Multiuser MIMO degrees of freedom with no CSIT

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***Abstract*— We provide the characterization of the degrees of freedom (DOF) region for a 2-user fading MIMO broadcast channel when perfect channel knowledge is available to the receivers and no channel knowledge is available to the transmitters. The results are applied to find the DOF region for some special cases of a 2-user MIMO interference channel. We also extend the outerbound of the DOF region to find the capacity region for a specific 2-user MIMO broadcast channel.**

I. INTRODUCTIONS

Multiple-input-multiple-output (MIMO) systems are capable of significantly higher capacity compared to traditional single-input-single-output systems. One of the key features of MIMO systems is the possibility of multiplexing signals in space. The ability of multiplexing signals in space is measured by spatial multiplexing gain [1], which is called capacity prelog or degrees of freedom (DOF). For the point-to-point MIMO communication systems, it has been shown that the availability of the channel state information at the transmitter (CSIT) does not affect the spacial multiplexing gain [2].

Unlike the point-to-point case, in a network with distributed processing units, it is well known [3], [4] that in the absence of channel knowledge, spatial multiplexing gain is lost. For example, the DOF of the fading multiple-input-single-output broadcast channel with M antennas at the transmitter and 1 antenna at all M receivers is M when perfect channel state information is available at the transmitter (perfect CSIT) [5], [6], [7]. However, the DOF of the same system is only 1 when channel state information is not available at the transmitter (no CSIT) [3]. Further understanding about the availability of channel knowledge and its effect on the degrees freedom of the networks can provide insights into the design and optimization of the wireless networks. A natural goal is to extend the results in the previous example to a more general MIMO broadcast channel with arbitrary number of users and antennas. In this paper, we make progress on this problem by studying the DOF region of a 2-user MIMO broadcast channel where the transmitter is equipped with M antennas and receivers $1, 2$ are equipped with N_1, N_2 antennas, respectively, under the assumption of perfect channel state information at the receivers (perfect CSIR) and no CSIT. An exact characterization of the DOF region of the channel is given, and our result shows that a simple time division between the two users is DOF-region optimal. We then use the result of the MIMO broadcast channel to find the DOF region for some special cases of a 2-user MIMO interference channel under the same channel state information setting. We also extend the outerbound of the DOF region to find the capacity region for a specific 2-user MIMO broadcast channel.

II. SYSTEM MODEL

Consider the 2-user Gaussian MIMO broadcast channel where the transmitter is equipped with M antennas and receivers $1, 2$ are equipped with N_1, N_2 antennas, respectively. The channel is described by the input-output relationship:

$$\mathbf{Y}^{[1]}(t) = \mathbf{H}^{[1]}(t)\mathbf{X}(t) + \mathbf{Z}^{[1]}(t) \quad (1)$$
$$\mathbf{Y}^{[2]}(t) = \mathbf{H}^{[2]}(t)\mathbf{X}(t) + \mathbf{Z}^{[2]}(t) \quad (2)$$

where at the t^{th} channel use, $\mathbf{Y}^{[i]}(t)$, $\mathbf{Z}^{[i]}(t)$ are the $N_i \times 1$ vectors representing the channel output and additive white Gaussian noise at receiver i, $\mathbf{H}^{[i]}(t)$ is the $N_i \times M$ channel matrix corresponding to receiver i, $i \in \{1, 2\}$, and $\mathbf{X}(t)$ is the $M \times 1$ input vector. The elements of $\mathbf{H}^{[i]}(t)$ and $\mathbf{Z}^{[i]}(t)$, $i = 1, 2$, are independent identically distributed circularly symmetric complex Gaussian random variables with zero mean and unit variance. We assume perfect CSIR, i.e., each receiver has perfect knowledge of all channel matrices at each instant, and no CSIT, i.e., the transmitter has no knowledge of the instantaneous values taken by the channel coefficients. To avoid cumbersome notation, we will henceforth suppress the channel use index t where it does not cause ambiguity.

The transmit power constraint is expressed as:

$$\mathrm{E}[||\mathbf{X}||^2] \leq P. \quad (3)$$

There are two independent messages W_1, W_2, associated with rates R_1, R_2, to be communicated from the transmitter to receivers $1, 2$, respectively. The capacity region $C(P)$ is the set of all rate pairs (R_1, R_2) for which the probability of error can be driven arbitrarily close to zero by using suitably long codewords. The degrees of freedom region is defined as follows:

$$\mathbf{D} \triangleq \Big\{(d_1, d_2) \in \mathbb{R}_2^+ : \exists (R_1(P), R_2(P)) \in C(P) \text{ s.t. } d_i = \lim_{P\to\infty} \frac{R_i(P)}{\log(P)}, i = 1, 2.\Big\}. \quad (4)$$

III. Degrees of Freedom of MIMO BC with No CSIT

Theorem 1: The degrees of freedom region of the MIMO BC with no CSIT, as defined in Section II is the following:

$$\mathbf{D} = \left\{ (d_1, d_2) \in \mathbb{R}_2^+ : \frac{d_1}{\min(M,N_1)} + \frac{d_2}{\min(M,N_2)} \leq 1 \right\}. \tag{5}$$

Proof: Without loss of generality, let us assume $N_1 \leq N_2$. The case where $M \leq N_1 \leq N_2$ is trivial, because in this case the degrees of freedom region, even with perfect CSIT, is given by:

$$\mathbf{D} = \{(d_1, d_2) \in \mathbb{R}_2^+ : d_1 + d_2 \leq M\} \tag{6}$$

which is clearly achievable even without CSIT, by simple time-division between the two users.

For the remainder of this section, we consider $M \geq N_1$.

Since the MIMO BC with no CSIT, as defined above, falls in the class of degraded broadcast channels [8], [9], its capacity region $C(P)$ is the set of rate pairs (R_1, R_2) given by:

$$\begin{aligned} R_1 &\leq I(U; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}) & (7)\\ &= I(\mathbf{X}; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}) - I(\mathbf{X}; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U) & (8)\\ R_2 &\leq I(\mathbf{X}; \mathbf{Y}^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U) & (9) \end{aligned}$$

where $U \rightarrow \mathbf{X} \rightarrow (\mathbf{Y}^{[1]}, \mathbf{Y}^{[2]})$ forms a Markov chain. Since the channel between the transmitter and receiver 1 cannot have more than $\min(M, N_1) = N_1$ degrees of freedom, we have:

$$I(\mathbf{X}; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}) \leq N_1 \log(P) + o(\log(P)). \tag{10}$$

Let us define r as the degrees of freedom for the term $I(\mathbf{X}; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U)$, i.e.,

$$I(\mathbf{X}; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U) = r \log(P) + o(\log(P)). \tag{11}$$

From (11) we obtain the following useful inequality:

$$\begin{aligned} &I(\mathbf{X}; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U)\\ &\quad = \sum_{i=1}^{N_1} I(\mathbf{X}; \mathbf{Y}_i^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U, \mathbf{Y}_{(i+1:N_1)}^{[1]})\\ &\quad \geq N_1 I(\mathbf{X}; \mathbf{Y}_1^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U, \mathbf{Y}_{(2:N_1)}^{[1]}). \end{aligned} \tag{12}$$

Equation (12) implies that

$$\frac{r}{N_1} \log(P) + o(\log(P)) \geq I(\mathbf{X}; \mathbf{Y}_1^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U, \mathbf{Y}_{(2:N_1)}^{[1]}). \tag{13}$$

Now we can write the upperbound (9) for R_2 as:

$$\begin{aligned} &R_2\\ &\leq I(\mathbf{X}; \mathbf{Y}^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U)\\ &= I(\mathbf{X}; \mathbf{Y}_{(1:\min(M,N_2))}^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U)\\ &\quad + I(\mathbf{X}; \mathbf{Y}_{(\min(M,N_2)+1:N_2)}^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U, \mathbf{Y}_{(1:\min(M,N_2))}^{[2]})\\ &= I(\mathbf{X}; \mathbf{Y}_{(1:\min(M,N_2))}^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U) + o(\log(P))\\ &= I(\mathbf{X}; \mathbf{Y}_{(1:N_1)}^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U) + o(\log(P))\\ &\quad + I(\mathbf{X}; \mathbf{Y}_{(N_1+1:\min(M,N_2))}^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U, \mathbf{Y}_{(1:N_1)}^{[2]})\\ &= I(\mathbf{X}; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U) + o(\log(P))\\ &\quad + \sum_{i=N_1+1}^{\min(M,N_2)} I(\mathbf{X}; \mathbf{Y}_i^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U, \mathbf{Y}^{[1]}, \mathbf{Y}_{(N_1+1:i-1)}^{[2]})\\ &\leq r \log(P) + o(\log(P))\\ &\quad + \sum_{i=N_1+1}^{\min(M,N_2)} I(\mathbf{X}; \mathbf{Y}_i^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U, \mathbf{Y}_{(2:N_1)}^{[1]})\\ &= r \log(P) + o(\log(P))\\ &\quad + (\min(M, N_2) - N_1) I(\mathbf{X}; \mathbf{Y}_1^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U, \mathbf{Y}_{(2:N_1)}^{[1]})\\ &\leq r \log(P) + (\min(M, N_2) - N_1) \frac{r}{N_1} \log(P) + o(\log(P))\\ &= \frac{\min(M, N_2)}{N_1} r \log(P) + o(\log(P)) \end{aligned} \tag{14}$$

Thus, for $0 \leq r \leq N_1$, an outerbound on the boundary of the degrees of freedom region is characterized as follows.

$$(d_1, d_2) = \left(N_1 - r, \frac{\min(M, N_2)}{N_1} r \right), \tag{15}$$

which implies that

$$\mathbf{D} \subset \left\{ (d_1, d_2) \in \mathbb{R}_2^+ : \frac{d_1}{N_1} + \frac{d_2}{\min(M, N_2)} \leq 1 \right\}. \tag{16}$$

Achievability of this outerbound follows trivially by time division between the two users, and the proof of Theorem 1 is complete. ∎

IV. Degrees of Freedom of the MIMO Interference Channel with No CSIT

A. System Model

Consider the 2-user Gaussian MIMO interference channel where transmitters $1, 2$ are equipped with M_1, M_2 antennas, respectively, and receivers $1, 2$ are equipped with N_1, N_2 antennas, respectively. The channel is described by the input-output relationship:

$$\mathbf{Y}^{[1]}(t) = \mathbf{H}^{[11]}(t)\mathbf{X}^{[1]}(t) + \mathbf{H}^{[12]}(t)\mathbf{X}^{[2]}(t) + \mathbf{Z}^{[1]}(t) \tag{17}$$

$$\mathbf{Y}^{[2]}(t) = \mathbf{H}^{[21]}(t)\mathbf{X}^{[1]}(t) + \mathbf{H}^{[22]}(t)\mathbf{X}^{[2]}(t) + \mathbf{Z}^{[2]}(t) \tag{18}$$

where at the t^{th} channel use, $\mathbf{Y}^{[j]}(t)$, $\mathbf{Z}^{[j]}(t)$ are the $N_j \times 1$ vectors representing the channel output and additive white Gaussian noise at receiver j, $\mathbf{H}^{[ji]}(t)$ is the $N_j \times M_i$ channel matrix corresponding to receiver j, and $\mathbf{X}^{[i]}(t)$ is the $M_i \times 1$ input vector, $i, j \in \{1, 2\}$. The following assumptions are similar to those in Section II. The elements of $\mathbf{H}^{[ji]}(t)$ and $\mathbf{Z}^{[j]}(t)$, $i, j \in \{1, 2\}$, are independent identically distributed circularly symmetric complex Gaussian random variables with zero mean and unit variance. We assume perfect CSIR and no CSIT.

The transmit power constraint is expressed as:

$$\mathrm{E}[||\mathbf{X}^{[i]}||^2] \leq P, \quad i = 1, 2. \tag{19}$$

There are two independent messages W_1, W_2, associated with rates R_1, R_2, to be communicated from the transmitter 1 to receiver 1 and from the transmitter 2 to receiver 2, respectively. The standard definitions of the capacity region and the DOF region are the same with those in Section II.

B. Main Results

Theorem 2: If $M_1 \leq N_2$ and $M_2 \leq N_1$, the degrees of freedom region of the MIMO interference channel with no CSIT is the following:

$$\mathbf{D} = \left\{ (d_1, d_2) \in \mathbb{R}_2^+ : \begin{array}{l} d_1 \leq \min(M_1, N_1), \\ d_2 \leq \min(M_2, N_2), \\ d_1 + d_2 \leq \min(N_1, N_2) \end{array} \right\} \quad (20)$$

Proof: Let a genie provide the transmitters with perfect channel state information. Since giving CSIT does not hurt, the converse argument is still valid. Then, the outerbound follows directly from the results of [10]. Achievability of this outerbound follows trivially by receiver zeroforcing. ■

Theorem 3: If $M_1 \geq N_1$ and $M_2 \geq N_2$, the degrees of freedom region of the MIMO interference channel with no CSIT is the following:

$$\mathbf{D} = \left\{ (d_1, d_2) \in \mathbb{R}_2^+ : \frac{d_1}{N_1} + \frac{d_2}{N_2} \leq 1 \right\} \quad (21)$$

Proof: Let a genie provide transmitter 1 with W_2 and transmitter 2 with W_1. Since the resulting channel is equivalent to a broadcast channel, the outerbound follows directly from Theorem 1. Achievability of this outerbound follows trivially by time division between the two users, and the proof is complete. ■

V. Capacity Region of a Class of Broadcast Channels with No CSIT

A. Models

Consider the 2-user Gaussian MIMO broadcast channel where the transmitter is equipped with M antennas and receivers $1, 2$ are equipped with N_1, N_2 antennas, respectively. M, N_1, and N_2 are assumed to satisfy

$$N_1 \leq M \quad (22)$$
$$N_2 \leq M. \quad (23)$$

The channel is described by the input-output relationship:

$$\mathbf{Y}^{[1]}(t) = \mathbf{H}^{[1]}\mathbf{Q}(t)\mathbf{X}(t) + \mathbf{Z}^{[1]}(t) \quad (24)$$
$$\mathbf{Y}^{[2]}(t) = \mathbf{H}^{[2]}\mathbf{Q}(t)\mathbf{X}(t) + \mathbf{Z}^{[2]}(t) \quad (25)$$

where the notation usage for $\mathbf{X}^{[i]}(t)$ and $\mathbf{Y}^{[i]}(t)$, the assumption for the noise term $\mathbf{Z}^{[i]}(t)$, and the assumption that the channel is equipped with perfect CSIR and no CSIT are the same with those in Section II. However, different from the previous assumption, $\mathbf{H^{[i]}}$ is assumed to be a time-invariant $N_i \times M$ channel matrix with N_i orthonormal rows, $i \in \{1, 2\}$. Note that this is possible only when $N_1 \leq M$ and $N_2 \leq M$. $\mathbf{Q}$ is an $M \times M$ isotropically random unitary matrix, normalized so that

$$\mathrm{E}[\mathbf{Q}\mathbf{Q}^\dagger] = M\mathbf{I}, \quad (26)$$

where $\mathbf{I}$ is a $M \times M$ identity matrix. The transmit power constraint and the standard definition of the capacity region are the same with those in Section II and we omit them for brevity.

B. Main Result

Theorem 4: The capacity region of the MIMO BC with no CSIT, as defined in Section V-A is the following:

$$\mathbf{C} = \left\{ (R_1, R_2) \in \mathbb{R}_2^+ : \frac{R_1}{N_1} + \frac{R_2}{N_2} \leq \log(1+P) \right\}. \quad (27)$$

Proof: The proof follows the similar lines in the proof of Theorem 1 and we omit the parts that are the same with those given in Section III for brevity. Without loss of generality, let us assume $N_1 \leq N_2$. Following (7), we have

$$R_1 \leq I(\mathbf{X}; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}) - I(\mathbf{X}; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U), \quad (28)$$

where $U \rightarrow \mathbf{X} \rightarrow (\mathbf{Y}^{[1]}, \mathbf{Y}^{[2]})$ forms a Markov chain. Denote the capacity of the point-to-point link from the transmitter to receiver 1 as C_1 and let

$$\gamma = I(\mathbf{X}; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U). \quad (29)$$

Following (28), we have

$$\begin{aligned} R_1 &\leq C_1 - \gamma \\ &= \mathrm{E}_{\mathbf{Q}} \log \left| \mathbf{I} + \mathbf{H}^{[1]}\mathbf{Q}(\frac{P}{M}\mathbf{I})\mathbf{Q}^\dagger {\mathbf{H}^{[1]}}^\dagger \right| - \gamma \\ &= \log \left| \mathbf{I} + \frac{P}{M}\mathbf{H}^{[1]}\mathrm{E}[\mathbf{Q}\mathbf{Q}^\dagger]{\mathbf{H}^{[1]}}^\dagger \right| - \gamma \\ &= \log \left| \mathbf{I} + \frac{P}{M}\mathbf{H}^{[1]}(M\mathbf{I}){\mathbf{H}^{[1]}}^\dagger \right| - \gamma \\ &= \log \left| \mathbf{I} + P\mathbf{H}^{[1]}{\mathbf{H}^{[1]}}^\dagger \right| - \gamma \\ &= N_1 \log(1+P) - \gamma. \end{aligned} \quad (30)$$

Following (12), we have following useful inequality:

$$\frac{1}{N_1} I(\mathbf{X}; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U) \geq I(\mathbf{X}; \mathbf{Y}_1^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U, \mathbf{Y}_{(2:N_1)}^{[1]}). \quad (31)$$

Now, following (9), we can write the upperbound for R_2 as follows.

$$\begin{aligned} R_2 &\leq I(\mathbf{X}; \mathbf{Y}^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U) \\ &= I(\mathbf{X}; \mathbf{Y}_{(1:N_1)}^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U) \\ &\quad + I(\mathbf{X}; \mathbf{Y}_{(N_1+1:N_2)}^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U, \mathbf{Y}_{(1:N_1)}^{[2]}) \\ &= I(\mathbf{X}; \mathbf{Y}^{[1]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U) \\ &\quad + \sum_{i=N_1+1}^{N_2} I(\mathbf{X}; \mathbf{Y}_i^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U, \mathbf{Y}^{[1]}, \mathbf{Y}_{(N_1+1:i-1)}^{[2]}) \\ &\leq \gamma + \sum_{i=N_1+1}^{N_2} I(\mathbf{X}; \mathbf{Y}_i^{[2]} | \mathbf{H}^{[1]}, \mathbf{H}^{[2]}, U, \mathbf{Y}_{(2:N_1)}^{[1]}) \\ &\leq \gamma + \frac{N_2 - N_1}{N_1}\gamma \\ &\leq \frac{N_2}{N_1}\gamma. \end{aligned} \quad (32)$$

Thus, for $0 \leq \gamma \leq N_1 \log(1+P)$, an outerbound on the boundary of the capacity region is characterized as follows.

$$(R_1, R_2) = \left(N_1 \log(1+P) - \gamma, \frac{N_2}{N_1}\gamma \right), \quad (33)$$

which implies that

$$\mathbf{C} \subset \left\{ (R_1, R_2) \in \mathbb{R}_2^+ : \frac{d_1}{N_1} + \frac{d_2}{N_2} \le \log(1+P) \right\}. \quad (34)$$

To provide the achievability of this outerbound, we first prove that $(0, N_2 \log(1+P))$ is achievable. Denote the capacity of the point-to-point link between the transmitter and receiver 2 as C_2, and we have

$$\begin{aligned} C_2 &= \mathrm{E}_{\mathbf{Q}} \log \left| \mathbf{I} + \mathbf{H}^{[2]} \mathbf{Q} (\frac{P}{M} \mathbf{I}) \mathbf{Q}^\dagger {\mathbf{H}^{[2]}}^\dagger \right| \\ &= \log \left| \mathbf{I} + \frac{P}{M} \mathbf{H}^{[2]} \mathrm{E}[\mathbf{Q}\mathbf{Q}^\dagger] {\mathbf{H}^{[2]}}^\dagger \right| \\ &= \log \left| \mathbf{I} + \frac{P}{M} \mathbf{H}^{[2]} (M\mathbf{I}) {\mathbf{H}^{[2]}}^\dagger \right| \\ &= \log \left| \mathbf{I} + P \mathbf{H}^{[2]} {\mathbf{H}^{[2]}}^\dagger \right| \\ &= N_2 \log(1+P). \end{aligned} \quad (35)$$

Thus, $(0, N_2 \log(1+P))$ is achievable. Using time division between the two users, and the proof is complete. ■

VI. Conclusions

In this paper, we explore the effect of the absence of channel state information for MIMO networks. Throughout the paper, we assume perfect CSIR and no CSIT. We provide the characterization of the DOF region for a 2-user MIMO broadcast channel. We then find the DOF region for some special cases of a 2-user MIMO interference channel by using the broadcast channel outerbound. We also extend the outerbound of the DOF region to find the capacity region for a specific 2-user MIMO broadcast channel.

References

[1] L. Zheng and D. N. Tse, "Packing spheres in the Grassmann manifold: A geometric approach to the non-coherent multi-antenna channel,," *IEEE Trans. Inform. Theory*, vol. 48, pp. 359–383, Feb 2002.

[2] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. on Telecomm. ETT*, vol. 10, pp. 585–596, November 1999.

[3] S. Jafar and A. Goldsmith, "Isotropic fading vector broadcast channels: the scalar upperbound and loss in degrees of freedom," *IEEE Trans. Inform. Theory*, vol. 51, pp. 848–857, March 2005.

[4] A. Lapidoth, "On the high-SNR capacity of non-coherent networks," *IEEE Trans. on Information Theory*, vol. 51, pp. 3025–3036, Sep. 2005.

[5] W. Yu and J. Cioffi, "Sum capacity of Gaussian vector broadcast channels," *IEEE Trans. on Information Theory*, vol. 50, pp. 1875–1892, Sept. 2004.

[6] P. Viswanath and D. Tse, "Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality," *IEEE Trans. Inform. Theory*, pp. 1912–1921, Aug 2003.

[7] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of MIMO broadcast channels," *IEEE Trans. Inform. Theory*, pp. 2895–2909, Oct. 2003.

[8] P. Bergmans, "A simple converse for broadcast channels with additive white gaussian noise," *IEEE Trans. Inform. Theory*, vol. 20, pp. 279–280, March 1974.

[9] P. Bergmans, "Random coding theorem for broadcast channels with degraded components," *IEEE Trans. Inform. Theory*, vol. 19, pp. 197–207, March 1973.

[10] S. Jafar and M. Fakhereddin, "Degrees of freedom for the mimo interference channel," *IEEE Transactions on Information Theory*, vol. 53, pp. 2637–2642, July 2007.