Cognitive Radio Networks: How much Spectrum Sharing is Optimal?

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***Abstract*—We explore the performance tradeoff between opportunistic and regulated access inherent in the design of multiuser cognitive radio networks. We consider a cognitive radio system with sensing limits at the secondary users and interference tolerance limits at the primary and secondary users. Our objective is to determine the optimal amount of spectrum sharing, i.e., the number of secondary users that maximizes the total deliverable throughput in the system. We begin with the case of perfect primary user detection and zero interference tolerance at each of the primary and secondary nodes. We find that the optimal fraction of licensed users lies between the two extremes of fully opportunistic and fully licensed operation and is equal to the traffic duty cycle. For the more involved case of imperfect sensing and non-zero interference tolerance constraints, we provide numerical simulation results to study the tradeoff between licensing and autonomy and the impact of primary user sensing and interference tolerance on the deliverable throughput.**

I. Introduction

In recent years, the increasing popularity of diverse wireless technologies has generated a huge demand for more bandwidth. While the traditional 'divide and set aside' approach to spectrum regulation ensures that the licensed (primary) users cause minimal interference to each other, it has created a crowded spectrum with most frequency bands already assigned to different licensees [1]. The term 'cognitive radio' encompasses several techniques [2]–[5] that seek to overcome the spectral shortage problem by enabling secondary (unlicensed) wireless devices to communicate without interfering with the primary users. Our work will exclusively focus on the 'interweave' (interference avoidance) approach [5], [6] to cognitive radio, wherein the secondary radio periodically monitors the radio spectrum, intelligently detects occupancy in the different frequency bands and then opportunistically communicates over the spectrum holes with minimal interference to the active primary users.

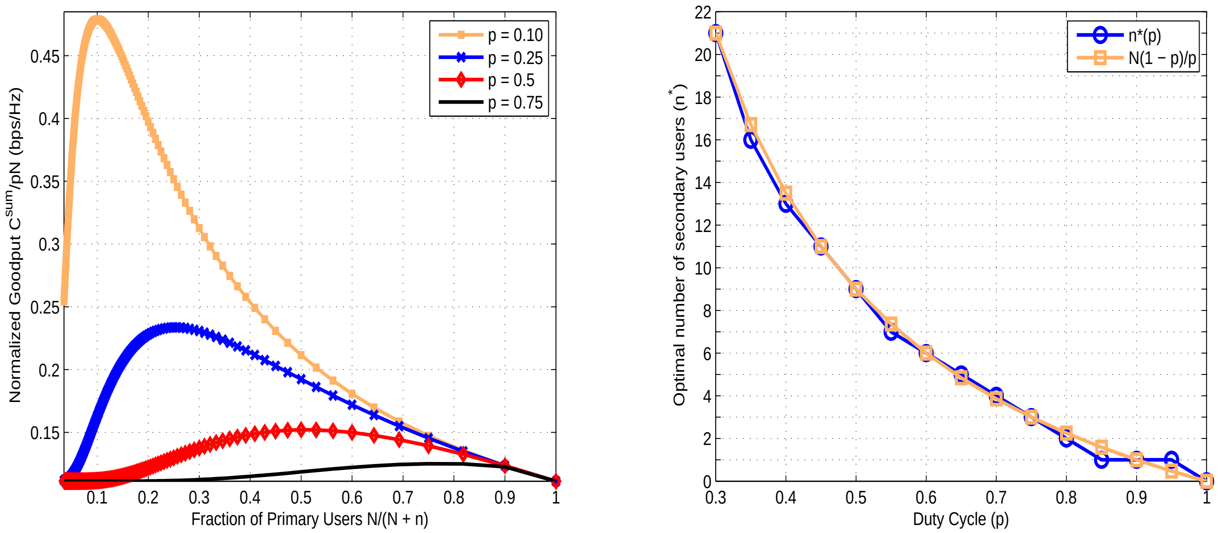
Opportunistic communication with the interweave technique faces a multitude of challenges in the detection of primary systems and spectrum access, coexistence and sharing in multiuser environments. The literature for the study of spectral sensing for cognitive radio systems is extensive [4] (and references therein). A major issue in a *multiple* secondary user environment is dynamic spectrum access and sharing, a topic that has generated a lot of research interest in the recent past [7]–[13]. This problem is similar to that of multiple access in multichannel wireless networks - in both these cases multiple independent transmitters need to access a set of shared channel resources. Many access protocols for cognitive networks have therefore been derived from conventional MAC protocols like ALOHA and CSMA [9], [10].

In practical multiuser environments, cognitive radio operation is governed by interference tolerance and sensing limits at the primary and secondary users. The interference limits at the primary and secondary users indicate the amount of protection needed at each (primary or secondary) user from the multiuser interference to maintain a certain rate. On the other hand, the sensing limits (minimum SNR needed for detection) at the secondary users reflect the amount of protection that each secondary user is individually able to provide to the primary users. In these scenarios, the key is to strike a balance between the two conflicting goals - minimizing the interference to the primary users, and maximizing the performance of the entire system - by limiting the number of secondary users. Therefore, the natural question that arises is: *What is the optimal number of secondary users (opportunistic access) relative to the number of primary users (licensed access) that maximizes the sum throughput in the system?* This is reminiscent of the familiar debate of licensing versus autonomy, a tradeoff that is fundamental to many areas of systems and control theory. The generality of this tradeoff is evident through an analogy with traffic control: Too much regulation, i.e., too few secondary users (traffic lights at every intersection) and the system is inefficient due to unoccupied spectral holes. On the other hand, too much autonomy/opportunistic behavior, i.e., too many secondary users, (no traffic lights) and the system becomes self-disruptive due to collisions between the secondary users.

The main goal of this work is to characterize the optimal amount of opportunistic use that maximizes the *sum* throughput in the system given the sensing and the tolerable interference limits. We begin with the mathematically tractable case of perfect sensing at the secondary systems and zero interference tolerance at the primary and secondary users in Section II, a case for which we analytically identify the optimal number of secondary users. In Section III, we explore the general scenario of imperfect sensing and non-zero interference tolerance at the primary and secondary receivers. For this more involved case, we develop throughput expressions and present some numerical results in Section III-A.

II. Perfect Sensing, Zero Interference Tolerance

Consider a certain channel resource that is equally shared among N primary users (primary transmitter-receiver pairs), i.e., each primary radio is licensed to transmit on a subchannel that spans $\left(\frac{1}{\mathsf{N}}\right)^{\mathsf{th}}$ of the available bandwidth. Data traffic arrives at each primary user in an i.i.d fashion with a probability of arrival p. To allow for higher spectral efficiencies, the channel is also open to be used *opportunistically* by n



(a) Normalized goodput versus fraction of primary users

(b) Optimal number of secondary users versus p

Fig. 1: Figure 1(a) plots the normalized goodput ($\frac{C^{sum}}{pN}$) versus the fraction of licensed users ($\frac{n}{N}$) with $N = 9$ users for different values of p. The optimal fraction of primary users can be seen to be equal to the duty cycle p. For different values of p, figure 1(b) compares the optimal number of secondary users $n^*(p)$ with the value of $\frac{N(1-p)}{p}$.

secondary users. We assume that delay intolerant data arrives at the secondary users with the same arrival probability p.

We first consider *perfect* primary radio detection at the secondary users and zero interference tolerance at the primary and secondary receivers, assumptions that will be relaxed in the next section. Any secondary user that has data to transmit monitors the N subchannels for primary users and randomly chooses one (if any) of the available channels for secondary communication.

Transmission at the primary and secondary users takes place at a rate[1] C. While the primary users can reliably transmit their data at this rate, the data of the secondary users is considered lost if either

- no free channel is available for secondary transmissions, or
- two or more secondary users select the same unoccupied licensed channel (i.e., when a *collision* occurs).

The performance metric of interest to us is the total amount of data (primary and secondary) that is *successfully delivered* per unit time, which we refer to as the *goodput*.

Perfect sensing at the secondary users precludes collisions between primary and secondary users. Therefore, the sum goodput of the primary users is:

$$C_p^{sum} = CN\text{Prob}\left[\text{PU is active}\right] = CNp \quad (1)$$

The sum goodput of the secondary users depend on the number of unoccupied subchannels. The sum secondary goodput can be written as

$$C_s^{sum} = \sum_{i=1}^{N} \binom{N}{i} p^{N-i} (1-p)^i \, C_s^{sum}(i), \quad (2)$$

where $C_s^{sum}(i)$ is the secondary goodput given that i of the N primary users do not have data to transmit (OFF). Conditioning on the number of secondary users having data to transmit (ON), $C_s^{sum}(i)$ can be expressed as

$$C_s^{sum}(i) = \sum_{j=1}^{n} \binom{n}{j} p^j (1-p)^{n-j} \, C_s^{sum}(i,j), \quad (3)$$

where $C_s^{sum}(i,j)$ is the secondary goodput given that i subchannels are unoccupied and j secondary users are ON. A secondary user's data is successfully transmitted only if there are no other secondary users in the slot it has chosen for transmission, the probability of which is $i\frac{1}{i}\left(1-\frac{1}{i}\right)^{j-1}$ and therefore $C_s^{sum}(i,j) = Cj\left(1-\frac{1}{i}\right)^{j-1}$. Substituting $C_s^{sum}(i,j)$ and equation (3) in equation (2) and combining with equation (1), the sum goodput $C^{sum} = C_p^{sum} + C_s^{sum}$ simplifies to

$$C^{sum} = pCN\left(1 + \frac{n}{N}\sum_{i=1}^{N}\binom{N}{i} p^{N-i}(1-p)^i\left(1-\frac{p}{i}\right)^{n-1}\right) \quad (4)$$

Figure 1(a) plots the normalized goodput ($\frac{C^{sum}}{pN}$) with increasing fraction of licensed users ($\lambda = \frac{N}{N+n}$ with decreasing n) for different values of the duty cycle p (with $N = 9$ primary users and $C = \frac{1}{N}$). The interesting observation from Figure 1(a) is that neither full autonomy ($\lambda = 0$, i.e., large n) nor fully regulated operation ($\lambda = 1$, i.e., $n = 0$) is goodput optimal. Instead the optimal fraction of primary users is an *intermediate value* $\lambda^*(p)$ that increases with the data arrival rate p. Consistent with intuition, we note that licensing is good for high duty cycle (always ON, $p \to 1$) traffic while opportunistic operation is more suited for low duty cycle (rarely ON, $p \to 0$) cases.

From Figure 1(a), it can be seen that the optimal fraction of licensed users ($\lambda^* = \frac{N}{N+n^*}$) is nearly equal to the duty cycle p, i.e., $\lambda^* \approx p$. Figure 1(b) demonstrates that the approximation is very tight, i.e., the optimal number of secondary users n^* (calculated from equation (4)) closely matches the fraction

[1]Notice that the transmit rate is assumed fixed and is independent of the locations of the secondary transmitter receiver pairs.

$\frac{N(1-p)}{p}$ for all p. This observation can be intuitively explained with a first order approximation as follows. The average number of primary users with data to transmit is Np. The average number of unoccupied subchannels is therefore $(N - Np)$. The average number of secondary users who have data to send is np. If $np > (N - Np)$, there is a high possibility of collisions. On the other hand, if $np < (N - Np)$, there is a high chance that some of the subchannels remain unoccupied. The best value would therefore be $n^* = \frac{(N-Np)}{p}$, as validated by Figure 1(b). The same trend is exhibited regardless of the value of N and p and C. This directly leads us to the following proposition:

Proposition 1: Given the number of primary users N, the optimal number of secondary users n^* that maximizes the sum goodput with perfect primary user detection and non-zero interference tolerance at each of the users is $n^* = \frac{N(1-p)}{p}$.

To determine the effects of sensing and interference tolerance on the goodput maximizing number of secondary users, we now consider a more general model in Section III.

III. Imperfect Sensing, Non-zero Interference Tolerance

We scale space and consider $(N + n)$ independent users (node-pairs) distributed uniformly in a circular area with unit radius, i.e, the probability that any node is located at a distance $r \in (0,\ 1)$ from the center of the disc is given by $p_R(r) = 2r$.

We assume that each node-pair is a time-slotted half duplex system, i.e., communication can take place in both directions in a node-pair, albeit not simultaneously. Each time slot is considered to be long enough that arbitrary rates lower than the channel capacity can be achieved over a single slot. In any time slot, a delay intolerant data packet arrives at the transmitting node of a node-pair (the node that is designated to transmit in that particular time slot) with a probability p, i.e., the nodes within a node-pair have data to exchange for a fraction p of the time.

a) **Primary and secondary users:** The available channel bandwidth is divided into N equal subchannels and is licensed to N of the users (primary users). When a primary node has any data to be sent, it transmits it in the associated subchannel. The rest of the users (n secondary users) only have opportunistic access to the spectrum and have to monitor all the N subchannels for primary activity before data transmission.

b) **Sensing Limit:** We assume that each secondary node can detect primary nodes within a radius R_s around it, as shown in Figure 2. A subchannel is assumed to be free if *both* the secondary transmitter and receiver do not detect any primary users within their respective sensing regions. Since we consider nodes within a unit radius disc, a sensing radius $R_s = 2$ corresponds to perfect primary detection, while $R_s = 0$ corresponds to no detection. The observation time for sensing primary users is assumed to be very small compared to the length of the time slot.

c) **Spectrum access model:** When more than one subchannel is detected to be free, the secondary node pair *randomly chooses one* of the subchannels for secondary communication. Since there is no cooperation between the secondary users, two or more secondary radios can choose the same subchannel.

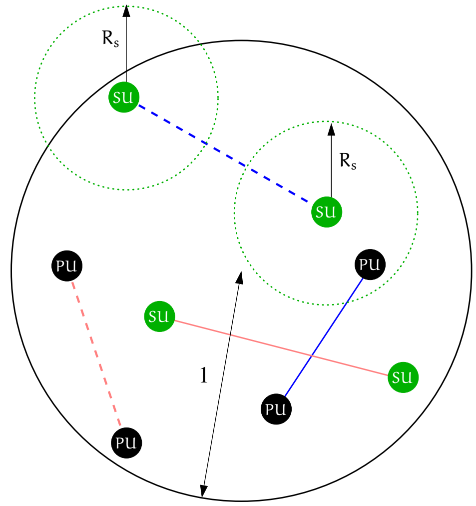


Fig. 2: PU and SU represent the primary and secondary users respectively. The circles around the secondary nodes are the sensing regions within which primary users are detected perfectly. The different subchannels are distinguished with colored links. Dotted lines indicate that the corresponding primary/secondary user does not have data to transmit (OFF).

d) **Interference Limit:** Signal propagation is modeled with path loss of the form d^{-2} with distance d. The transmitting nodes use independent Gaussian codebooks with an average power constraint P. Transmission between any node-pair takes place at a data rate set so that the receiving node can tolerate a total interference of I, i.e., the data rate is given by $\log\left(1 + \frac{P/d^2}{1+I}\right)$, where d is the distance between the nodes. If the interference at the receiving node is larger[2] than I, the data packet is considered lost. Further, the data packet of a secondary user is dropped if no free slots are detected.

We capture the locations of the primary node-pairs in the random variables $D_{p,i} = \left\{r_i^{(1)}, \theta_i^{(1)}, r_i^{(2)}, \theta_i^{(2)}, d_{p,i}\right\}, \quad 1 \leqslant i \leqslant N$, where $d_{p,i}$ is the link distance[3]. Similarly we define $D_{s,j} = \left\{r_j^{(1)}, \theta_j^{(1)}, r_j^{(2)}, \theta_j^{(2)}, d_{s,j}\right\}, \quad 1 \leqslant j \leqslant n$ for the secondary node-pairs. We also collect $\mathcal{D} = \{D_{p,1}, \cdots, D_{p,N}, D_{s,1}, \cdots, D_{sn}\}$. The set of all secondary users transmitting in subchannel i is captured in the set $\mathcal{B}_i$.

The sum goodput of the primary and the secondary users can be expressed as in equations (5) and (6), where the indicator functions $\mathbb{I}[\cdot]$ determine whether or not there is an outage. x_{ij} is the *minimum* distance (worst-case interference) between the i^{th} primary and j^{th} secondary users, and similarly, y_{ij} between the i^{th} and j^{th} secondary users. The binary variable $q_i = \mathbb{I}\left[i^{th}\ \text{PU is ON}\right]$ indicates whether or not the i^{th} primary user is active.

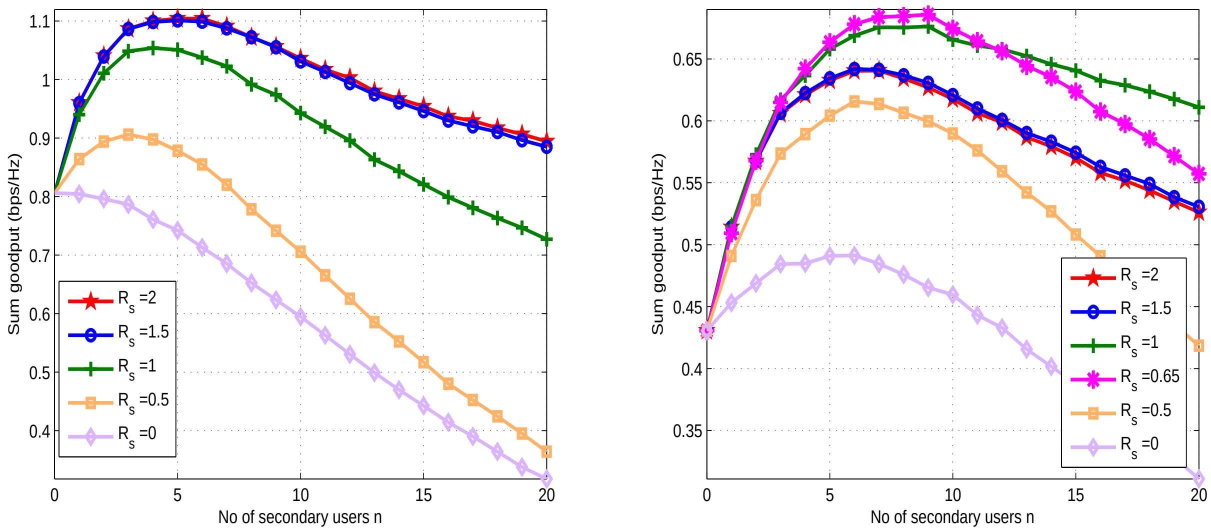
While equations (5) and (6) express the sum goodput in non-ideal, realistic scenarios with imperfect sensing and non-zero interference limits, the additional complexity makes analysis very difficult. Therefore, we numerically evaluate equations

[2] We emphasize that the interference limit I is *not* a constraint imposed on the system, i.e., there is no guarantee that the interference at the receivers is less than I.

[3] In terms of the polar coordinates of the two nodes, $d_{p,i} = \sqrt{r_i^{(1)2} + r_i^{(2)2} - 2r_i^{(1)} r_i^{(1)} \cos\left(\theta_i^{(1)} - \theta_i^{(2)}\right)}$

$$C_p^{sum} = \mathsf{E}_{\mathcal{D}}\left[\mathsf{E}_{\{\mathcal{B}_1,\ \mathcal{B}_2,\ \cdots\mathcal{B}_N\}|\mathcal{D}}\left[\sum_{i=1}^{N} p\mathbb{I}\left[\sum_{j\in\mathcal{B}_i}\frac{P}{x_{ij}^2}\leqslant I\right]\log\left(1+\frac{P/d_{p,i}^2}{1+I}\right)\right]\right], \tag{5}$$

$$C_s^{sum} = \mathsf{E}_{\mathcal{D}}\left[\mathsf{E}_{\{\mathcal{B}_1,\ \mathcal{B}_2,\ \cdots\mathcal{B}_N\}|\mathcal{D}}\left[\sum_{i=1}^{N} p\left(\sum_{k\in\mathcal{B}_i}\mathbb{I}\left[\sum_{j\in\mathcal{B}_i/k}\frac{P}{y_{kj}^2}+q_i\frac{P}{x_{ik}^2}\leqslant I\right]\log\left(1+\frac{P/d_{s,k}^2}{1+I}\right)\right)\right]\right]. \tag{6}$$



(a) Sum goodput vs. n for different R_s $(I = 0)$

(b) Sum goodput vs. n for different R_s $(I = 2)$

Fig. 3: Figures 3(a) and 3(b) plot the goodput versus increasing number of secondary users (n) for different sensing radii (R_s) for $I = 0$ and $I = 2$ respectively.

(5) and (6) to gain insights into the optimal number of secondary users in scenarios with imperfect sensing and non-zero interference tolerance limits.

A. Simulation Results

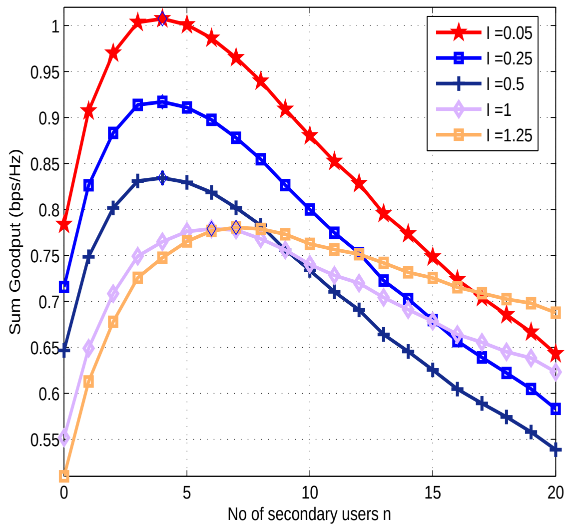
We consider the system model discussed above with $N = 5$ primary users and plot the goodput for different values of the interference tolerance I and the sensing radius R_s. We assume a traffic arrival probability $p = 0.5$ at each of the primary and secondary users.

Figure 3(a) plots the sum goodput with increasing number of secondary users for different values of the sensing radius R_s. The interference tolerance I is equal to 0, i.e., even a small amount of interference to the primary or secondary users results in undecodable data and goodput loss. For a given number of secondary users n, the probability of a secondary user colliding with other primary users increases as the sensing radius R_s decreases. The goodput is therefore maximum for $R_s = 2$ (perfect sensing), as Figure 3(a) shows. Notice that the optimal number of secondary users $n^*(R_s)$ decreases as R_s decreases. This reflects the importance of sensing in a zero interference tolerance environment - for the specific case of $R_s = 0$ (no sensing), the presence of even a single secondary user introduces sufficient interference to the primary users to cause a decrease in the goodput, i.e., $n^*(R_s = 0, I = 0) = 0$. Since sensing takes place at both the secondary transmitter and receiver, most of the primary users are detected even with a moderate sensing radius $(R_s > 1)$. The sum goodput difference between the $R_s = 1$ case and perfect sensing is therefore not very large.

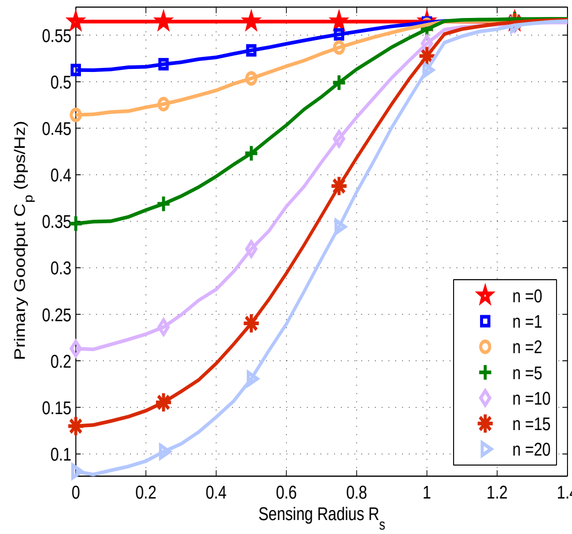
Figure 3(b) considers a scenario where the primary and secondary users have a interference tolerance $I = 2$. Since each of the users is transmitting at a lower rate $(I = 2)$, the sum goodput for the same n and sensing radius R_s is lower than that for the $I = 0$ (zero tolerance) case. Further, the higher interference tolerance at each of the users implies that, compared to the $I = 0$ case, more secondary users can be accommodated for the same sensing radius R_s, i.e., $n^*(R_s, I = 2) > n^*(R_s, I = 0)$. The interesting observation from Figure 3(b) is that secondary systems can *exploit the tolerance of the primary links to transmit more aggressively*, i.e. use a lower sensing radius. As Figure 3(b) shows, a smaller sensing radius $(R_s = 0.65)$ provides the secondary users with more opportunities to transmit while maintaining the interference level below the limit. Notice that for $n \leqslant 11$, the optimal sensing radius is $R_s = 0.65$. Similar trends are shown even at higher interference tolerance values.

Figure 4(a) compares the sum throughput with increasing number of secondary users for different interference tolerance values. It can be seen that the throughput advantages of transmission at higher rates (lower I) decrease as the number of secondary users increase because of the higher number of collisions between the primary and secondary users. Specifically, at higher loads $(n > 15)$, a larger sum goodput can be achieved by increasing the interference tolerance.

To study the effect of the sensing radius on the primary



(a) Sum goodput vs. n for different I ($R_s = 0.9$)



(b) Sum primary goodput vs. R_s for different n ($I = 1$)

Fig. 4: Figure 4(a) plots the sum goodput with increasing number of secondary users n for a sensing radius $R_s = 0.9$. Figure 4(b) shows the primary user goodput vs. sensing radius R_s for different number of secondary users n. The interference margin is fixed at $I = 1$.

users, Figure 4(b) plots the primary goodput C_p with decreasing R_s for a tolerance level of $I = 1$. It can be seen that even with $n = 20$ secondary users, a sensing radius of $R_s = 1.1$ is sufficient to guarantee that the interference caused to primary users is minimal.

IV. Conclusion

Spectrum sharing and access are important issues facing opportunistic communication in multiuser cognitive radio systems. Because of the presence of user priority (primary and secondary), they pose unique design challenges that are not faced in conventional wireless systems. In an environment with multiple primary and secondary users, the tradeoff between sum throughput maximization and primary user interference minimization is a result of the well known interplay between regulation and autonomy. In scenarios with perfect primary user sensing and transmissions at the channel capacity, we characterize this tradeoff and identify the optimal amount of spectrum sharing that maximizes the total system throughput. We observe that the optimal fraction of primary users is equal to the duty cycle of the data traffic.

The more general case of imperfect sensing and finite interference tolerance at each of the users manifests similar trends and the optimal number of secondary users is again found to be between the two extremes of complete regulation and complete autonomy. Numerical results show that in a zero interference tolerance environment, the optimal number of secondary users increases as the sensing ability of the secondary nodes increases. Overall, the sensitivity of the sum goodput to primary user sensing is found to decrease as the interference tolerance at the primary and secondary users increases.

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