Beyond Links - Soft Information Optimization for Memoryless Relay Networks

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***Abstract*— The problem of soft information optimization for memoryless relay networks is motivated and studied through AF relay networks, and in particular the AF relay multiple access (MAC) and broadcast (BC) channels. Relay optimization, capacity region characterization and optimal scheduling are addressed.**

I. Introduction

Current wireless network design principles are largely borrowed from wired networks. The borrowed design principles may be quite inefficient due to the vast differences in the nature of wired and wireless signal propagation. Nowhere is this mismatch more visible than in the link abstraction. Unlike wired networks which can be viewed as independent point to point links, the wireless propagation medium is inherently a broadcast medium which gives rise to tight interactions between concurrent transmissions. The link abstraction, quite fundamental to wired network design, creates artificial bottlenecks when imposed upon wireless networks. There is, therefore an urgent need to rethink the relevance of the link based model and to find the optimal design principles that are specialized for wireless networks.

A. What is soft information?

We use the term *hard information* to refer to *reliable* information bits that are transmitted across point to point *links*. *Soft information*, on the other hand, refers to possibly *unreliable* information in the form of *noisy* signals that percolate through multiple hops across a wireless network and interact with each other before converging at their intended destinations where reliable decoding is required. While hard information is motivated by the wired propagation model of independent capacitated links, soft information is motivated by the unbounded nature of the wireless medium, the inherent channel uncertainties arising out of its time varying nature, and the mixing of signals in the air through the superposition of electromagnetic waves that is the cause of interference. Hard information represents a view of a network as a collection of reliable data pipes. Soft information represents a view that the entire network between the source and destination of each message is one channel and reliability needs to be guaranteed only in the end-to-end sense between the initial sources and the final destinations. Under the hard information perspective, each link should be operated at its individual capacity which is similar to optimizing the mutual information for a point to point link over all possible input distributions that satisfy the power constraints. Under the soft information perspective, it is not only the input distribution, but the channel itself that is optimized for the end-to-end capacity region of all concurrent messages in the network.

B. Why is soft information needed ?

Reliable communication, which is the goal of a communication network, implies that every message must be decoded reliably by its intended destination. It is generally not required, it is sometimes not possible and it may be not even desirable that intermediate nodes should be able to decode the information that they participate in relaying. The relay node's ability to decode information is especially limited when the relay node is exposed to the information only through a limited window in space and/or time either by necessity or by design.

Space limited window of relay observation:

Consider the wireless network shown in Figure 1 where the information flows from the source to the destination through multiple paths. Each relay node is exposed to only one of many paths. While the relay may be active from the beginning to the end of transmission it may not have enough diversity to be able to decode the message. On the other hand the destination node is able to collect information from multiple paths and improve its signal to noise ratio (SNR) due to coherent combining gain as well as the diversity gain which allows it to decode its intended message.

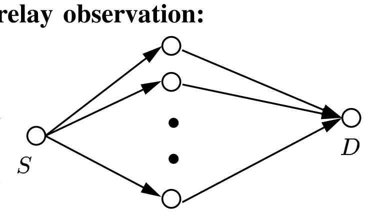

Fig. 1a: Multiple Paths

Time limited window of relay observation:

While the source transmits a long codeword a relay node may be exposed to only a part of the codeword for a number of reasons. In Fig. 2, while the source transmits the encoded symbols from a long codeword, the relay is only exposed to $k+1$ symbols. Thus, the relay transmitted symbol is only a function of its last $k+1$

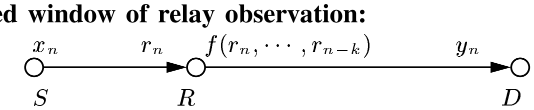

Fig. 1b: Relay memory of $k+1$ symbols

observations. For example, the relay may be active only part of the time to conserve its own power. Or, if the delay constraints are tight, the relay may not be allowed to wait until it receives the entire codeword before it forwards the information. The extreme example of time limited observation is a memoryless relay which can not have access its past observations so that its transmitted symbol is only a function of its current observation $f(r_n)$.

Security: For secrecy reasons it may be required that the intermediate nodes are not capable of decoding the end-to-end message. While collectively the relay nodes deliver enough information for the destination to decode the message, each relay node, by design, may not be exposed to enough information to be able to decode the message.

In all of these examples, the relay's limited exposure to the information significantly limits its capacity to reliably decode information. Requiring each relay to decode, i.e. make hard decisions, would imply one of two possibilities. First, if reliable decoding at the relay is desired then the informaiton rate should be lower than each relay's capacity based on its limited window of observation. This would create artificial bottlenecks for the capacity between the source and the final destination of the message. The second possibility is that the rate is higher than the relay's limited observation capacity. In this case the relay cannot make a reliable decision. Forwarding an unreliable hard decision in a network can be quite suboptimal. For example, consider the simple scenario where the destination needs to decode a simple binary message as reliably as possible. Suppose it is aided by two relays, one of which makes an error so that the destination receives two conflicting messages from the two relays. With only the relay's hard decisions available to the destination it must make a blind guess. However, if the relays forward some form of soft information that tells the destination about the degree of uncertainty at the relay then it can agree with the relay whose information appears more certain and thus reduce its own probability of error. Therefore, if the relays are unreliable then it is better to forward soft estimates of the relays' received signals in a way that the destination is able to collect these pieces of information and reliably decode the message.

II. Exploring Soft Information for Point to Point Channel with One Relay:

Starting with the simplest scenario, we consider the case of a single message propagating through a two hop network with the assistance of a single memoryless relay. The case of binary inputs is studied first, followed by a generalization to continuous alphabet.

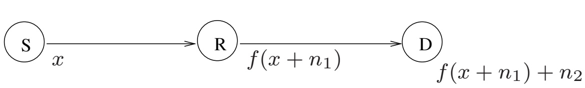

Fig. 2. Elemental Relay Channel

1) Binary inputs and the optimality of EF (Estimate-and-Forward) Relays: A single memoryless relay channel is shown in Fig. 2 where the direct link between the source and the destination is also omitted for simplicity. The source transmits a codeword over a binary alphabet using BPSK (Binary Phase Shift Keying) modulation. For each transmitted symbol $x \in \{+\sqrt{P}, -\sqrt{P}\}$, the relay observes a noisy version $r = x + n_1$. Based on the observation r, the relay transmits a symbol $f(r)$ which is received at the destination along with additional noise n_2. All noise powers are normalized to unity. Note that the relay transmitted symbol is only a function of its current received symbol. Thus the relay functionality does not utilize memory of past received symbols. This is the case we refer to as "memoryless relays" and it represents the simplest form of relay optimization for soft information.

The class of memoryless relays includes many common relay functionalities, such as demodulate-and-forward (DemF) and amplify-and-forward (AF). With DemF, the relay can demodulate each symbol and forward a hard decision about each symbol, so the the relay function $f_{DemF}(r) = \sqrt{P_R}\text{sgn}(r)$, where P_R is the relay's power constraint. Because of symbol-wise processing, the probability of symbol error at the relay would be the same as for uncoded transmission even though the symbols observed by the relay are a part of a codeword transmitted by the source. With AF, the relay simply forwards a scaled version of its received symbol where the scaling factor is chosen to satisfy the relay's power constraint. The relay functionality with AF is $f_{AF}(r) = \sqrt{\frac{P_R}{P+1}}r$.

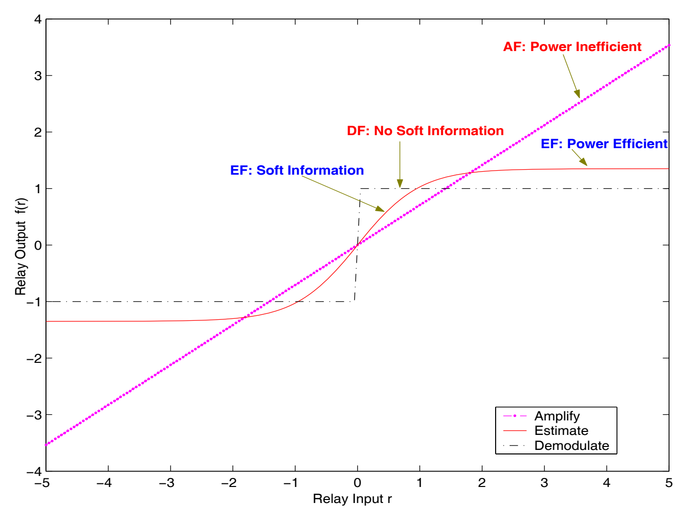

Fig. 3. Relay functions of common forwarding schemes for BPSK (P=1)

The relay functions are shown in Figure 3. It can be seen that the magnitude of the received symbol remains irrelevant during the demodulation process although it contains reliability or soft information. Due to the hard decision made in demodulation, the relay transmitted signal carries no information about the degree of uncertainty in the relays choice of the optimal demodulated symbol. Evidently with AF, the relay tries to provide soft information to the destination.

While the reliability information is greatly beneficial in the uncertainty region around 0, the power constraint comes into play especially when r is high because then significant power is expended. From the relay functions of AF and DF, one can argue that an optimal relay function should provide soft information when there is an uncertainty in the received symbol, and it should conserve power when the cost of power outweighs the value of soft information. The natural question therefore is: what is the most cost effective way of conveying soft information? In other words, the problem is simply to determine the optimal relay function $f(\cdot)$ that maximizes the mutual information between the source and the destination subject to the power constraint $\mathcal{E}_r[|f(r)|^2] \leq P_R$.

The optimization problem is stated as

$$C = \max_{f(.)} \ I(X;Y)$$
$$\text{s.t} \ \ \mathcal{E}|f(x+n_1)|^2 = P_R. \quad (1)$$

Using calculus of perturbations we are able to numerically verify that the solution to the optimization problem above is for the relay to forward the MMSE estimate

$$f^{\star}(r) = \kappa\mathcal{E}[x|r],$$

where κ satisfies the power constraint. We call this forwarding strategy estimate-and-forward (EF) as it combines the advantages of AF and DF. Interestingly EF is also optimal for maximizing SNR at the destination. The SNR optimality of EF can be established even with multiple parallel relays.

Optimality of Gaussian inputs and AF relays: The first generalization of this basic scenario is to enrich the input alphabet form BPSK to continuous alphabet and in particular for Gaussian inputs. Interestingly, while EF and AF are distinct for binary inputs, they are identical for Gaussian inputs. This is because the MMSE estimate for Gaussian symbols is simply the linear estimate. Forwarding the linear estimate corresponds to a simple scaling of the relay observation and is therefore identical to the AF relay functionality. Finally, we also notice that if the relay functionality is AF then Gaussian inputs are also optimal. Thus, we have three useful observations.

- Based on numerical evidence, estimate and Forward is the capacity optimal relay functionality with binary inputs.
- Estimate and Forward is identical to Amplify and Forward for Gaussian inputs.
- For AF relay Gaussian inputs are optimal.

These three observations lead us to the conjecture that: *Gaussian inputs and AF relays are capacity optimal for the point to point memoryless relay channel.*

III. Soft Information Optimization for AF relay MAC and BC

While much of the work on relay networks has focused on point to point communication, multiuser relay networks are increasingly gaining attention as well [1]–[5]. In [6] gains for AF relays in a multiuser parallel network are determined to achieve a joint minimization of the MMSE of all the source signals at the destination. Tang et. al. [3] consider a MIMO relay broadcast channel, where a multiple antenna transmitter sends data to multiple users via a relay with multiple antennas over two hops. They find different algorithms for computing the transmit precoder, relay linear processing matrix and the sum rate under the assumption of zero-forcing dirty paper coding and Gaussian signals. Capacity bounds are used to establish that the performance loss is not significant. Capacity with cooperative relays has been explored for the multicast problem by Maric and Yates [7], [8], for the broadcast problem by Liang and Veeravalli [5], and for the mixed multiple access and broadcast problem by Host-Madsen [9]. Maric and Yates explore an accumulative multicast strategy where nodes collect energy from previous transmissions, while Liang and Veeravalli [5] and Host-Madsen [9] address the general question of optimal relay functionality which may not be an amplify and forward scheme. Azarian et. al. [10] explore the diversity-multiplexing tradeoff in half-duplex, cooperative multiple access and broadcast scenarios for various AF and DecF protocols. With channel knowledge available only to the receiving nodes, the diversity multiplexing tradeoff for AF is shown to be dominated by DecF. In the absence of channel knowledge at the transmitters, AF protocols cannot utilize the array gain from coherent combining of signals at the receiver and therefore nothing is to be gained from more than one relay transmitting the same symbol simultaneously. The results of [11], [12] have shown that if channel knowledge is available at the transmitters as well as the receivers then AF relay networks can benefit significantly from coherent combining of simultaneous transmissions. It is in this coherent scenario that we explore the benefits of soft information for AF relay MAC and BC.

2) Two hop AF relay multiple access channel: In the two hop multiple access channel, transmitters with powers P_1 and P_2 send messages to a common destination with the assistance of several AF relay nodes that have a collective power constraint of P_R. The superposition of the signals from different users over the same bandwidth presents an interesting relay optimization problem. In the point to point case it is obvious that the relays should co-phase their signals to achieve maximum coherent combining gain. With multiple users, the simultaneous coherent combining of all users' signals is not possible. Therefore the relays must optimally choose their power and phase allocations to achieve the optimal compromise, i.e. a point on the boundary of the capacity region.

Following a similar approach to [11], the optimization problem for each point on the boundary of the capacity region can be expressed into the Rayleigh quotient form and using Cauchy-Schwartz inequality. Thus, we are able to characterize the capacity region of the two hop AF relay multiple access channel for any number of users, and relays, with complex signals and channels. The two user example is shown in Figure 4. It is found that all relays participate with their powers distributed according to the channel strengths so that relays with better channels are allocated more power than those with weak channels. The parameter θ in the figure indicates how the relays transition from helping user 2 to helping user 1 as θ changes from 0 to $\pi/2$. Intermediate values of θ represent optimal compromise points where the relays help both users.

3) Duality between multihop AF relay multiple access and broadcast communications: The duality relationship is summarized in the following theorem.

Theorem 1: The multihop AF relay broadcast channel illustrated in Figure 5 has the **same capacity region** as the multihop AF relay multiple access channel of Figure 6 obtained by switching the roles of source and destination nodes and with power constraints as indicated in the figure.

Notice that the power constraints in the dual multiple access and broadcast channels are modified so that each hop carries the same total transmit power. For example, the first hop of the broadcast channel is the last hop of the dual multiple access channel. So the first hop in the broadcast channel and the last hop of the multiple access channel are associated with the same total transmit power P_Q^R even though the transmitting nodes are different in the two cases. Similarly, the first hop of the multiple access channel and the last hop of the broadcast channel are associated with total transmit power of P_0^R.

The duality relationship between the AF relay broadcast and multiple access channels extends the one hop multiple access and broadcast channel duality shown in [13] to multiple hops. It is especially useful for characterizing the broadcast channel capacity region in terms of the multiple access channel capacity regions as explained below.

A. Two hop AF relay broadcast channel

The two hop AF relay broadcast channel consists of one source with power P sending *independent* messages to several destination nodes with the assistance of multiple AF relay nodes with a total relay power constraint of P_R. The capacity region of the AF relay broadcast channel is easily obtained from the characterization of the multiple access capacity region described in Figure 4 and the duality relationship illustrated in Figures 5 and 6.

Figure 7 shows the broadcast channel capacity region as the union of the multiple access channel capacity regions for different power allocations P_1, P_2 to users 1 and 2 such that $P_1 + P_2 = P$.

B. Optimal Scheduling for AF relay MAC

Figure 8 shows the sum capacity for the AF relay multiple access channel on the vertical axis while the power per source P is shown on the horizontal axis. Clusters of plots represent different number of relay nodes M while the total relay power is fixed. We find that simultaneously scheduling more sources leads to higher throughput when the power per source is small but it is better to have schedule only one source at a time if the source power is sufficiently high. As the number of relays increases the time division policy becomes increasingly optimal.

IV. Conclusion

We investigate soft information optimization for memoryless relay networks. Based on relay funcationality optimization for a single relay point to point communication channel, AF relay networks are chosen as representative scenarios for studying soft information propagation. In particular we study relay optimization, the capacity region and scheduling aspects for the AF relay MAC and BC. A duality relationship is found to exist between the multihop AF relay MAC and BC. The capacity region of the AF relay MAC can be found directly by convex optimization using the Rayleigh quotient form and the duality relationship can be used to find the capacity region of the AF BC.

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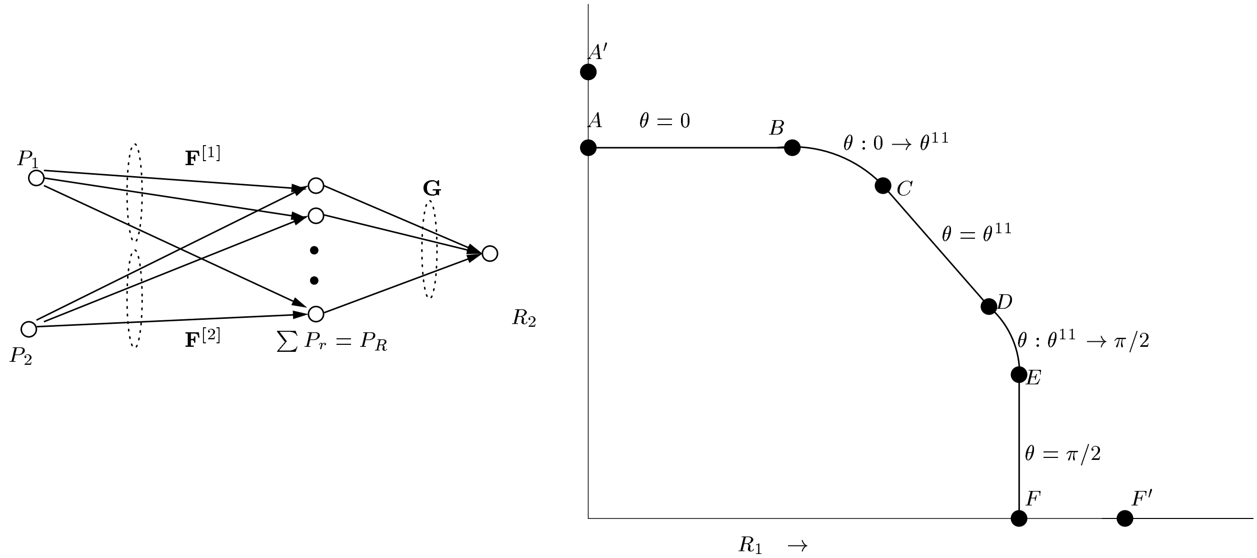

Fig. 4. Rate region of two user AF relay multiple access channel with user 1 and 2's transmit powers *equal* to P_1, P_2, respectively.

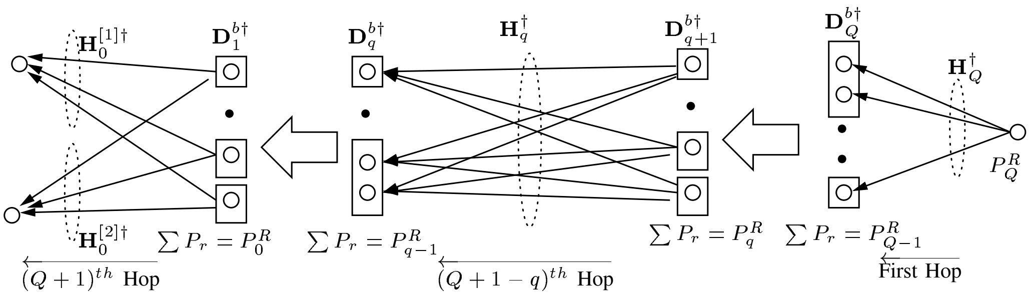

Fig. 5. AF relay broadcast (one-to-many) communication over multiple hops and multiple paths

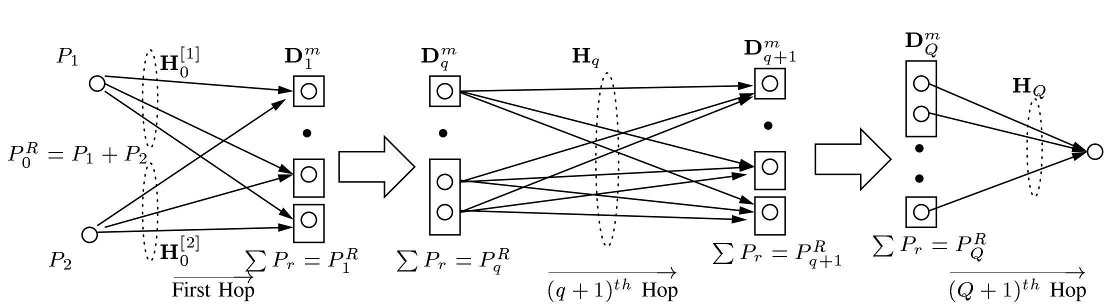

Fig. 6. Dual AF relay multiple access (many-to-one) communication over multiple hops and multiple paths

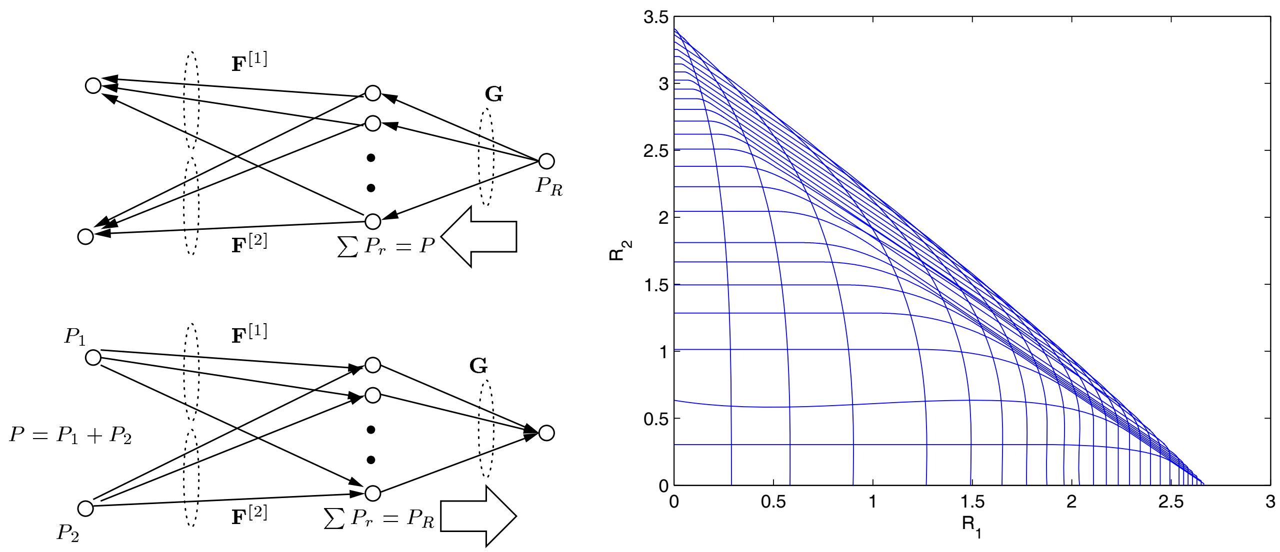

Fig. 7. 2-hop AF relay BC rate region as the union of AF relay MAC rate regions

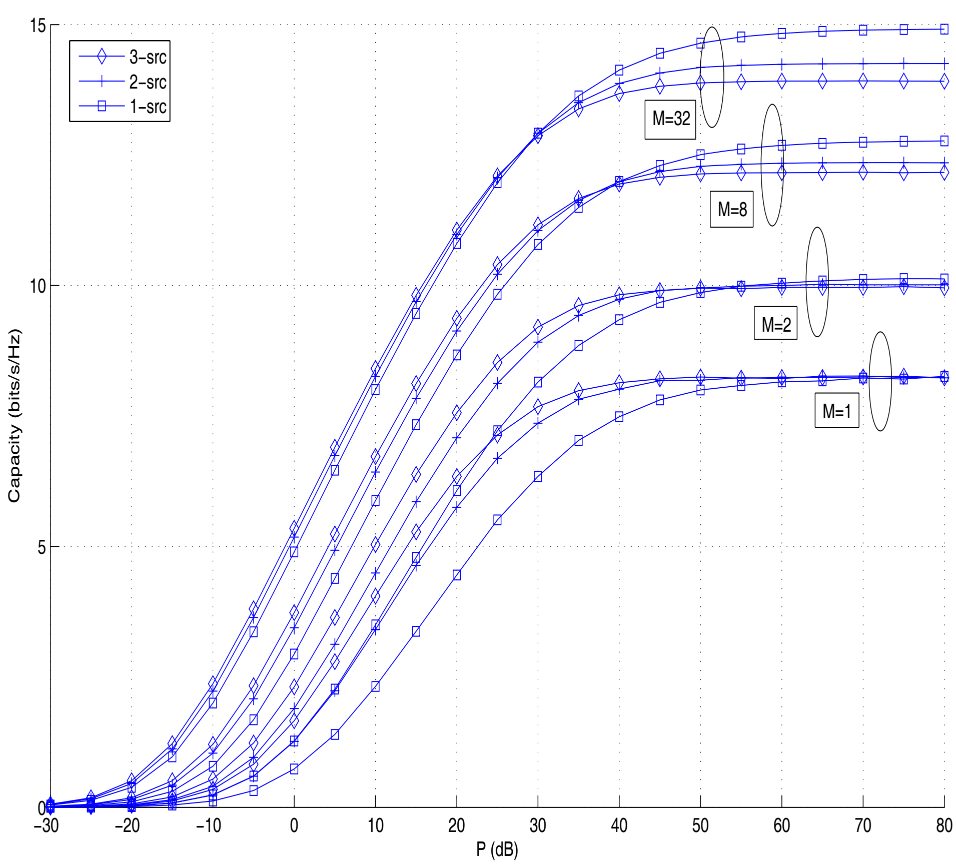

Fig. 8. Sum Capacity Optimal Scheduling for AF relay MAC