Abstract—We study stability regions of multi-rate Gaussian multiple access (MAC) and broadcast (BC) networks with centralized scheduling algorithms. Techniques are presented to characterize stability regions of BC and MAC networks with peak power constraints and average power constraints. The duality property that relates the MAC and BC information theoretic capacity regions is found to extend to their stability regions as well, in the average power constraint case.

I. INTRODUCTION

Traditionally, information theorists and the networking community have taken two different approaches to the study of communication networks. The information theoretic approach explores the capacity region of a network by optimally designing the encoding and decoding schemes while abstracting the higher layers to infinitely backlogged queues. The networking approach on the other hand explores the stability region of a network by optimizing the scheduling and routing algorithms while abstracting the physical layer encoding and decoding schemes to server processes.

The networking and information theory perspectives have lead to many important results. For example, the networking approach has found stability-optimal scheduling and routing algorithms [1], [2] such as the maximum weight match scheduling and dynamic back-pressure routing algorithms. One of the remarkable successes of information theory, on the other hand, has been the successful characterization of the capacity regions of the Gaussian multiple access (MAC) and broadcast (BC) channels and discovery of an elegant duality relationship between their capacity regions [3]. Both networking and information theory perspectives have their own limitations. A common limitation of the networking research has been that sophisticated encoding/decoding techniques introduced by information theory, e.g. successive decoding, are overlooked. The utility of information theoretic results on the other hand is limited by the numerous underlying ideal assumptions such as infinite backlog, availability of of infinitely many codebooks and infinitely long codewords etc.

In this work, we combine the strengths of information theory and networking approaches to study wireless broadcast and multiple access networks. Motivated by the networking approach, we study the stability regions of wireless multiple access and broadcast networks when only limited (finite) sets of rates are available to the physical layer. Motivated by the information theoretic approach, we allow the physical layer to use some of the optimal multiuser encoding and decoding schemes for the MAC and BC. In particular we allow coding schemes based on independent encoding and successive decoding with successive interference cancellation. Note that independent encoding and successive decoding suffice to achieve all points within the information theoretic capacity region of the Gaussian MAC and BC. Also, motivated by the information theoretic duality of the MAC and BC capacity regions we explore the existence of similar duality relationships for the stability regions of the multi-rate MAC and BC networks.

II. BACKGROUND

A. Stability region of a network

A stationary stochastic process $Q(t)$ is defined to be stable if

$$\lim_{x \to \infty} \limsup_{t \to \infty} \Pr\left[Q(t) > x\right] = 0.$$ 

A communication network is defined to be stable if the stochastic processes representing its individual queue states are stable. The stability region of a network is defined to be the closure of the set of all arrival rate matrices that can be stabilized by some scheduling and routing policy with complete knowledge of the arrival statistics, the queue state and any other parameters of the network such as channel conditions, distance between nodes etc.

B. Equivalent definition based on infinitely backlogged nodes

The network layer capacity region is defined as the set of all average departure rate matrices that can be achieved in a network assuming that all nodes are infinitely backlogged taking into consideration all possible scheduling routing and resource allocation strategies in the network with possibly complete knowledge of the parameters of the network such as channel conditions, distance between nodes etc.

The stability region defined in the previous subsection is shown to be equivalent to the network capacity region of a network in [4]. The equivalence is important because the latter is formulated under the assumption of infinitely backlogged nodes. The infinite backlog model eliminates the dependence on instantaneous queue states from the stability optimal scheduling and routing algorithms.

The equivalence of the stability region and the ergodic capacity region has been explored for the multiple access channel in [5]. However, unlike this work, [5] does not assume a finite number of codebooks.
III. THE SYSTEM MODEL

We shall study the stability regions of multi-rate Gaussian multiple access (MAC) and broadcast networks (BC) (Figure 1). We first describe the generic system model that is common to both the models of multiple access and broadcast networks in this work.

Let $N$ indicate the number of users (and hence, the number of links) in the network. The channel is a Rayleigh fading channel with additive white Gaussian noise of unit variance at the receiver/s (one receiver in the MAC network and multiple receivers in a BC network). The channel fading is represented using the gain vector $\tilde{h}(t) = (h_1(t), h_2(t), \ldots, h_N(t))$ with each component of the vector corresponding to a particular link in the network. The following are important aspects of the system model.

A. Encoding

The transmitters transmit a single fixed-length codeword per time slot (i.e., the length of one time slot is normalized to the length of the codeword). The codeword is chosen from a fixed finite set of Gaussian codebooks. Let $\mathcal{R}_i = \{0, R_0, R_1, \ldots, R_{M_i-1}\}, i = 1, 2$ indicate the rates of the codebooks available for transmission corresponding to user $i$ (rate '0' indicating that the user does not transmit). Therefore, the transmitted rate vector in a particular time slot lies in $\mathcal{R}_1 \times \mathcal{R}_2$. For example, in a MAC network with both users having a fixed codebook of rate $R_0$, the rate vector that can be transmitted in a particular time slot belongs to $\{(0, 0), (R_0, 0), (0, R_0)\}$ (indicated by $\times$ symbol in Figure 2). We assume availability of a finite number of codebooks since it is a more reasonable assumption in practice, as compared to the information theoretic model which assumes availability of an uncountable number of codebooks.

B. Slow fading - long coherence times

$\tilde{h}(t)$ is ergodic and remains constant in the duration of a time slot. The coherence time of the channel is assumed to be long enough (i.e., the time slot is long enough) so that codewords of sufficient length can be transmitted to guarantee error free communication over the link, if at all possible at that rate.

C. Decoding

The decoding strategy at the receivers is restricted to successive decoding and interference cancellation from the received signal. The receiver can decode one complete codeword at a time, cancel the corresponding interference and then proceed to decode the next codeword. This successive decoding procedure is carried on until the receiver decodes its desired codeword.

For example, in a two user multiple access network, there are only two different ways of decoding corresponding to two possible decoding orders. The codeword corresponding to user 1 can be decoded first, treating the complete codeword corresponding to user 2 as noise. This is followed by interference cancellation and subsequent decoding of user 2’s codeword. The receiver could alternately choose to decode using the reversed order i.e., user 2 can be decoded first, followed by interference cancellation and then user 1 can be decoded.

Consider a fixed-codebook MAC network, i.e., a MAC network of two users with each user having a fixed codebook of rate $R_0$. As mentioned previously, one of the four points indicated by \( \times \) symbol in Figure 2 are transmitted. At time slot $t$, for a particular channel state $\tilde{h}(t)$ and a particular codeword power let the pentagon ABCDO represents the capacity region of this MAC channel. Then the assumption on decoding implies that the rate pairs that can be reliably transmitted over this network in this time slot, only if it belongs to rectangle ABEO (corresponding to decoding user 1 first) or rectangle FCDO (i.e. user 2 is decoded first). In other words, the rate pairs that can be transmitted reliably over the network with this power allocation lie in the ‘L-shaped’ region (traced in thick lines in Fig. 2). Note that in Figure 2, although $(R_0, R_0)$ lies in the pentagon 1 - the pentagon representing the MAC capacity region, it cannot be achieved with either decoding order and is therefore not achievable at time slot $t$ with this particular codeword power allocation.

Note that in a BC network, in a particular time slot, all points in the BC capacity region can be achieved with successive interference and decoding.

IV. PROBLEM DEFINITION

We pursue the following objectives in this paper.

1) Optimal Rate Regions: We find the optimal mapping of channel states to the set of supported rates available for a scheduler.

2) Stability Regions: We use the region mapping to characterize the stability regions of multiple access and broadcast networks.

3) MAC-BC Stability Region Duality: We use the stability region characterization to explore duality relationships between the stability regions of multi-rate MAC and BC.

For ease of exposition, we assume $N = 2$ and $\mathcal{R}_1 = \mathcal{R}_2 = \{0, R_0\}$, i.e., we restrict our networks to 2 user fixed codebook networks. The approach used in solving the problems under these assumptions and the duality results presented can be directly applied to general multi-rate multi-user MAC and BC networks. A solution to the general case is presented in the extended version of this paper [6].

The rates of transmitted codewords are limited by constraints on the power of the transmitted codewords. Power constraint can, in general, occur in three forms. First, fixed power constraints are constraints of the form $P(t) = P_{\text{max}}$. Analysis of systems with power constraint of this form is useful, particularly for systems being studied for a short period of time. Power constraints of the form $P(t) \leq P_{\text{max}}, \forall t$ are called peak power constraints and are used to model limits on the instantaneous power of the transmitted signal imposed by the limits of the power source and the amplifiers in the system. Average power constraints are of the form $E[P(t)] \leq P_{\text{max}}$ and are used to model limits on battery life at the transmitters.

An interesting consequence of the finite rate set assumption is that, at certain channel states, it may be possible to achieve a larger set of rates by reducing the power of transmission. For example, in figure 2, pentagon 2 represents the MAC capacity region with a power allocation strictly smaller than that which achieves pentagon 1 at the same channel state.
lies in the ‘L-shaped’ region associated with pentagon 2 (plotted with broken lines in figure 2) it is achievable with the corresponding power allocation. This implies that, unlike in the information theoretic model, a fixed power constraint is not equivalent to a peak power constraint since it is possible that better rates are achieved by transmitting less than the maximum possible power. In this work, we focus on MAC and BC stability regions with peak and average power constraints.

V. STABILITY REGION OF MAC AND BC NETWORKS WITH PEAK POWER CONSTRAINTS

The stability region of a general single hop network with a finite number of channel states and peak power constraint is known ([7]) to be:

$$\Gamma = \sum_S \pi(S) \text{Co}(C(S))$$

(1)

where $$\pi(S)$$ denotes the probability with which the channel takes state $$S$$, $$C(S)$$ denotes the set of all rate vectors that can be transmitted in the network reliably, when the channel state is $$S$$ and $$\text{Co}(X)$$ denotes the convex hull of set $$X$$. For the Gaussian MAC and BC networks, while the Rayleigh fading channel vector $$\vec{h}(t) = (h_1(t), h_2(t))$$ takes values over a continuum, a finite state space is naturally defined by the partitioning of the channel space based on the rate pairs that can be supported. Let $$C = 2^{R_1} \times 2^{R_2}$$ denote the set of all subsets of $$R_1 \times R_2$$ with cardinality $$K = |C|$$. Let $$C_i \in C, i = 1, 2 \ldots K$$ denote all possible sets of supported rates (or equivalently all possible elements of $$2^{R_1} \times 2^{R_2}$$). Then define the state space $$S_i, i = 1, 2 \ldots K$$ to be:

$$S_i = \{(h_1, h_2) : C_i \in 2^{R_1} \times 2^{R_2}\}$$

The set of supported rates is unique to each channel state and equation 1 can now be applied to obtain stability regions.

Notation: In the description below, we use the notation $$F(a, b) = \log(1 + a^2b)

A. Fixed codebook MAC network

Consider a MAC network with transmit powers constrained by $$P_i(t) \leq P_i, i = 1, 2$$. Let $$\mathcal{R} = \mathcal{R}_1 = \mathcal{R}_2 = \{0, R_0\}$$. Therefore we have $$2^2 = 16$$ possible subsets of $$\mathcal{R} \times \mathcal{R}$$. However, it is easy to see that, except for five of these subsets, the channel partitions corresponding to all other subsets are empty. These five subsets, represented as $$C_i, i = 1, 2, 3, 4, 5$$ and their corresponding channel partitions $$S_i$$ are described below.

1) $$S_1$$: This represents the state where the channel gains are so low that reliable transmission is not possible on either channel, i.e $$F(h_1, P_1) < R_0, i = 1, 2$$ with $$C_1 = \{(0, 0)\}$$

2) $$S_2$$: Transmission is possible only on channel 1 i.e $$C_2 = \{(R_0, 0), (0, 0)\}$$ and therefore $$F(h_1, P_1) \geq R_0, F\left(h_2, P_2\right) < R_0$$

3) $$S_3$$: Transmission on channel 2 $$C_3 = \{(0, R_0), (0, 0)\}$$ and $$F(h_1, P_1) < R_0, F\left(h_2, P_2\right) \geq R_0$$

4) $$S_4$$: Transmission is possible individually on both channels AND simultaneously on both channels i.e $$C_4 = \mathcal{R} \times \mathcal{R} = \{(0, 0), (R_0, 0), (0, R_0)\}$$ and $$F(h_1, P_1) \geq R_0, i = 1, 2$$, $$F(h_2, P_2) \geq R_0$$ can be achieved with two possible decoding orders. If user 1 is decoded first, user 2 transmits at the minimum power required to decode a codeword of rate $$R_0$$ so that the user causes the minimum possible interference. Therefore, the condition for successful decoding of $$(R_0, R_0)$$ with user 1 decoded first can be written as:

$$\frac{P_1}{1 + h_2^2 P_{\text{min}}} \geq R_0$$

Similarly, if user 2 is decoded first, the condition for successful decoding is:

$$\frac{P_2}{1 + h_1^2 P_{\text{min}}} \geq R_0$$

where $$F(h_1, P_{\text{min}}) = R_0$$ and $$F(h_2, P_{\text{min}}) = R_0$$. Therefore, the condition for $$\vec{h}$$ to lie in $$S_4$$ can be described by the following equations $$F(h_1, P_1) \geq R_0, i = 1, 2$$, $$F(h_1, P_{\text{min}}) \geq R_0$$, or $$F(h_2, P_{\text{min}}) \geq R_0$$.

5) $$S_5$$: Transmission is possible individually in both channels, but simultaneous transmission is not possible. i.e $$C_5 = \{(0, 0), (R_0, 0), (0, R_0)\}$$ and $$F(h_1, P_i) \geq R_0, i = 1, 2$$, $$F(h_1, P_{\text{min}}) < R_0$$, $$F(h_2, P_{\text{min}}) < R_0$$.

Figure 3 shows the partitioning $$S_i$$ of the $$(h_1, h_2)$$ space and the sets of supported rates $$C_i$$ corresponding to those partitions.

The stability region of this network can be found analytically from the definition in (1) and is plotted in Fig. 5 for the Rayleigh fading MAC with $$R_0 = 1$$ and power constraints $$P_1$$ and $$P_2$$ satisfying $$P_1 + P_2 = 2$$. (The variance of the channel fade is assumed to be unity). It can be shown that as long as the probabilities $$\pi(S_i)$$ are greater than zero, the stability region is a pentagon.

B. Fixed codebook broadcast network

The two user broadcast network with fixed codebooks of rate $$R_0$$ and the transmit power governed by $$P(t) \leq P$$ is handled similar to the MAC. We refer the reader to [6] for a description of the partitioning in terms of $$h_1, h_2$$. Figure 5 has
VI. STABILITY REGIONS OF MAC AND BC NETWORKS WITH AVERAGE POWER CONSTRAINTS

The stability region of the networks are convex, since a centralized scheduler can use time division multiplexing to stabilize any finite convex combination of arrivals vectors in the stability region. Therefore finding the boundary of the stability region is equivalent to maximizing \(< \vec{w}, \vec{R} >\), \(\forall 0 < w \leq 1\), under the given power constraint, where \(\vec{R} = (\vec{R}_1, \vec{R}_2)\) represents the average rate vector in the stability region, \(\vec{w} = (w, 1 - w)\), and \(< \vec{x}, \vec{y} >\) represents the dot product of vectors \(x\) and \(y\). Note that this formulation is due to the convex nature of the stability region and is independent of the form of the power constraint. In general, the optimal rate allocation strategy may involve transmitting different rate vectors (possibly randomly with different probabilities) at a particular channel state. For example, in the fixed power case, \(P = 2\) and \(R_0 = 1\), it was optimal to multiplex between \((R_0, 0)\) and \((0, R_0)\) when the channel state was in \(S_5\). However in networks with average power constraints, if the channel comes from an continuous state space, the optimal strategy transmits a single unique rate vector at a given channel state. We state this formally below.

Lemma 6.1: In MAC and BC networks with average power constraints, if the components of \(\vec{h}\) take values from an continuous state space, the optimal rate allocation strategy that maximizes \(< \vec{w}, \vec{R} >\) associates with a channel state \(\vec{h}\) a unique rate vector \(\vec{R}(\vec{h}) \in \mathcal{R}_1 \times \mathcal{R}_2\). And therefore, the optimization problem may be expressed as

\[
\max_{\vec{R}(\vec{h}) \in \mathcal{R}_1 \times \mathcal{R}_2} \int_{\vec{h}} < \vec{w}, \vec{R}(\vec{h}) > f(\vec{h})d\vec{h}
\]

\[
s.t \int_{\vec{h}} P_i(\vec{h})f(\vec{h})d\vec{h} \leq P_i, i \in \{\text{set of transmitting nodes}\}
\]

and \(\vec{R}(\vec{h})\) can be reliably transmitted at channel state \(\vec{h}\) with powers \(P_i(\vec{h})\).


We now treat the MAC and BC networks separately. (In the discussion below, we indicate the channel state by the power of the fade as \(\vec{x} = \vec{h} \circ \vec{h}\) where \(\vec{a} \circ \vec{b}\) indicates the Hadamard product between vectors \(\vec{a}\) and \(\vec{b}\).)

A. Fixed codebook MAC Network

In the MAC network, for a given channel state, a rate vector may be achieved with different transmit powers based on the decoding order. Let us denote two possible decoding orders of a two-user MAC network by \(\Pi = \{\pi_1, \pi_2\}\). \(\pi_1\) represents the order of decoding user 1 first and then user 2, and \(\pi_2\), vice-versa. Let \(P_{i}^{\pi_j}(\vec{r}, \vec{\chi}), i = 1, 2\) denote the transmit power required at user \(i\) to reliably transmit rate vector \(\vec{r}\) at channel state \(\vec{\chi}\) when the decoding order is fixed at \(\pi_j\). The solution to the optimal rate and power allocation problem is given by

\[
(\vec{R}^*(\vec{\chi}), \pi^*(\vec{\chi})) = \arg \max_{\vec{r} \in \mathcal{R}_1 \times \mathcal{R}_2, \pi \in \Pi} \{< \vec{w}, \vec{r} >
\]

\[
-\kappa_1 P_{\vec{r}}^\pi(\vec{r}, \vec{\chi}) - \kappa_2 P_{\vec{r}}^\pi(\vec{r}, \vec{\chi})\}
\]

with the Lagrangian multipliers \(\kappa_1\) and \(\kappa_2\) chosen so that the power constraints of the system are met.

For example, when the channel state is \(\vec{h}\), the optimal scheduler would choose, the maximum of the following:

1) \(0\) (No transmission)

2) \(wR_0 - \kappa_1(\frac{2R_0 - 1}{h^2_1})\) (Transmission only along channel 1)

3) \((1 - w)R_0 - \kappa_2(\frac{2R_0 - 1}{h^2_2})\) (Transmission along channel 2)

4)

\[
R_0 - \kappa_1(\frac{2R_0 - 1}{h^2_1}) - \kappa_2(\frac{2R_0 - 1}{h^2_2})
\]

\((\text{channel } 1)\) is transmitted with user 2 decoded first

5)

\[
R_0 - \kappa_2(\frac{2R_0 - 1}{h^2_2}) - \kappa_1(\frac{2R_0 - 1}{h^2_1})
\]

\((\text{channel } 2)\) is transmitted with user 1 decoded first

Figure 4 shows the mapping of the optimal rate to the channel state for \(w = 0.5\). (Note: for \(w = 0.5\), \(\kappa_1 = \kappa_2\) because of symmetry). Note that the boundaries between these channel states are linear in \(\vec{x}^{-1}\) - the Hadamard reciprocal of \(\vec{x}\).

The stability region of a fixed codebook MAC networks with \(R_0 = 1\) and average power constraints \(P_1, P_2\) satisfying \(P_1 + P_2 = 2\) are found by performing the above optimization for all \(0 \leq w \leq 1\), and presented in Figure 6.
B. Fixed codebook BC network

In the broadcast network, the channel state imposes a decoding order. Therefore, for a given channel state $\chi$, the optimal rate allocation is given by

$$\tilde{R}^*(\chi) = \arg \max_{\vec{r} \in R_1 \times R_2} \{ <\vec{w}, \vec{r}> - \kappa P(\vec{r}, \vec{h}) \}$$

with $\kappa$ chosen so that the transmitter power constraint is met. The optimum scheduler for a given channel state is constructed in a manner similar to the MAC network. We refer the reader to [6] for the details. The stability region of a fixed codebook BC network with $R_0 = 1$ and $E[P(t)] \leq 2$ is presented in Figure 6.

VII. DUALITY RELATIONSHIPS BETWEEN STABILITY REGIONS OF THE MAC AND BC NETWORKS

As discussed in the introduction of this paper, the duality relationship between the information theoretic capacity regions of MAC and BC channels in [3] provides a motivation to explore similar results with our system model. In the discussion that follows, a MAC network is defined to be a dual of a BC network with identical number of users if

1) The power constraints of the MAC and BC networks have the same form i.e. either both have average power constraints or peak power constraints. Furthermore, the maximum transmit power (average or peak, as the case may be) of the broadcast network is equal to the sum of all the maximum transmit powers of all users of the MAC network (For example, the dual MAC network of a two user broadcast network with power constraints $P(t) \leq P$ has power constraints $P_1(t) \leq P_1$ and $P_2 \leq P_2$ such that $P_1 + P_2 = P$)

2) Their channel states have identical statistics and the noise variance at all the receivers in the BC network are identical to the noise variance at the receiver in the MAC.

3) The set of rates of the codebooks available at all the MAC users are identical to each other and are identical to the set of rates at the transmitter in the broadcast channel.

Note that for a given BC network, there exist infinitely many MAC networks.

**Theorem 7.1:** The union of stability regions of all MAC networks that are dual to a particular BC network with peak power constraint is strictly contained in the stability region of that BC network. Refer [6] for a proof.

**Theorem 7.2:** The stability region of a broadcast network with an average transmit power constraint is equal to the union of the stability regions of all its dual MAC networks. Interestingly, the duality result holds in spite of restricting our system to finite rate sets and disallowing partial codeword decoding at the receivers. Refer [6] for a proof.

The plots of stability regions of BC and dual MAC networks with peak and average power constraints (figures 5 and 6) confirm the results established in Theorem 7.1 and Theorem 7.2.

**VIII. CONCLUSION**

We have characterized the stability regions of multi-rate multiple access and broadcast networks with both average and peak power constraints. In the process of doing so, we observed some interesting implications of the finite rate set assumption. The stability regions of MAC and BC networks with average power constraints satisfy a duality relationship similar to the one between information theoretic capacity regions of these networks. Interestingly, the duality holds in spite of restricting users to finite rate-sets and disallowing rate splitting. An interesting area of future work is the design of the optimal finite rate set for the MAC and BC networks.

**REFERENCES**


