocation optimization.

Successive decoding means that users are decoded sequentially, and that the user to be decoded treats all the users yet to be decoded as noise, and subtracts out the symbols transmitted by the users already decoded from the received codeword. Since each user is transmitting at an arbitrarily small probability of error, we can subtract out the users already decoded without introducing additional errors. For any given channel state, the optimality of successive decoding follows directly from the chain rule for mutual information [3]. To show that the optimal ordering of successive decoding is independent of the channel state, we state the expression for the capacity region of this system for a given a power allocation policy $\mathbf{P}(\mathbf{h})$ that satisfies $\mathbb{E}_{\mathbf{h}}[P_i(\mathbf{h})] = \bar{P}_i$, $1 \leq i \leq N$ without proof (please see [6] for proof) as

$$\begin{aligned} & C_{side}(\mathbf{h}, \bar{\mathbf{P}}) \\ &= \{\mathbf{R} : \mathbf{R}(S) \leq \tfrac{1}{2}\mathbb{E}_{\mathbf{h}}[\log(1 + \textstyle\sum_{i\in S} h_i P_i(\mathbf{h}))],\ \forall S\}, \end{aligned} \quad (1)$$

were S is any subset of $\{1, 2, \ldots N\}$ and $R(S) = \sum_{i\in S} R_i$. The meaning of this expression is that the sum of the rates of any subset S of users in this system is less than the rate obtained as if one user had the entire received power $\sum_{i\in S} h_i P_i(\mathbf{h})$. Suppose the transmitter has no side information (i.e. knows nothing about the channel) but the receiver has perfect side information, then the capacity region is given by [3]

$$C_{noside}(\mathbf{h}, \bar{\mathbf{P}}) = \{\mathbf{R} : \mathbf{R}(S) \leq \frac{1}{2}\mathbb{E}_{\mathbf{h}}[\log(1 + \sum_{i\in S} h_i \bar{P}_i)]\}. \quad (2)$$

In this case, we cannot allocate any resources dynamically, since the transmitter has no side information. After looking at Equations (1) and (2) carefully, we note that the only difference between the two is the time varying power allocation in (1). Thus, if we treat the variation in power $P_i(\mathbf{h})$ as part of the channel [1], i.e. if we were to consider a new channel $h_i P_i(\mathbf{h})$ for user $i, 1 \leq i \leq n$ with no transmitter side information and an average power of unity for each user, then

$$C_{noside}(\mathbf{h} \otimes \mathbf{P}(\mathbf{h}), \mathbf{1}) = C_{side}(\mathbf{h}, \bar{\mathbf{P}}), \quad (3)$$

where $\otimes$ denotes the Hadamard product of the two vectors. As pointed out before, when the transmitters of all the users have no side information, they must transmit at a constant rate and power and cannot change codebook from state to state to achieve a point in the capacity region. Since the receiver decodes one user at a time, and since the codeword of each user spans many different channel states, the receiver cannot change decoding order from one channel state to another. Thus, using a constant codebook (and hence decoding order) and varying the power allocation with the channel state achieves any point in the capacity region, and as a special case, the boundary.

In the next section, we employ Theorem 1 to solve the problem of finding the optimal rate and power allocations that achieve the boundary of the capacity region.

4. Problem Formulation and Solution

It is well known that a point on the capacity region boundary can be obtained by solving the equivalent convex problem [6]

$$\max_{\mathbf{P}(\mathbf{h})} \mathbb{E}_{\mathbf{h}}[\boldsymbol{\mu} \cdot \mathbf{R}(\mathbf{h})] \quad (4)$$

subject to $\mathbb{E}_{\mathbf{h}}[P_i(\mathbf{h})] = \bar{P}_i$ $1 \leq i \leq N$, where $\cdot$ represents a dot product. We call $\boldsymbol{\mu} = \{\mu_1, \ldots, \mu_N\}$, a vector of priorities[1], where $\sum_{i=1}^{N} \mu_i = 1 \text{and} \mu_i \geq 0$. In this formulation, each user is assigned a priority value μ. In Equation (4), the rate policy vector $\mathbf{R}(\mathbf{h})$ is implicitly a function of the power policy vector $\mathbf{P}(\mathbf{h})$ (and vice versa). If the optimal decoding order at the receiver were not unique but differed from state to state, it would be extremely difficult to characterize $\mathbf{R}(\mathbf{h})$ in terms of the $\mathbf{P}(\mathbf{h})$ and using utility arguments as in [6] is the only known approach for solving this problem. But using the unique decoding order property and the following theorem, we can explicitly write $\mathbf{R}(\mathbf{h})$ in terms of $\mathbf{P}(\mathbf{h})$, and hence rewrite (4) in a simpler form.

Theorem 2 *The unique decoding order is the reverse order of the priorities μ_i. If we label the users in the decreasing order of their priorities, i.e. users $1, \ldots N$ have priorities $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_N$ then the decoding order is $N, N-1, \ldots, 1$. Thus, the user with the lowest priority is decoded first, the user with the next lowest priority is decoded second and so on with the highest priority user decoded last.*

To prove this we assume an optimal decoder ordering of the form $\pi(1), \pi(2), \ldots, \pi(N)$. Then the maximal rate achievable by User i given a power policy $\mathbf{P}(\mathbf{h})$ is

$$\frac{1}{2}\mathbb{E}_{\mathbf{h}} \log\left(1 + \frac{h_i P_i(\mathbf{h})}{1 + \sum_{\pi^{-1}(j) > \pi^{-1}(i)} h_j P_j(\mathbf{h})}\right), \quad (5)$$

since all the users to be decoded after User i are noise, and all the users already decoded have been subtracted out.

[1] Geometrically, $\boldsymbol{\mu} \cdot \mathbf{R} = c$ represents a hyperplane in n dimensional space. This hyperplane will be a tangent to the convex capacity region for the maximum value of c.

Thus, for a given power policy, $\mathbb{E}_{\mathbf{h}}[\boldsymbol{\mu} \cdot \mathbf{R}(\mathbf{h})]$ has a maximum value given by

$$\frac{1}{2}\mathbb{E}_{\mathbf{h}} \sum_i \mu_{\pi(i)} \log\left(1 + \frac{h_{\pi(i)} P_{\pi(i)}(\mathbf{h})}{1 + \sum_{j>i} h_{\pi(j)} P_{\pi(j)}(\mathbf{h})}\right). \quad (6)$$

If in (6), there are two users such that for $i < j$,$\pi(i) < \pi(j)$, then by interchanging their decoding orders, we can increase (6). Thus, the optimal decoding order must be $\pi(i) = N - i$. A complete proof of this can be found in [7].

The results obtained above allow us to rewrite Equation (4) as the follows: Given $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_N$,

$$\max_{\mathbf{P}(\mathbf{h})} \sum_{i=1}^{N} \frac{\mu_i}{2} \mathbb{E}_{\mathbf{h}} \log\left(1 + \frac{h_i P_i(\mathbf{h})}{1 + \sum_{j<i} h_j P_j(\mathbf{h})}\right) \quad (7)$$

subject to $\mathbb{E}_{\mathbf{h}}[P_i(\mathbf{h})] = \bar{P}_i, \;\; 1 \leq i \leq N$. This problem definition is, as promised, an explicit function of the N power policies $P_i(\mathbf{h})$, $1 \leq i \leq N$, and of nothing else. Also, since the decoding order is fixed, the rate policy of User i is $1/2 \log\left(1 + \frac{h_i P_i(\mathbf{h})}{1+\sum_{j<i} h_j P_j(\mathbf{h})}\right)$, which is a function of $\mathbf{P}(\mathbf{h})$. Thus, we can concentrate on solving for the optimal $\mathbf{P}(\mathbf{h})$, which is turn gives us $\mathbf{R}(\mathbf{h})$. For this optimization we use standard methods: forming the Lagrangian (Equation (8)), differentiating the Lagrangian with respect to each of the N variables $P_i(\mathbf{h})$ and setting the derivatives to zero. Specifically, the Lagrangian is:

$$\begin{aligned} L(\mathbf{P}(\mathbf{h}), \boldsymbol{\lambda}) &= \sum_{i=1}^{N} \frac{\mu_i}{2} \mathbb{E}_{\mathbf{h}} \log\left(1 + \frac{h_i P_i(\mathbf{h})}{1 + \sum_{j\leq i} h_j P_j(\mathbf{h})}\right) \\ &- \sum_{i=1}^{N} \lambda_i P_i(\mathbf{h}) \end{aligned} \quad (8)$$

and we wish to solve

$$\frac{\partial L(\mathbf{P}(\mathbf{h}), \boldsymbol{\lambda})}{\partial \mathbf{P}(\mathbf{h})} = 0 \quad (9)$$

Equation (9) yields N equations in $\mathbf{P}(\mathbf{h})$. We label the equation in (9) obtained by differentiating with respect to $P_i(\mathbf{h})$ as $\partial L(i)$. Note that until now we have ignored the positivity constraint on $\mathbf{P}(\mathbf{h})$, i.e. that the power of each user is non-negative. Including this constraint divides the space of all possible $\mathbf{h}$ vectors (which is $\mathbb{R}_+^N$ in general) into 2^N disjoint regions, since each user can be transmitting or not and there are N users. Since $P_i(\mathbf{h}) > 0$ in a region implies that $\partial L(i)$ gives us a non-negative solution for $P_i(\mathbf{h})$ in that region, this is equivalent to dividing $\mathbb{R}_+^N$ into 2^N disjoint regions based on where the subsets of the equation set $\{\partial L_i\}$ have a non-negative solution for $P_i(\mathbf{h})$. We denote these regions using $\mathfrak{R}_j$, with the binary expansion of j determining which users transmit in the region $\mathfrak{R}_j$. Thus, if the binary expansion of j is $[j_1, \ldots, j_k, \ldots, j_N]$, then $j_k = 1$ implies that user k transmits in region $\mathfrak{R}_j$ and $j_k = 0$ implies that he does not. Similarly, let $\mathfrak{G}_j$ denote a subset of the N equations $\{\partial L(i)\}_{i=}^{N}$ such that $\partial L(k) \in \mathfrak{G}_j$ if and only if $j_k = 1$ (Equivalently,$\partial L(k) \in \mathfrak{G}_j$ if solving $\mathfrak{G}_j$ yields $P_k(\mathbf{h}) > 0$. By solving the subset of equations $\mathfrak{G}_j$ simultaneously, we obtain the power policy of each active user in $\mathfrak{R}_j$ in terms of $\boldsymbol{\lambda}$.

We find the region boundaries of $\mathfrak{R}_j$ using two facts: that the powers obtained by solving $\mathfrak{G}_j$ are positive, and that all the other powers are zero. After performing this for all the 2^N equation subsets $\mathfrak{G}_j$ and finding the corresponding boundaries of the regions $\mathfrak{R}_j$, we obtain $\boldsymbol{\lambda}$ by using the N power constraints in the problem definition. Say $\mathfrak{G}_j$ were the subset $\{\partial L(i_1), \partial L(j_2), \ldots, \partial L(i_n)\}$ with $i_1 < \cdots < i_k < \ldots < i_n$ representing the positions where the binary expansion of j equals unity. Then the optimum power and rate allocation policies in region $\mathfrak{R}_j$ obtained by solving $\mathfrak{G}_j$ is found to be:

$$\begin{aligned} P_{i_1}(\mathbf{h}) &= \frac{\mu_{i_1} - \mu_{i_2}}{\lambda_{i_1} - \lambda_{i_2} h_{i_1}/h_{i_2}} - \frac{1}{h_{i_1}} \\ &\vdots \\ P_{i_k}(\mathbf{h}) &= \frac{\mu_{i_k} - \mu_{i_{(k+1)}}}{\lambda_{i_k} - \lambda_{i_{(k+1)}} \frac{h_{i_k}}{h_{i_{(k+1)}}}} - \frac{\mu_{i_{(k-1)}} - \mu_{i_k}}{\lambda_{i_{(k-1)}} \frac{h_{i_k}}{h_{i_{(k-1)}}} - \lambda_{i_k}} \\ &\vdots \\ P_{i_n}(\mathbf{h}) &= \frac{\mu_{i_n}}{\lambda_{i_n}} - \frac{\mu_{i_{(n-1)}} - \mu_{i_n}}{\lambda_{i_{(n-1)}} h_{i_n}/h_{i_{(n-1)}} - \lambda_{i_n}} \\ P_l(\mathbf{h}) &= 0 \qquad \forall\; l \notin \{i_1, \ldots, i_n\} \end{aligned} \quad (10)$$

where h_{i_k} represents the channel of user i_k. The power policies in (10) show that the power of each user increases with his own channel gain. In Section 4.2 we shall provide an intuitive understanding of the power policies obtained above for a two user system based on a waterfilling argument. As mentioned before, the boundaries of $\mathfrak{R}_j$ are obtained by using positivity constraints on the power policies in (10), and that $P_k(\mathbf{h}) = 0$, $\forall\, k \notin \{i_1, \ldots, i_n\}$.

4.1 Characteristics of the Optimum Power Policy

Based on Equations (10), we can draw the following conclusions

Observation 1 *The power policy of User i_k given by (10) within each region $\mathfrak{R}_k$, is a function only of h_{i_k},$h_{i_{k-1}}$ and $h_{i_{k+1}}$. Moreover, $P_{i_k}(\mathbf{h})$ is a strictly increasing function of h_{i_k} and a strictly decreasing function in $h_{i_{k-1}}$ and $h_{i_{k+1}}$.*

In other words, the power policy of a given user for channel state $\mathbf{h}$ depends only on his channel gain and the channel gains of the users decoded immediately before and after him. Specifically, if the channel of User i improves with the channels of other users remaining fixed, his power must increase and the powers of those adjacent to him in decoding order must decrease to allow User i a higher rate and an overall increase in system throughput. If the channel of those adjacent in decoding order improves, his power must decrease to allow their throughputs to increase. Thus, across all regions, User i_k "waterfills" to the channel, except that the waterfill level depends on which users are active in region $\Re_j$, and within each region on the channels of the users decoded immediately before and after.

For the special case of the user being decoded first, we state the following theorem

Theorem 3 *The optimal power and rate allocation for the user being decoded first is "waterfilling" to the SIR it observes, where waterfilling is used in the same sense as in [4], i.e. for $\sigma_I^2 = 1 + \sum_{1\le k<N} h_k P_k(\mathbf{h})$, the optimum power is of the form*

$$P_N = \begin{cases} 0 & \frac{h_N}{\sigma_I^2} < \gamma \\ \frac{1}{\gamma} - \frac{\sigma_I^2}{h_N} & \frac{h_N}{\sigma_I^2} > \gamma \end{cases}$$

This theorem is proved by rewriting $\partial L(N)$ in this form.

4.2 Two User Example

For illustrative purposes, we consider a two user example with $\mu_1 > \mu_2$. In this case, we have 4 different $\Re_j$ regions. We can combine the power policies of User 1 in all the regions as:

$$P_1(\mathbf{h}) = \begin{cases} 0 & \frac{h_1}{N_0} < \frac{\lambda_1}{\mu_1} \\ \frac{\mu_1}{\lambda_1} - \frac{N_0}{h_1} & \frac{h_1}{N_0} > \frac{\lambda_1}{\mu_1} \text{ and } P_2(\mathbf{h}) = 0 \\ \frac{\mu_1 - \mu_2}{\lambda_1 - \lambda_2 (h_1/h_2)} - \frac{N_0}{h_1} & \frac{h_1}{N_0} > \frac{\lambda_1}{\mu_1} \text{ and } P_2(\mathbf{h}) \neq 0 \end{cases} \tag{11}$$

Note that when User 2 is not transmitting, User 1 waterfills to the channel with a constant waterlevel. When User 2 transmits, User 1 still waterfills to the channel, but the waterlevel increases with h_1. The power policy of User 2 is

$$P_2(\mathbf{h}) = \begin{cases} 0 & \frac{h_2}{N_0 + h_1 P_1(\mathbf{h})} < \frac{\lambda_2}{\mu_2} \\ \frac{\mu_2}{\lambda_2} - \frac{N_0 + h_1 P_1(\mathbf{h})}{h_2} & \frac{h_2}{N_0 + h_1 P_1(\mathbf{h})} > \frac{\lambda_2}{\mu_2} \end{cases} \tag{12}$$

i.e., User 2 waterfills to the SIR he observes, with a constant waterlevel regardless of User 1. The capacity region for this two-user example in the case of Rayleigh flat fading, i.e. when h_i are exponential distributed as e^{-h_i}, and with $\bar{\mathbf{P}} = \{1, 1\}$ is shown in Figure 1. The region boundaries for the system are plotted in Figure 2. Notice that P_1 and P_2 cutoff below a certain value of h_1 and h_2 respectively, and that this cutoff value is independent of the channel state of the other user. Also note that beyond a certain value of h_1, User 1 always transmits regardless of h_2, but that for every value of h_2 there is an h_1 such that User 1 alone transmits, i.e that User 2 is cutoff.

Now we revisit Equation (9) to explain the practical implications of this approach. Differentiating $L(\mathbf{P}(\mathbf{h}), \boldsymbol{\lambda})$ with respect to $P_i(\mathbf{h})$ and setting it to zero gives us the optimal power distribution of User i, *given* the power policies of the remaining users. Thus, if the system preassigns priority values to each user, and performs successive interference cancellation [2] at the receiver, then the optimal power policy of User i is obtained from $\partial L(i)$, even when the other users in the system allocate powers in an arbitrary fashion. This result suggests an interpretation of our result based on competitive non-cooperative equilibrium, where the users compete among themselves to maximize the objective function (7). The end result of such a game will be the joint optimal power allocation as described in (10).

In the next section, we present a simple technique for computing the Lagrangian $\boldsymbol{\lambda}$ which is needed to find the capacity region.

5. Iterative Algorithm for Obtaining Capacity Region Boundary

Since solving for the vector $\boldsymbol{\lambda}$ is complicated in general, we suggest a simple iterative algorithm that finds the λ_i one at a time. We consider a two user example for illustrative purposes, with $\mu_1 > \mu_2$.

Step 1: Find the single user optimum power distribution for User 1, i.e., find λ_1 such that $\mathbb{E}_{h_1}[\mu_1/\lambda_1 - N_0/h_1]^+ = P_1$ where $[f(x)]^+ = \text{maximum}(f(x), 0)$. Denote this estimate of the two user optimum power distribution of User 1 by $\hat{P}_1$.

Step 2: Find the power distribution of User 2 according to Equation (12) satisfying the power constraint for User 2, using the $\hat{P}_1$ found earlier for $P_1(\mathbf{h})$.This process gives us λ_2. Denote this estimate of the optimum power distribution of User 2 by $\hat{P}_2$.

Step 3: Find λ_1 and the power distribution for User 1 using Equation (11), the power constraint for User 1 and the λ_2 obtained in Step 2.

Step 4: Terminate if $|\lambda_1(n) - \lambda_1(n-1)| < \epsilon, |\lambda_2(n) - \lambda_2(n-1)| < \delta$, where ϵ, δ are design parameters (chosen to be suitably small) and n the iteration number. Else go to Step 2.

Since $\mu_1 > \mu_2$, the iterative algorithm begins by assuming that User 1 achieves the best possible performance on the channel (effectively assuming that $\mu_2 = 0$), and then adjusts the power distribution of the users towards the true values of μ_1 and μ_2. This algorithm is useful since finding λ_1 and λ_2 one at a time is not as difficult as solving for them simultaneously. We have not yet proved the convergence of this algorithm, but in our calculations it always converges. The example capacity region for Rayleigh flat fading shown in Figure 1 uses this algorithm.

6. Rate-Sum Capacity

Rate-sum capacity is the point on the capacity region where all the users have equal priority ($\mu_i = \mu_j \; \forall i, j$ in Equation (7)), i.e. when we wish to maximize $\sum_{i=1}^{N} R_i$. This case has already been studied in [5] and [6], where the authors show that TDMA is optimal for this scenario. We provide an alternative argument based on the optimality of an arbitrary decoding order.

Using the strict concavity of the logarithm in Equation (7), it is easy to see that the rate-sum ($\sum_{i=1}^{N} R_i$) will be maximized at a unique point. Revisiting the proof of Theorem 2, we find that any decoding order at the receiver will maximize the sum of rates. This can also be understood by noticing that the rate-sum point on the boundary of the capacity region can be approached from different directions, each of them corresponding to a different decoding order. Hence, the power policies $\mathbf{P}$ are such that the rates achieved by the users ($\mathbf{R}$) is the same for all decoding orders. This implies that, for User i

$$\mathbb{E}_{\mathbf{h}} \log \left(1 + \frac{h_i P_i(\mathbf{h})}{N_0 + \sum_{j \in S} h_j P_j(\mathbf{h})}\right) \tag{13}$$
$$= \mathbb{E}_{\mathbf{h}} \log \left(1 + \frac{h_i P_i(\mathbf{h})}{N_0 + \sum_{j \in S'} h_j P_j(\mathbf{h})}\right)$$

for any two subsets $S, S' \subset \{1, 2, \ldots, i-1, i+1, \ldots, N\}$ and for all i. In particular, this will hold true when $S = \{1, 2, \ldots, i-1, i+1, \ldots, N\}$ and $S' = \{\}$, which by (13) implies

$$\mathbb{E}_{\mathbf{h}} \log \left(1 + \frac{h_i P_i(\mathbf{h})}{N_0 + \sum_{j \neq i} h_j P_j(\mathbf{h})}\right) = \mathbb{E}_{\mathbf{h}} \log \left(1 + \frac{h_i P_i(\mathbf{h})}{N_0}\right) \tag{14}$$

Since $h_j P_j(\mathbf{h}) \geq 0$ for all j and $\mathbf{h}$, $N_0 + \sum_{j \neq i} h_j P_j(\mathbf{h}) > N_0$ unless $P_j(\mathbf{h})$ is equal to zero for all $j \neq i$ (Note that if $h_j = 0$, $P_j(\mathbf{h})$ must equal zero). In other words, the only way Equation (6) can hold is if at most one of the $P_j(\mathbf{h}) \neq 0$, $1 \leq j \leq N$. Therefore, TDMA is the optimum power allocation scheme for achieving rate-sum capacity.

7. Conclusions

In an N user uplink scenario, we have shown that successive decoding among users with the decoding order being independent of the channel state is Shannon optimal. We use this result to obtain the optimum power and rate allocation policies for each user in the system, using a straightforward convex optimization. We also propose a simple iterative algorithm to evaluate the expressions obtained for the optimum power and rate allocation, and provide an alternate proof showing that TDMA is optimal for maximizing the sum of rates.

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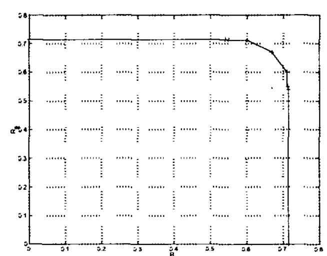

Figure 1. Capacity region for the two user flat fading channel

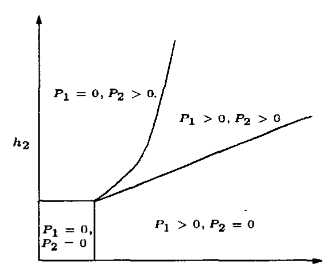

Figure 2. Regions $\mathfrak{R}_j$ for the two user example