Vector MAC Capacity Region with Covariance Feedback

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Introduction

- Single user MIMO channel capacity with partial CSIT and perfect CSIR studied extensively.
  - Visotsky & Madhow ISIT'00 - Optimal input distributions for MISO.
  - Jafar & Goldsmith ICC'01 - Optimal input distributions for MIMO, beamforming etc.
- MIMO MAC system capacity with partial CSIT and perfect CSIR unexplored.

CSIT/CSIR : Channel State Information at Transmitter/Receiver.
MIMO/MISO:Multiple Input Multiple/Single Output.
MAC: Multiple Access Channel.

System Model

MIMO MAC system:

$$\mathbf{y}(k) = \sum_{i=1}^{K} \mathbf{H}_i(k)\mathbf{x}_i(k) + \mathbf{n}(k)$$

Covariance Feedback:

- Each user's channel $\mathbf{H}_i$ known at each instant to the common receiver.
- All spatial correlations known to each transmitter.

Channel Fade Correlations

Across users channel fades are uncorrelated.

For each user, channel fades associated with different antennas decorrelate as

- separation between antennas increases.
- density of scatterers in the vicinity increases.

Typically, mobile is surrounded by more scatterers but also has a more stringent size constraint than the BS.

BS : Base Station.

MIMO Fading Models

- **Model 1:** I.i.d. columns and correlated rows.

$$H_i[\cdot c] \sim N(\mathbf{0}, \Sigma_i).$$

Correlated receive antennas at the BS and uncorrelated transmit antennas at mobile.

- **Model 2:** I.i.d. rows and correlated columns.

$$H_i[r \cdot] \sim N(\mathbf{0}, \Sigma_i).$$

Uncorrelated receive antennas at the BS and correlated transmit antennas at mobile.

- **Special Case:** I.i.d. rows & columns. $\Sigma_i = \mathbf{I}$.

Gaussian MAC Capacity Region

- Optimal input distribution for each user is zero mean, vector Gaussian.
- Capacity region can be derived as

$$\mathrm{CO}_{\mathcal{Q}}\left\{\mathbf{R} : \sum_{i\in S} R_i \leq \mathrm{E}_{\mathbf{H}}\left[\frac{1}{2}\log\left|\sum_{i\in S}\mathbf{H_i}\mathbf{Q_i}\mathbf{H_i^\dagger} + \mathbf{I}\right|\right]\right\}$$

$$\forall \;\; S \subset \{1, 2, \ldots, K\}$$

Objectives

We need to determine

- optimal successive decoding order
- shape of capacity region
- optimal input covariance matrix for each user

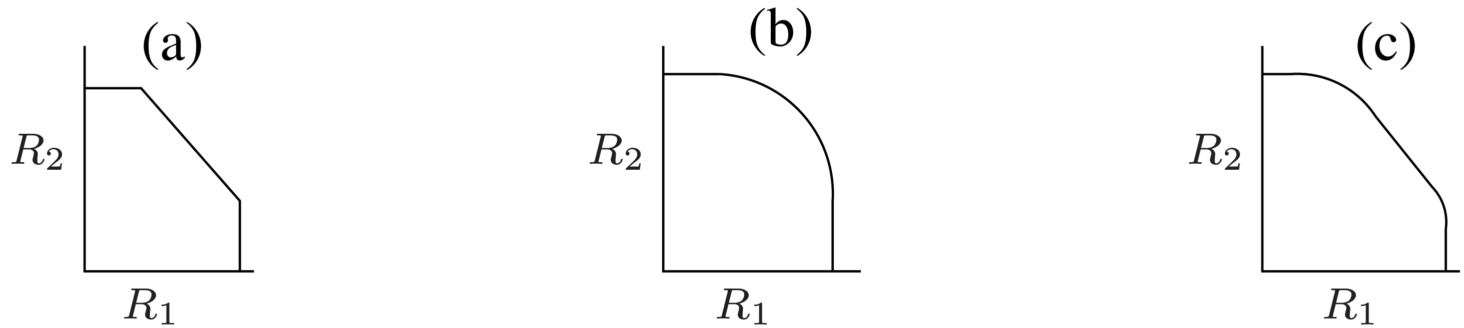
Decoding order for Boundary points

- Boundary points of the capacity region obtained by maximizing $\boldsymbol{\mu}.\mathbf{R}$.
- The associated successive decoding order depends only on the ordering of $\boldsymbol{\mu}$
  - for all $\boldsymbol{\mu}$ such that $\mu_1 < \mu_2 \cdots < \mu_K$, decoding order: user 1$\rightarrow$ user 2 $\rightarrow \cdots \rightarrow$ user K.
  - independent of fade correlations

Decoding Order : Sketch of Proof

- Consider 2 users, $\mu_1 < \mu_2$ and $\boldsymbol{\mu}.\mathbf{R}$ maximized with Q_1, Q_2.
- With **Perfect** CSIT and CSIR and **fixed** Q_1, Q_2.
  - Capacity region is a pentagon for each channel state.
  - Optimal decoding order $1 \to 2$ is independent of channel state.
- Covariance Feedback: Decoding order 1$\to$2 achieves the same R_1, R_2. Can't do better than perfect CSIT.

MAC Capacity Regions



(a) Fading scalar MAC with no CSIT and perfect CSIR. Also non-fading scalar MAC.

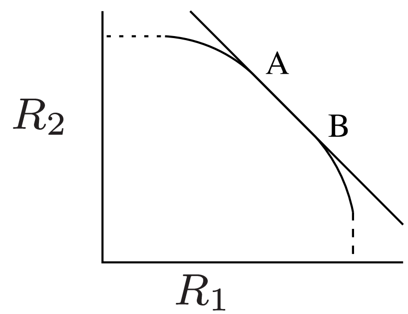
(b) Fading scalar MAC with perfect CSIT and perfect CSIR.

(c) Non fading vector (MIMO) MAC channel.

Fading MIMO MAC Capacity Region

- What is the general shape of the MIMO MAC capacity region with covariance feedback ?
- What is the shape for some specific fade correlation models?

Results : Shape of Capacity Region

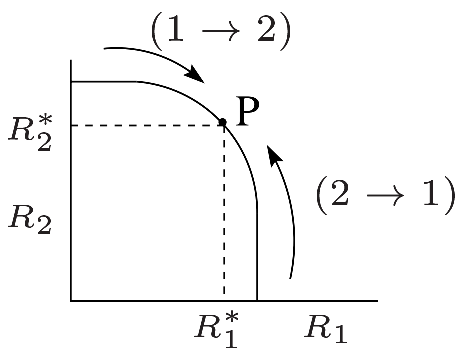


- Boundary traced by $\boldsymbol{\mu} > 0$ is not strictly convex.
  - Sum rate maximizing point is not unique.

Sum Rate Maximizing Points : Proof

Proof by contradiction

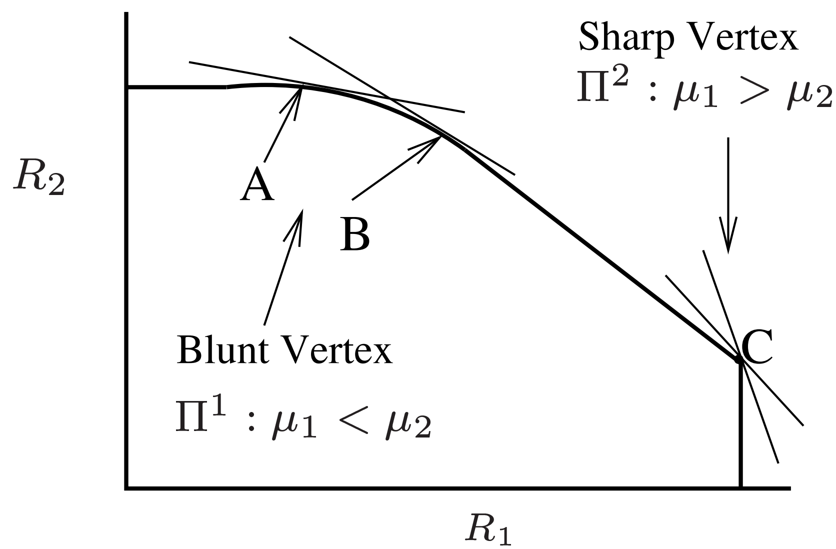
- **Suppose sum rate point is unique**, with optimum transmit covariance matrices Q_1, Q_2.



Must have $R_1^*(1 \to 2) = R_1^*(2 \to 1)$.

Leads to a contradiction.

Region Vertices



- Sharp Vertex : All $\boldsymbol{\mu}$ such that $\mu_1 > \mu_2$ lead to the same unique boundary point C.
- Otherwise : Blunt Vertex.

Shape of Vertices : Sharp or Blunt

- For 2 users, and $\mu_1 > \mu_2 >$, $\boldsymbol{\mu}.\mathbf{R}$ becomes

$$(\mu_1 - \mu_2) \quad \mathrm{E} \quad \left[\log\left|I + H_1 Q_1 H_1^\dagger\right|\right] +$$
$$\mu_2 \quad \mathrm{E} \quad \left[\log\left|I + H_1 Q_1 H_1^\dagger + H_2 Q_2 H_2^\dagger\right|\right]$$

- In general optimal Q_i depend on $\boldsymbol{\mu}$ (Blunt vertices).

If each term is maximized by same Q_i, then optimal Q_i is independent of $\boldsymbol{\mu}$ and the decoding order (Sharp vertices).

Key Lemmas

Consider $f(\mathbf{Q}) = \mathbb{E}_{\mathbf{H}} \log |\mathbf{A} + \mathbf{H}\mathbf{Q}\mathbf{H}^{\dagger}|$, where trace$(\mathbf{Q}) \leq P$ and $\mathbf{A}$ is *any* positive semidefinite matrix.

- If the columns of $\mathbf{H}$ are i.i.d., $(\mathbf{H}[\cdot c] \sim N(\mathbf{0}, \Sigma_{\mathbf{H}}))$ then $f(\mathbf{Q})$ is maximized when $\mathbf{Q} = \frac{P}{n}\mathbf{I}$.
- If the rows of $\mathbf{H}$ are i.i.d., $(\mathbf{H}[r\cdot] \sim N(\mathbf{0}, \Sigma_{\mathbf{H}}))$, then $f(\mathbf{Q})$ is maximized when $\mathbf{Q}$ has the same eigenvectors as $\mathbf{\Sigma}_{\mathbf{H}}$.

Optimal Q_i and Shape of Capacity Region

- If i^{th} user's channel $\mathbf{H}_i$ consists of i.i.d. fades, his optimal $Q_i = \frac{P_i}{n}\mathbf{I}$ for all $\boldsymbol{\mu}$ and for all correlations of the rest of the users' channels.
- If all users see i.i.d. fades, capacity region is a polytope.
- If $\mathbf{H}_i$ has i.i.d columns (uncorrelated transmit antennas), $Q_i = \frac{P_i}{n}\mathbf{I}$ for all $\boldsymbol{\mu}$ and for all correlations of the rest of the users.
- If all channels have i.i.d. columns, capacity region is a polytope.

Optimal Q_i and Shape of Capacity Region

- If $\mathbf{H}_i$ has i.i.d rows (uncorrelated receive antennas), then for all $\boldsymbol{\mu}$ and for all correlations of the rest of the users Q_i, Σ_i have the same eigenvectors.
- General Correlated Channels
  - Capacity region has blunt vertices in general.
  - In special cases, it can be a polytope, but cannot be strictly convex.

Disparate-Modes Channels

Disparate-modes channels: Channels for which single user capacity may be achieved with a unit rank transmit covariance matrix Q. (ISIT'01).

- If user i in vector MAC has a disparate-modes channel, then optimal Q_i has unit rank for all $\boldsymbol{\mu}$ and all correlation matrices of other users.
- If every user in vector MAC has disparate modes-channel, capacity region is a polytope.

Conclusions

For the MIMO MAC channel with covariance feedback,

- the optimum decoding order for capacity region boundary points inverse of $\boldsymbol{\mu}$ ordering.
- general shape of capacity region defined by non-unique rate sum point and shape of vertices.
- Capacity region vertices blunt for general channels.
- For independent fades on mobile antennas, can completely characterize optimal input distribution and the capacity region vertices are sharp.
- For independent fades on BS antennas, can partially characterize optimal input distribution.