Too Much Mobility Limits the Capacity of Wireless Ad-hoc Networks

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***Abstract*— We consider a K user isotropic fast fading ad-hoc network with no channel state information at any transmitter or receiver. Assuming that the users' channels are identically distributed we determine the capacity region of this ad-hoc network for any partition of the users into transmitters and receivers. The optimal strategy is such that only one pair of transmit-receive nodes is active at a time while all the other nodes are inactive. There is no benefit from cooperation and the total throughput grows at most double-logarithmically with the number of nodes. Even if the channel variations are slow enough that the receiver can track the channel perfectly the inability of the transmitter to track the network topology limits the total throughput growth rate to no more than logarithmic in the number of nodes. Our analysis extends Hochwald and Marzetta's single user Rayleigh fading AWGN channel result [1] to show that under the more general model of an *isotropic* fading ad-hoc network with arbitrary distribution of additive noise [2] [3] there is no capacity benefit from increasing the number of transmit antennas beyond the channel coherence time T_c.**

I. Introduction

In recent years, the capacity of ad-hoc networks has been the subject of increasing attention. In the seminal work of Gupta and Kumar [4] it is shown that the capacity of fixed ad-hoc networks does not scale linearly with the number of users, so that the capacity per user goes to zero as the number of users becomes large. Information theoretic justification for this result is provided by Leveque and Telatar in [5]. These works do not assume any mobility in the network and perfect channel knowledge is assumed. In the absence of mobility the fundamental limitation is that communication between distant nodes causes too much interference. Therefore most of the communication must happen between only nearest neighbors and multiple hops are required to carry a message to a distant node. Allowing mobility in the network has interesting implications. Grossglauser and Tse [6] show that with sufficient mobility an ad-hoc network can exploit a form of multiuser diversity via packet relaying. The nodes use mobile relays that can hand off the packets to the destination when they are close to it. The benefit of mobility is striking as it is shown that the total throughput in a mobile adhoc network can increase *linearly* with the number of nodes. A caveat of this result is that the delays incurred are of the order of the time-scale of node mobility. In the light of these results it might seem as if the faster the nodes move the better the performance of the ad-hoc network can be. However, these results assume perfect channel knowledge. For highly mobile wireless ad-hoc networks we need a different set of assumptions. Mobility introduces many challenges: e.g., with fast moving nodes the channel changes rapidly and channel estimation is harder. In this paper, we explore the capacity of highly mobile ad-hoc networks with multiple antennas at each node. In particular we investigate the main implications of high mobility: *channel uncertainty*, *network homogeneity* and the associated *loss in degrees of freedom*. These notions are further explained as follows.

Channel Uncertainty: The price for mobility is that the channel varies rapidly and reliable channel estimation is not possible. The amount of channel uncertainty is determined by the coherence time T_c of the channel. For the extreme case of $T_c = 1$ neither the transmitters nor the receivers are able to track the channel. Another relevant scenario is where T_c is large enough so that the receivers are able to track the channel, while there is no mechanism for the transmitters to acquire channel state information.

Network Homogeneity: Mobility blurs the distinction between nearest neighbors and distant nodes as the topology of the network is constantly changing, allowing all nodes to approach each other at different times. This can be perceived both as an advantage as well as a disadvantage. It is an advantage in the sense that the connectivity of the network is improved. In the absence of mobility multiple hops are required to communicate between distant nodes. With enough mobility all nodes can directly communicate with each other. The disadvantage is that when the distinction between near and far nodes is blurred, all transmissions interfere with each other. Thus, nodes are no longer protected from the interference caused by distant nodes.

Loss in Degrees of Freedom: In a multiple antenna ad-hoc network, the available channel state information (CSI) is directly connected to the *usable* degrees of freedom. Multiple users and multiple antennas offer additional degrees of freedom in the spatial domain. If it is possible to use these degrees of freedom then remarkable throughput gains are possible due to the multiplicity of antennas and users [7] [8]. However, the ability to use these degrees of freedom depends critically upon the amount of channel knowledge at the transmitters and receivers. With insufficient channel knowledge the degrees of freedom are lost and the capacity benefits quickly disappear. This observation has been made previously in the context of single user multiple antenna systems [9] as well as multiple antenna multiple access [10] and broadcast channels [11] [2]. In this paper we will characterize the loss in degrees of freedom due to the channel uncertainty and network homogeneity in highly mobile ad-hoc wireless networks with multiple antennas at each node. We build upon the results of previous work in [9] [11] and [2].

II. Isotropic Fading Ad-hoc Network

A. Channel Model

Let K be the total number of users in the ad-hoc network. The index set of all users is denoted by $\mathcal{K} = \{1, 2, \cdots, K\}$. For simplicity we will assume that all nodes, whether transmitting, receiving or inactive, have M antennas each. The channel is block fading with coherence time T_c (measured in number of channel uses). Block fading means that the matrix channels between all transmit-receive node pairs are assumed to remain constant for T_c channel uses, after which they change to an independent realization. As in [9] we will consider the T_c symbol extension of the channel. In other words, instead of dealing with the channel inputs and outputs over each channel use, we will work with the channel inputs and outputs over each channel fade block of T_c channel uses. The advantage of considering the extended channel is that over successive fade blocks the channel becomes memoryless. During the n^{th} channel fade block, let $\mathcal{T}_x(n)$ and $\mathcal{R}_x(n)$ be the index sets of nodes that are transmitting and receiving respectively. The signal received by a node $r \in \mathcal{R}_x(n)$ is:

$$\mathbf{Y}^{[r]}(n) = \sum_{t \in \mathcal{T}_x(n)} \mathbf{H}^{[t,r]}(n)\mathbf{X}^{[t]}(n) + \mathbf{Z}^{[r]}(n), \qquad (1)$$

where $\mathbf{X}^{[t]}(n)$ is the $M \times T_c$ matrix of symbols transmitted by node $t \in \mathcal{T}_x(n)$ on the M transmit antennas over the T_c channel uses in the n^{th} fade block, $\mathbf{H}^{[r,t]}(n)$ is the $M \times M$ wireless channel matrix between nodes t and r, and $\mathbf{Z}^{[r]}(n)$ is $M \times T_c$ matrix of the additive noise experienced by node r. To keep the notation concise, the fade block index n will not always be explicitly mentioned. The underlying assumptions of our channel model are listed next.

B. Isotropic Fading for MIMO Channels

The fading multiple antenna channel $\mathbf{H}^{[j,k]}$, is said to be **isotropic fading** if, $\mathbf{H}^{[j,k]} = \overline{\mathbf{H}}^{[j,k]}\mathbf{\Phi}^{[j,k]}$, where $\mathbf{\Phi}^{[j,k]}$ is an isotropically random unitary matrix and $\overline{\mathbf{H}}^{[j,k]}$ is a random matrix *independent* of $\mathbf{\Phi}^{[j,k]}$.

Isotropic fading captures the scenario where the amount of channel knowledge is insufficient to discriminate between different directions in the M dimensional space of transmitted signals.

C. Further Assumptions and Definitions

Simplex Communication: At any instant we assume that each user can be in only one of two states: transmitting or receiving. Thus, $\mathcal{T}_x(t) \cap \mathcal{R}_x(t) = \phi, \mathcal{T}_x(t) \cup \mathcal{R}_x(t) = \mathcal{K}$. Note that a transmitter or receiver can still be classified as such even if it is inactive.

Channel and Noise: We make no assumptions on the distribution of the additive noise process $\mathbf{Z}^{[\cdot]}$ for each receiver except that it is assumed to be a memoryless, ergodic and stationary stochastic process, independent of the channel fade matrix $\mathbf{H}^{[\cdot,\cdot]}$ and the input signal $\mathbf{X}^{[\cdot]}$. The additive noise may be *non-AWGN*. We also make no assumptions on the distribution of the channel fade process $\mathbf{H}^{[\cdot,\cdot]}$ except that it is *isotropic fading*, memoryless, ergodic and stationary over successive fade blocks, independent of the channel input signal $\mathbf{X}^{[\cdot]}$ and constant within each T_c symbol fade block. Note that the noise and channel fade processes corresponding to different nodes may have *different* distributions.

Channel State Information : We assume no channel state information at either the transmitter or the receiver. The channel and noise are ergodic, stationary and memoryless stochastic processes with the probability densities *known* to the transmitter and receiver.

Transmit Power Constraint : For user k, we assume an average transmit power constraint of $P^{[k]}$ per fade block. Mathematically $\mathrm{E}\left[||\mathbf{X}^{[k]}||^2\right] \leq P^{[k]}$, where $||\cdot||$ is the Frobenius norm. $\mathbf{X}^{[k]}(n)$ is defined to be zero if $k \notin \mathcal{T}_x(n)$. The expectation is over all the different partitions of $\mathcal{K}$ into $\mathcal{T}_x$ and $\mathcal{R}_x$.

Point-to-point Transmission : We assume that each message is encoded by exactly one transmitter and is destined for exactly one receiver. Under this assumption we denote the message being transmitted by user j and destined for user k as $U^{[j,k]}$.

Definition 1: [Rate-Tuples $\vec{R}$] With each message $U^{[i,j]}$ we associate a rate $R^{[i,j]}$ such that over N channel uses the message $U^{[i,j]}$ is a sequence of $2^{NR^{[i,j]}}$ Bernoulli (1/2) bits. $R^{[i,j]}$ is defined to be zero when either $i \notin \mathcal{T}_x$ or $j \notin \mathcal{R}_x$. These definitions allow us to define a $K \times K$ rate-tuple

$$\vec{R} \triangleq \{R^{[i,j]} : i, j \in \mathcal{K}\}, \qquad (2)$$

Definition 2: [Configuration Capacity Region $C(\mathcal{T}_x, \mathcal{R}_x)$] Each partition of $\mathcal{K}$ into $\mathcal{T}_x$ and $\mathcal{R}_x$ is called a *configuration*. Associated with each configuration is its capacity region $C(\mathcal{T}_x, \mathcal{R}_x)$, defined as the closure of the set of rate-tuples $\vec{R}$ at which *reliable* transmission is possible while satisfying the transmit power constraints.

We mean *reliable* transmission in the conventional ergodic capacity sense, i.e. the error probabilities can be made arbitrarily small by using the channel many times ($N \to \infty$).

Definition 3: [Feedback-Free Capacity Region C] We define the feedback-free capacity region as the convex hull of the union of the configuration capacity regions over all configurations:

$$C \triangleq \mathrm{co}\left[\cup_{\mathcal{T}_x, \mathcal{R}_x} C(\mathcal{T}_x, \mathcal{R}_x)\right], \qquad (3)$$

where co$[\cdot]$ denotes the convex hull operation.

The reason we call this the "feedback-free" capacity region is the following. Feedback is not possible in a configuration capacity region because the set of transmitting nodes never receive and the receiving nodes never transmit, i.e., there is no feedback. C is simply achieved by time sharing between the rate-tuples from different configuration capacity regions. The feedback-free capacity region is simply a convenient way to summarize our results in a way that is not configuration specific.

III. Channel Uncertainty

In this section we generalize a result shown in [9] for a single user Rayleigh fading AWGN channel to the general isotropic fading ad-hoc network.

Theorem 1: *Given any partition of the set of nodes $\mathcal{K}$ into mutually exclusive and collectively exhaustive subsets of transmitting and receiving nodes, $\mathcal{T}_x$ and $\mathcal{R}_x$ (respectively), the capacity region $C(\mathcal{T}_x, \mathcal{R}_x)$ of the isotropic fading ad-hoc network described in Section II is unaffected as the number of antennas at any transmitting node is increased beyond the channel coherence time T_c.*

Theorem 1 is configuration specific. In other words the partition of nodes into transmitting and receiving nodes is assumed fixed for all time. However, since the feedback free capacity region is simply obtained by time sharing between different configurations, the result of Theorem 1 is also true for the overall feedback-free capacity region. The proof of Theorem 1 follows from the following lemmas. The proofs for the lemmas are presented in [3].

Lemma 1: For any given configuration $(\mathcal{T}_x, \mathcal{R}_x)$, ad-hoc networks that have the same marginal distributions $p\left(\mathbf{Y}^{[r]} \middle| \mathbf{X}^{\mathcal{T}}\right)$ for all $r \in \mathcal{R}_x$, have the same capacity region $C(\mathcal{T}_x, \mathcal{R}_x)$. $\mathbf{X}^{\mathcal{T}}$ is the set of all transmitted signals over a fade block, defined as:

$$\mathbf{X}^{\mathcal{T}} \triangleq \{\mathbf{X}^{[t]} : t \in \mathcal{T}_x\} \tag{4}$$

Lemma 1 is an extension of the corresponding observation for broadcast channels due to Cover [12]. The second lemma presents another basic observation. In simple words, it states that any change in the input signal that does not affect the output statistics, does not affect the capacity.

Lemma 2: Let $f(\cdot) : \mathbb{C}^{M \times T_c} \to \mathbb{C}^{M \times T_c}$ be a mapping from the space of input signals to the same space such that $f(\cdot)$ does not violate the power constraint. That is, $||f\left(\mathbf{X}^{[\cdot]}\right)|| \leq ||\mathbf{X}^{[\cdot]}||$. Suppose, instead of transmitting $\mathbf{X}^{[\cdot]}$, each node $t \in \mathcal{T}_x$ transmits $f\left(\mathbf{X}^{[\cdot]}\right)$. Then, the capacity is unaffected by this change in the input signal, if

$$p\left(\mathbf{Y}^{[r]} \middle| \mathbf{X}^{\mathcal{T}} = \mathbf{x}^{\mathcal{T}}\right) = p\left(\mathbf{Y}^{[r]} \middle| \mathbf{X}^{\mathcal{T}} = f\left(\mathbf{x}^{\mathcal{T}}\right)\right), \quad \forall r \in \mathcal{R}_x. \tag{5}$$

Here, $f\left(\mathbf{X}^{\mathcal{T}}\right) \triangleq \{f(\mathbf{X}^{[t]}) : t \in \mathcal{T}_x\}$.

Proof of Theorem 1: Suppose the number of transmit antennas M at each node is greater than the coherence time T_c. To prove Theorem 1 we need to show that, capacity-wise, only T_c antennas are needed at each transmitting node: i.e., the remaining $M - T_c$ antennas are redundant and can be eliminated without affecting the capacity region. Define the mapping $f(\cdot) : \mathbb{C}^{M \times T_c} \to \mathbb{C}^{M \times T_c}$ as

$$f(\mathbf{X}) \triangleq \mathbf{R}_{\mathbf{x}}, \tag{6}$$

where $\mathbf{X} = \mathbf{Q}_{\mathbf{x}}\mathbf{R}_{\mathbf{x}}$ is the QR factorization of the tall $(M \times T_c)$ matrix $\mathbf{X}$, such that $\mathbf{Q}_{\mathbf{x}} \in \mathbb{C}^{M \times M}$ is a unitary matrix and $\mathbf{R}_{\mathbf{x}} \in \mathbb{C}^{M \times T_c}$ is an upper-triangular matrix.

The mapping $f(\cdot)$ does not affect the power constraint because

$$\begin{aligned} ||\mathbf{X}||^2 &= \text{Trace}(\mathbf{X}\mathbf{X}^\dagger) = \text{Trace}(\mathbf{Q}_{\mathbf{x}}\mathbf{R}_{\mathbf{x}}\mathbf{R}_{\mathbf{x}}{}^\dagger\mathbf{Q}_{\mathbf{x}}{}^\dagger) & (7)\\ &= \text{Trace}(\mathbf{R}_{\mathbf{x}}\mathbf{R}_{\mathbf{x}}{}^\dagger) = ||f(\mathbf{X})||^2. & (8) \end{aligned}$$

Next we show that the mapping $f(\cdot)$ does not affect the marginal distribution of the channel outputs at any receiving node in the ad-hoc network given the channel inputs of all the transmitting nodes, $p\left(\mathbf{Y}^{[r]} \middle| \mathbf{X}^{\mathcal{T}}\right)$. Starting with the channel model description (1) and using the definition of isotropic fading we have

$$\begin{aligned} \mathbf{Y}^{[r]} &= \sum_{t \in \mathcal{T}_x} \overline{\mathbf{H}}^{[t,r]}\mathbf{\Phi}^{[t,r]}\mathbf{Q}_{\mathbf{x}}{}^{[t]}\mathbf{R}_{\mathbf{x}}{}^{[t]} + \mathbf{Z}^{[r]}, & (9)\\ &\sim \sum_{t \in \mathcal{T}_x} \overline{\mathbf{H}}^{[t,r]}\mathbf{\Phi}^{[t,r]}\mathbf{R}_{\mathbf{x}}{}^{[t]} + \mathbf{Z}^{[r]}, & (10)\\ &= \sum_{t \in \mathcal{T}_x} \mathbf{H}^{[t,r]}f(\mathbf{X}^{[t]}) + \mathbf{Z}^{[r]}. & (11) \end{aligned}$$

Equations (9) through (11) imply that

$$p\left(\mathbf{Y}^{[r]} \middle| \mathbf{X}^{\mathcal{T}=\mathbf{x}^{\mathcal{T}}}\right) = p\left(\mathbf{Y}^{[r]} \middle| \mathbf{X}^{\mathcal{T}=f(\mathbf{x}^{\mathcal{T}})}\right), \quad \forall r \in \mathcal{R}_x. \tag{12}$$

Combining (6), (8) and (12) with the result of Lemma 2 we obtain the result that transmitting the input signal $f(\mathbf{X})$ instead of $\mathbf{X}$ does not affect the capacity region of the ad-hoc network. However, note that $f(\mathbf{X})$ is a *tall* upper triangular matrix of dimensions $M \times T_c$. The main diagonal of this matrix ends at row T_c, and therefore rows $T_c + 1$ through M contain only zeros. But each row of the transmitted signal matrix corresponds to the signal sent on the corresponding antenna. Therefore, we obtain the result that only zeros are transmitted on transmit antennas number $T_c + 1, T_c + 2, \cdots, M$. In other words, only T_c transmit antennas are needed at each node $t \in \mathcal{T}_x$ and the remaining $M - T_c$ antennas can be eliminated without affecting the capacity region. This completes the proof of Theorem 1. ■

The result of Theorem 1 is especially interesting for its generality. No assumptions are needed on the distributions of the additive noise $\mathbf{Z}^{[\cdot]}$ seen by each receiving node, or the channel fade matrix $\overline{\mathbf{H}}^{[\cdot,\cdot]}$ between each pair of nodes. Thus, the result of Theorem 1 applies when the noise is not AWGN, the fading is not Rayleigh, and different nodes have different channel fade and noise statistics. Moreover, although we assume equal number of antennas at each node for simplicity, one can easily verify that the result of Theorem 1 holds even when the number of antennas at each node is different.

Theorem 1 characterizes the loss in the usable degrees of freedom due to mobility in an ad-hoc wireless network. As the mobility increases and the nodes move faster, the channel coherence time decreases. Theorem 1 shows that the number of *useful* transmit antennas decreases as well.

Thus, although multiple transmit and receive antennas provide additional degrees of freedom that can increase throughput remarkably, the ability to use these degrees of freedom is limited by the mobility of the network as manifested in the form of increased channel uncertainty.

IV. Network Homogeneity

In this section we investigate another consequence of mobility: network homogeneity. With enough mobility the statistics of the channels between different node pairs over the duration of the codeword may be assumed similar. Therefore, in this section we model the channels between all node-pairs as statistically equivalent.

$$\mathbf{H}^{[t_1,r_1]} \sim \mathbf{H}^{[t_2,r_2]}, \quad \forall t_1, t_2 \in \mathcal{T}_x, \quad r_1, r_2 \in \mathcal{R}_x. \tag{13}$$

Instead of individual power constraints we use a common power constraint $\mathrm{E}\left[\sum_{t\in\mathcal{T}_x} ||\mathbf{X}^{[t]}||^2\right] \leq PK$, with the understanding that since the users are statistically equivalent, the overall average transmit power constraint of KP translates to an individual power constraint of P per user.

Definition 4: [Single User Capacity C_0] Define

$$C_0(P) \triangleq \max_{p(\mathbf{X}):\mathrm{E}[||\mathbf{X}||^2]\leq P} I(\mathbf{X};\mathbf{HX}+\mathbf{Z}), \tag{14}$$

In other words, $C_0(P)$ is the single user channel capacity with transmit power P.

A. Extreme Mobility: $T_c = 1$

We consider the limiting case of extreme mobility, i.e., $T_c = 1$, when the channel changes to an independent realization every symbol. No CSIT or CSIR is available.

Theorem 2: *For any given configuration $(\mathcal{T}_x, \mathcal{R}_x)$, the capacity region of the ad-hoc network described in Section II is:*

$$\begin{aligned} C(\mathcal{T}_x,\mathcal{R}_x) = \{\vec{R} \quad : \quad & \sum_{j\in\mathcal{T}_x}\sum_{k\in\mathcal{R}_x} R^{[j,k]} \leq C_0(KP), \\ & R^{[j,k]} = 0 \quad if \quad j\notin\mathcal{T}_x \quad or \quad k\notin\mathcal{R}_x\} \end{aligned}$$

The feedback-free capacity region of the ad-hoc network described in Section II is:

$$C = \{\vec{R} : \sum_{j=1}^{K}\sum_{k=1}^{K} R^{[j,k]} \leq C_0(KP)\}. \tag{15}$$

The entire capacity region is achievable with a combination of single user communication and time division multiplexing.

Notice that the theorem makes no assumptions about the additive noise distribution, or channel fade distribution except that it is isotropic fading. Thus, Theorem 2 is a fundamental result that generalizes the findings of [10] for the multiple access channel to the ad-hoc network scenario. It proves that all the degrees of freedom associated with multiple users and multiple transmit antennas are lost in a fast fading ($T_c = 1$) ad-hoc network due to the network homogeneity induced by extreme mobility. In the absence of channel state information, and with statistically equivalent nodes, there is nothing to be gained by simultaneous transmission by many users or simultaneous reception by more than one user. Also, the overall throughput of the ad-hoc network is the same as a single user system.

Achievability of the capacity region is quite straightforward by time division. We present the proof of converse.

Proof of Theorem 2 [Converse] : Let us assume that the rate-tuple $\vec{R}$ is achievable in the configuration $(\mathcal{T}_x, \mathcal{R}_x)$. Then we wish to show that the sum rate is less than the single user capacity.

$$\sum_{j\in\mathcal{T}_x}\sum_{k\in\mathcal{R}_x} R^{[j,k]} \leq C_0(KP). \tag{16}$$

A sketch of the proof is outlined as follows.

Step 1: Suppose we allow all the transmitters to co-operate. The resulting system is a vector broadcast channel with $M|\mathcal{T}_x|$ transmit antennas at the base station and M antennas at each of the $|\mathcal{R}_x|$ receivers. There is no CSIR or CSIT. Since cooperation does not hurt the capacity, the sum rate of this broadcast channel can not be smaller than the sum rate of the original ad-hoc network.

Step 2: Notice that in this broadcast channel each receiver is i.i.d. In other words, the received signals of the users are statistically equivalent. Since all users are alike, if one user is capable of decoding a message U, then each of the $|\mathcal{R}_x|$ users must be capable of decoding the message U. Thus, each user is capable of decoding all the transmitted messages. This in turn implies that the sum rate of this broadcast channel can not be more than the channel capacity between the multi-antenna base station and a single user with M antennas.

Step 3: Now consider the single user communication system, with the $|\mathcal{T}_x|$ transmit antennas and a single receive antenna. This is an isotropic fading system with coherence time ($T_c = 1$), i.e. the channel changes to an independent value every symbol. There is no CSIR or CSIT. In Theorem 1 we have shown that under these assumptions the capacity is unaffected if we discard all but one transmit antennas.

Step 4: Let us discard all but one transmit antennas. Then we are left with a single user communication system with one transmit antenna and M receive antennas. The power constraint is KP, the channel is isotropic fading and $T_c = 1$. We already defined the capacity of this system as $C_0(KP)$.

Combining steps 1 through 5, we conclude that the sum rate of the ad-hoc network with the configuration $(\mathcal{T}_x, \mathcal{R}_x)$ is not more than the single user capacity $C_0(KP)$. This proves the converse for the configuration capacity region. The converse for the overall capacity region follows directly because the overall capacity region is just the convex hull of the union of the configuration capacity regions. Since the sum rate in each configuration is not more than $C_0(KP)$, the sum rate in the convex hull can not be more than $C_0(KP)$ either. This completes the proof of the converse to Theorem 2. ■

B. Rate of Growth of Total Throughput with Nodes

We start with the case of fast fading: $T_c = 1$. The feedback free sum capacity of the ad-hoc network in this case is $C_0(KP)$. The asymptotic growth rate of $C_0(KP)$ has been studied previously by Lapidoth and Moser in [13]. A direct application of the results of [13] leads to the following corollary.

Corollary 1: The feedback-free sum capacity of the ad-hoc network defined in Section II grows at most double logarithmically with the number of nodes for large number of nodes.

$$\lim_{K\to\infty} \{C_0(KP) - \log\log(KP)\} < \infty. \tag{17}$$

Compare this to the growth rate of sum-capacity of a fixed ad-hoc network, which is of the order of the square root of the number of nodes [4]. Thus if the transmitters and receivers are unable to estimate the channel, the throughput of a highly mobile ad-hoc network is worse than that of a fixed mobile network. In other words, too much mobility limits the capacity of wireless ad-hoc networks.

Next we consider the case of $T_c > 1$. For any given coherence time T_c we can still use steps 1 and 2 of the proof of Theorem 2. As in Step 1 we let all the transmitters cooperate to form one transmitter with $M|\mathcal{T}_x|$ antennas. As in step 2, we eliminate all but one receiving nodes. Thus the feedback-free sum capacity of the ad-hoc network is bounded above by the capacity of a single user Rayleigh fading multiple antenna channel with $M|\mathcal{T}_x|$ transmitting antennas and M receiving antennas with transmit power KP, and no CSIT or CSIR. We assume that the number of transmitting nodes $|\mathcal{T}_x|$ goes to infinity as $K \to \infty$. The single user asymptotic growth rate of capacity with SNR for $T_c > 1$ has been explored by Zheng and Tse in [14]. Using the results of [14] we arrive at the following corollary.

Corollary 2: As the number nodes $K \to \infty$ the feedback-free sum capacity, C_Σ, of the ad-hoc network defined in Section II is bounded above as:

$$C_\Sigma < \min(\lfloor T_c\rfloor/2, M)\left(1 - \tfrac{\min(\lfloor T_c\rfloor/2, M)}{T_c}\right)\log(KP) + c + o(1).$$

Finally we consider the slow fading case where the receiver is able to track the channel. Allowing cooperation between transmitting nodes and eliminating all but one receiving nodes, followed by a simple application of Jensen's inequality leads to the following upperbound on the sum capacity.

Corollary 3: As the number nodes $K \to \infty$ the feedback-free sum capacity, C_Σ, of the ad-hoc network defined in Section II, with the additional assumption of perfect CSIR, is bounded above as $C_\Sigma < M\log(1 + KP)$.

Note that in this case we do not know the configuration capacity region and time-division may not be optimal.

V. Conclusion

We explore the implications of extreme mobility in a wireless ad-hoc network. The cost of high mobility is that the channel fluctuates too rapidly and can not be tracked at the transmitter or the receiver. We characterize the loss in degrees of freedom due to channel uncertainty. For the broad class of isotropic fading channels with completely general additive noise distributions we show that increasing the number of transmit antennas M at any node beyond the channel coherence time T_c (measured in units of channel uses) does not affect the capacity region of the ad-hoc network. In [3] we also explore spatially correlated Rayleigh fading channels. For these channels we show that the capacity region depends only on the $\min(M, T_c)$ largest eigenvalues of each users' transmit fade covariance matrix. Spatial correlation is shown to enhance the capacity region of the fast ($T_c = 1$) Rayleigh fading ad-hoc network.

Another cost of high mobility is that it blurs the distinction between the near and distant nodes. In other words, mobility makes the network more " homogeneous". Assuming statistically equivalent nodes and fast fading ($T_c = 1$) we are able to determine the capacity region of the ad-hoc network. It turns out that all the degrees of freedom corresponding to multiple users and transmit antennas are lost. The best strategy is to allow only one pair of nodes to communicate at a time. There is no benefit in letting transmit nodes cooperate. Thus, in a highly mobile wireless network, the loss in degrees of freedom due to the inability of the transmitter and receiver to track the channel limits the capacity. The network throughput growth in this case is shown to be no more than double-logarithmic with the number of users, which is even worse than fixed wireless networks. The sum throughput grows no more than logarithmically with the number of nodes even if we assume that all the receiving nodes are able to track the channel perfectly.

References

[1] T. Marzetta and B. Hochwald, "Unitary space-time modulation for multiple-antenna communications in Rayleigh flat fading," *IEEE Trans. Inform. Theory*, vol. 46, pp. 543–564, March 2000.

[2] S. Jafar and A. Goldsmith, "On the capacity region of the vector fading broadcast channel with no CSIT," in *Proceedings of International Conference on Communications (ICC), June 2004, Paris, France.*

[3] S. Jafar, "Too much mobility limits the capacity of wireless ad-hoc networks," Submitted. Preprint available upon request.

[4] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, pp. 388–404, August 2000.

[5] O. Leveque and E. Telatar, "Information theoretic upper bounds on the capacity of large extended ad-hoc wireless networks," *Submitted to IEEE Transactions on Information Theory*.

[6] M. Grossglauser and D. Tse, "Mobility increases the capacity of adhoc wireless networks," *IEEE/ACM Transactions on Networking*, vol. 10, pp. 477–486, August 2002.

[7] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun. : Kluwer Academic Press*, no. 6, pp. 311–335, 1998.

[8] B. Hochwald and S. Vishwanath, "Space-time multiple access: Linear growth in sum rate," in *Proceedings of 40th Annual Allerton Conference on Commun., Control and Computing*, Oct. 2002.

[9] T. Marzetta and B. Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading," *IEEE Trans. Inform. Theory*, vol. 45, pp. 139–157, Jan 1999.

[10] S. Shamai and T. L. Marzetta, "Multiuser capacity in block fading with no channel state information," *IEEE Trans. Inform. Theory*, vol. 48, pp. 938–942, April 2002.

[11] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Trans. Inform. Theory*, vol. 49, pp. 1691–1706, July 2003.

[12] T. M. Cover, "Broadcast channels," *IEEE Trans. Inform. Theory*, vol. 18, pp. 2–14, Jan. 1972.

[13] A. Lapidoth and S. M. Moser, "Capacity bounds via duality with applications to multi-antenna systems on flat fading channels," Submitted to IEEE Trans. Inform. Theory. See http://www.isi.ee.ethz.ch/ lapidoth.

[14] L. Zheng and D. N. Tse, "Packing spheres in the Grassmann manifold: A geometric approach to the non-coherent multi-antenna channel,," *IEEE Trans. Inform. Theory*, vol. 48, pp. 359–383, Feb 2002.