Optimal Use of Antennas in Interference Networks: A Tradeoff between Rate, Diversity and Interference Alignment

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***Abstract*—The tradeoff between diversity, interference alignment and rate for a K user multiple-antenna interference network is analyzed. It is assumed that the sources employ a space-time code in combination with linear precoding, while the receiving nodes use linear detectors. We show that interference alignment is needed if the system is operating at or close to the maximum achievable rate. For low rates the preferred strategy is to utilize all antennas in order to achieve high diversity gains, rather than using some of the antennas to align the interference. For the case of $K = 3$ users, an exact characterization of the tradeoff is provided. We also investigate the impact of channel estimation errors on the diversity of the system. It turns out that small channel estimation errors can be tolerated, while larger errors reduce the diversity gain significantly.**

I. Introduction

In recent times, multiple-input multiple-output (MIMO) systems are of high interest due to their capability of significantly increasing the capacity of a system in comparison to their single-input single-output (SISO) counterparts. There exists a huge number of transmission schemes utilizing the benefits of such MIMO systems, ranging from transmit diversity schemes to increase the reliability of the system to schemes which provide high spectral efficiency. Some well known schemes are the Alamouti scheme [1], space-time codes from orthogonal and quasi-orthogonal designs [2], [3], [4], and V-BLAST [5]. It was shown in [6] that there is a fundamental tradeoff in MIMO systems, i.e. maximum reliability and spectral efficiency are not achievable simultaneously. The tradeoff was useful to compare schemes achieving different points on the tradeoff region and for the design of tradeoff-optimal schemes like in [7], [8].

At the receiver side, an appropriate multi-user or multi-stream detection has to be applied in order to decouple the received superposition of the transmitted signals. Hereby, the receiver structures range from linear detection schemes such as matched filtering [9], zero-forcing, MMSE detector to successive interference cancelation [5], sphere decoding, lattice reduction aided detection [10], [11] and the optimal, however highly complex, ML detector. The reduction of receiver complexity is of particular interest for the research community as well as for the industry and ways to do that were explored in e.g. [12] .

Another way of reducing the complexity at both the transmitter and the receiver has been investigated in [13]. There, the transmit antennas were partitioned into groups and for each group a space-time trellis code was employed, while at the receiver multi-user detection strategies were used. Later on, it was shown in [14], [15] that by using OSTBC in a multiple-access channel the number of antennas required at the receiver can be reduced significantly. This was generalized later on to more than two antennas and multiple users in [16], [17]. It was also shown that by using an ML detector, the diversity achieved is the same as if the interference of other users is not present.

The phenomenon of interference itself appears in networks where multiple users are using the same resources, e.g. in adhoc and cellular networks. Thus, interference is and remains a fundamental issue in wireless networks. It was thought that interference would reduce the data rate achievable proportionally to the number of users in the system, until the idea of interference alignment was proposed in [18]. It turns out that in a K user interference network, where each node is equipped with a single antenna and the channels are time-varying, the achievable degrees of freedom (also known as multiplexing gain or pre-log) is given by $K/2$. For systems with M antennas at each node, an upper bound of $KM/2$ degrees of freedom was derived. For the $K = 3$ user case with M antennas at each node, the upper bound was shown to be achievable with interference alignment.

The availability of perfect channel channel state information (CSI), not necessarily global as shown in [19], seems to be of high importance for interference alignment. From a practical point of view, however, it is of interest to analyze channel knowledge imperfections. For example, the impact of imperfect channel knowledge on the capacity for single-user MIMO systems was investigated in [20], [21]. This was generalized in [22], [23] to multi-user systems. The combination of space-time coding and beamforming utilizing imperfect channel knowledge at the transmitter was analyzed in [24], [25], [26], [27], [28].

In this work, we take into account all the aspects discussed above, i.e. multiple antennas and users, space-time coding, simple decoding, interference alignment and imperfect CSI, to determine the achievable diversity in a multi-user multi-

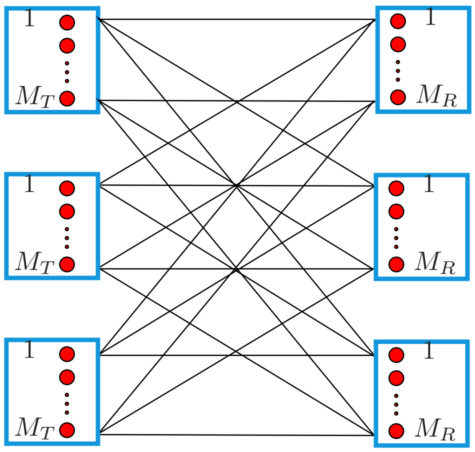

Fig. 1. $K = 3$ User Multi-Antenna Interference Channel

antenna interference channel. In more details, the questions we answer are: Given linear detectors at the destinations, what is the tradeoff between interference alignment, diversity, and rate? How does channel knowledge imperfection affect this tradeoff?

We would like to emphasize that the tradeoff investigated here is different from the tradeoff shown in [6]. Like [29], only finite and fixed constellations are considered here, while [6] is changing the constellation and the code as a function of the signal-to-noise ratio (SNR) in order to operate close to the channel capacity.

The rest of the paper is organized as follows. In the following section, the system model is introduced. The channel adaptive block of the transmit chain is discussed in Subsection II-A, followed by the discussion on the non-adaptive block of the transmit chain in Subsection II-B. The derivation of the diversity and the tradeoff is presented in Section III, followed by the discussion on receiver processing in Subsection III-A. Later on, the impact of imperfect channel knowledge on the diversity is investigated in Section IV, followed by some illustrations in Section V and concluding remarks in Section VI.

II. System Model

Consider a multi-antenna interference channel with K pairs where each transmitter has M_T transmit antennas while each receiver has M_R receive antennas. An example is shown in Fig. 1 for $K = 3$. The interference channel is characterized by the following input-output relationship at time instant t, $1 \leq t \leq T$

$$\mathbf{y}_d^t = \mathbf{H}_{dd}\mathbf{x}_d^t + \sum_{s=1, s\neq d}^{K} \mathbf{H}_{sd}\mathbf{x}_s^t + \mathbf{n}_d^t,$$

where $\mathbf{y}_d$ is the received signal at destination d, $\mathbf{x}_d$ is the desired signal from source d and $\mathbf{x}_s$ is the interference received from the source s. The channel of the desired link is given by $\mathbf{H}_{dd}$, while $\mathbf{H}_{sd}$ describes the channels from the interferer s to receiver d. The additive white complex zero-mean unit variance Gaussian noise at receiver d is denoted by $\mathbf{n}_d$. The received signals and the noises are M_R dimensional vectors,

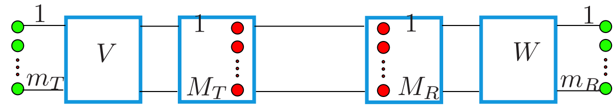

Fig. 2. Precoding at the transmitter with $m_T \leq M_T$ input streams and post-processing at the receiver with $m_R \leq M_R$ output streams

while the transmit signals are M_T dimensional vectors. The channels are $M_R \times M_T$ dimensional matrices. Combining the received vectors from time instant $1 \leq t \leq T$ in a received matrix results in

$$\mathbf{Y}_d = \mathbf{H}_{dd}\mathbf{X}_d + \sum_{s=1, s\neq d}^{K} \mathbf{H}_{sd}\mathbf{X}_s + \mathbf{N}_d, \tag{1}$$

where the dimensions of $\mathbf{Y}_d$ and $\mathbf{X}_s$ are given by $[\mathbf{Y}_d] = M_R \times T$, and $[\mathbf{X}_s] = M_T \times T$, respectively. The transmit signal of user s is given by

$$\mathbf{X}_s = \sum_{n=1}^{N} \mathbf{P}_n^{(s)} c_n^{(s)}, \tag{2}$$

where beamforming and space-time coding is done jointly by the matrices $\mathbf{P}_n$ and the $c_n^{(s)}$ are entries of the data vector $\mathbf{c}_s = [c_1^{(s)}, c_1^{(s)}, \ldots, c_N^{(s)}]^T$ of user s. The entries $c_i^{(s)}$ are drawn from an arbitrary but fixed constellation such as QAM or PSK. The rate r is defined as $r = {}^{N}\!/_{T}$, with $0 \leq r \leq \min(m_T, m_R)$, where m_T, $1 \leq m_T \leq M_T$, and m_R, $1 \leq m_R \leq M_R$, denotes the number of input and output streams as shown in Fig. 2, respectively. It should be mentioned that a more general transmit signal structure than (2) is given in [30]. For simplicity and clarity of presentation, however, the results in the paper are based on the structure in (2).

In the following, we assume that all users employ the same space-time code. Furthermore, for simplicity we assume that

$$\mathbf{P}_n^{(s)} = \mathbf{V}_s\mathbf{G}_n, \tag{3}$$

where $\mathbf{V}_s$ denotes the precoding (or beamforming) matrix and $\mathbf{G}_n$ is the codeword matrix with dimensions given as $[\mathbf{V}_s] = M_T \times m_T$ and $[\mathbf{G}_n] = m_T \times T$, respectively. Note that the space-time code is fully determined by the set of codeword matrices $\{\mathbf{G}_n\}_{n=1}^N$. The combination of precoding, the channel and the post-processing at the receiver creates an effective channel as shown in Fig. 2. This effective channel has m_T input elements and m_R receive elements.

The interpretation of (3) is that we decompose the processing at the transmitter in two parts. The first part (precoding) is optimized based on the channel state information at the transmitter (CSIT), while the codeword matrix $\mathbf{G}_n$ is designed based on the desired transmit diversity gain and rate without utilizing the CSIT. The separability of power allocation or/and beamforming and space-time coding was shown to be capacity-optimal in [31], [32] and in [33] for some single-user scalar channels and for the single-user MIMO channel, respectively.

Although we neither prove nor claim that the separation of space-time coding and precoding is optimal for our setup as well, intuitively one can argue as follows. Space-time codes are designed based on the fact that channel information is not available and thus they transmit the data isotropically over all available dimensions. Interference alignment on the other hand forces the interference to be in a certain subspace, in order to leave the remaining subspace for the desired signals. Thus, both techniques have opposite goals and thus separation seems natural and lossless. Certainly, this will change if the channel knowledge is not perfect and as a consequence the separation incurs a loss.

The decomposition allows to analyze the impact on the tradeoff of each block or module within the transmit and receive chain separately. Afterwards, all insights can be combined to arrive at the final result. The organization of the remainder of the paper reflects this idea. In the following two subsections, intuitive explanations are provided. Afterwards, in section III the main result is derived rigorously.

A. Channel adaptive processing- Interference alignment

The channel knowledge at the transmitter allows to align the interference at the receiver end of each pair, which is optimal in order to approach the capacity of interference networks at high SNR. The sum capacity of such networks is characterized as [18]

$$C = \frac{K}{2}\log \mathsf{SNR} + o\left(\log \mathsf{SNR}\right)$$

for $M_T = M_R = 1$, where the factor $K/2$ in front of the first $\log$ expression is referred to as degrees of freedom. The second term, where o is the little Landau symbol, grows slower than $\log \mathsf{SNR}$ and thus can be neglected asymptotically. In multiple antenna networks an upper bound on the degrees of freedom is given by $\frac{KM}{2}$, with $M_T = M_R = M$, which is known to be achievable in time-varying (fast fading) channels. The upper bound was also shown to be achievable in the three-user case MIMO case with M antennas at each node in flat fading channels. The three-user case also provides a lower bound on the total degrees of freedom given as $3M/2$ by using interference alignment for more than three users.

Let us now introduce a parameter η, which will be formally defined later in section III. η basically describes the quality of interference alignment and is given by

$$\eta = \frac{\text{rank}\left(\text{Interference}\right)}{\text{rank}\left(\text{Signal}\right)}.$$

The range of η is in general given by $1 \leq \eta \leq K-1$. Here, $\eta = 1$ means perfect interference alignment, while $\eta = K-1$ reflects the fact that the subspaces occupied by the interfering nodes at each destination are not aligned at all. The overlapping of the subspaces at receiving node $d = 1$ is also illustrated in Fig. 3, where each source has m_T input streams, with $m_T \leq M_T$. As a result, at the receiver $M_R - \eta m_T$ antennas are available for interference free detection.

Except for the $K = 3$ user case [18], so far it is unknown, how much interference can be aligned for the general case of K users and multiple antennas and how to achieve the upper bound of $\frac{KM}{2}$ on the degrees of freedom if signaling dimensions are finite, i.e. the channels are flat fading. Only for

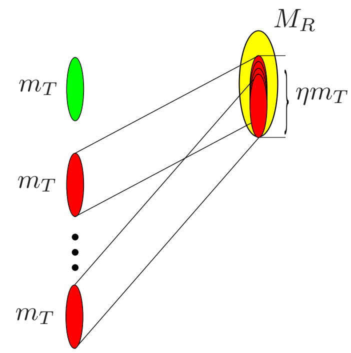

Fig. 3. Illustration of interference alignment at receiver $d = 1$.

the $K = 3$ user, we have a full characterization of η, which is given by

$$\eta = \begin{cases} 1 & m_T < \frac{M_T}{2} \\ \frac{M_T}{2m_T} + (K-1)\left(1 - \frac{M_T}{2m_T}\right) & m_T \geq \frac{M_T}{2}. \end{cases}$$

Thus, as long as $m_T \leq \frac{M_T}{2}$ holds, the interference can be perfectly aligned. Once $m_T > \frac{M_T}{2}$, leakage occurs and increases with m_T.

B. Transmit diversity and rate tradeoff

Suppose that after precoding at the transmitter m_T dimensions are left as shown in Fig. 2 for space-time coding. Since we are interested in the effects of space-time coding, we ignore the fact of interference alignment in this subsection, i.e. we assume that no alignment takes place at the destinations. The space-time coding block introduces redundancy and transmits the data isotropically over all available dimensions in order to achieve high transmit diversity, thereby unavoidably reducing the transmission rate. Thus, there is a tradeoff between rate r and achievable transmit diversity gain D_T, which is characterized in the following theorem.

Theorem 1 ([29]) *Given a desired transmit diversity gain D_T and m_T transmit antennas, the rate $r = \frac{N}{T}$ has the upper bound*

$$r \leq m_T - D_T + 1. \quad (4)$$

Rewriting (4), we have for the transmit diversity gain

$$D_T = \lfloor m_T - r + 1 \rfloor. \quad (5)$$

With space-time coding, the span of the total interference at each receiver is $r\left(K-1\right)$ (ignoring the effect of interference alignment), which leaves a subspace of dimension $M_R - r\left(K-1\right)$ antennas completely free of interference at the receiving ends.

III. Diversity and tradeoff analysis

As mentioned in the introduction, multiple antennas at transmitter and receiver can be used to increase the reliability of the system by providing spatial diversity. The average probability of error P_e of a system can in general be approximated at high SNR [34] by

$$P_e\left(\mathsf{SNR}\right) \approx \alpha_{m_T,m_R} \mathsf{SNR}^{-D_T m_R},$$

where α_{m_T,m_R} describes the horizontal shift of the P_e-curve in a log-log plot of the P_e versus the SNR and the transmit diversity D_T describes the slope of this P_e-curve. Note that α_{m_T,m_R} is a constant independent of the SNR. For example, the average error probability of a single-user system with the transmitter and receiver equipped with a single antenna each at high SNR can be approximated by

$$P_e(\mathsf{SNR}) \approx \alpha_{1,1}\mathsf{SNR}^{-1}.$$

If instead two antennas are used, the average error probability can be approximated by

$$P_e(\mathsf{SNR}) \approx \alpha_{1,2}\mathsf{SNR}^{-2}.$$

Thus, the slope of the average error probability as a function of SNR decreases faster by having multiple independent observations, e.g. multiple receive antennas. As a rule of thumb, the slope corresponds to the number of independent channel paths a symbol passes through. An important observation is that in order to determine the diversity gain of a system, analyzing the SNR exponent is sufficient. The goal of the paper is now to provide a high SNR analysis of the rate-diversity-interference alignment tradeoff with a fixed constellation.

We have the following theorem.

Theorem 2 *Consider a multiple antenna interference channel with K users with linear detectors at the receivers and separate space-time coding and interference alignment, where each source is equipped with M_T transmit and each destination is equipped with M_R antennas. The overall diversity of such a system is given by*

$$\begin{aligned} &D_{M_T,M_R,K}(\eta, r, m_T) = \\ &(\lfloor m_T - r + 1 \rfloor)(M_R - \min(m_T\eta, r(K-1))) \end{aligned} \quad (6)$$

where $1 \le m_T \le M_T$, $0 \le r \le \min(m_T, m_R)$, $1 \le m_R \le M_R$, $M_R > \min(m_T\eta, r(K-1))$ and $1 \le \eta \le K-1$.

Proof: Starting with (1) and applying the $\mathrm{vec}(\cdot)$ operation on both sides of (1) results in

$$\mathbf{y}_d = \mathcal{H}_{dd}\mathcal{G}\mathbf{c}_d + \sum_{s=1,s\neq d}^{K} \mathcal{H}_{sd}\mathcal{G}\mathbf{c}_s + \mathbf{n}_d,$$

where $\mathcal{G} = [\mathrm{vec}(\mathbf{G}_1), \mathrm{vec}(\mathbf{G}_2), \ldots, \mathrm{vec}(\mathbf{G}_N)]$, $\mathcal{H}_{sd} = \mathbf{I}_T \otimes \mathbf{H}_{sd}\mathbf{V}_s$ and $\otimes$ denotes the Kronecker product. The dimensions of the individual components are $[\mathcal{G}] = m_T T \times N$, $[\mathbf{y}_d] = M_R T \times 1$, $[\mathbf{n}_d] = M_R T \times 1$, $[\mathbf{H}_{sd}\mathbf{V}_s] = M_R \times m_T$, and $[\mathbf{c}_s] = N \times 1$. Without loss of generality, we focus on the first user in the following. Rewriting the above equation in a more compact form results in

$$\mathbf{y}_1 = \mathcal{H}_1\mathbf{c} + \mathbf{n}_1, \quad (7)$$

where $\mathcal{H}_1$, with $[\mathcal{H}_1] = M_R T \times KN$, is given by

$$\mathcal{H}_1 = [\mathcal{H}_{11}\mathcal{G}, \mathcal{H}_{21}\mathcal{G}, \ldots, \mathcal{H}_{K1}\mathcal{G}]$$

The interference matrix of user 1 is given by

$$\mathcal{H}_{IF} = [\mathcal{H}_{21}\mathcal{G}, \ldots, \mathcal{H}_{K1}\mathcal{G}],$$

where $[\mathcal{H}_{IF}] = M_R T \times KN - N$. The rank of $\mathcal{H}_{IF}$ is given by

$$\mathrm{rank}(\mathcal{H}_{IF}) \le \min(m_T T\eta, N(K-1)).$$

Note that η is defined as

$$\eta = \frac{\mathrm{rank}(\mathcal{H}_{IF})}{\mathrm{rank}(\mathcal{H}_{11}\mathcal{G})}.$$

It holds that

$$\dim(\mathcal{N}) + \mathrm{rank}(\mathcal{H}_{IF}) = M_R T,$$

where $\mathcal{N}$ is the null space of $\mathcal{H}_{IF}$. Thus

$$\dim(\mathcal{N}) \ge M_R T - \min(m_T T\eta, N(K-1)). \quad (8)$$

Dividing $\dim(\mathcal{N})$ by T gives the number of dimensions free of interference at the receiver. Combining this with (5) and (8) results in the overall diversity in (6). This completes the proof. ∎

From Theorem 2, we observe that there is a tradeoff between interference alignment, diversity, and rate. That is, using some of the antennas for interference alignment reduces m_T and thus the diversity of the system, depending on the rate r of the system.

A. Processing at the receiver

In this subsection, we briefly describe how to achieve the diversity in (6) by using a linear detector. Let $\mathbf{W}_1$ be a $[M_R T - \min(m_T T\eta, N(K-1)) \times M_R T]$ matrix with

$$\mathbf{W}_1 = \left[\mathbf{w}_1^{(1)}, \mathbf{w}_2^{(1)}, \ldots, \mathbf{w}_{M_R T - \min(m_T T\eta, N(K-1))}^{(1)}\right]^T, \quad (9)$$

where $\left\{\mathbf{w}_1^{(1)}, \mathbf{w}_2^{(1)}, \ldots, \mathbf{w}_{M_R T - \min(m_T T\eta, N(K-1))}^{(1)}\right\}$ is a set of orthonormal vectors in $\mathcal{N}$. With (7) and (9), we have

$$\underbrace{\mathbf{W}_1\mathbf{y}_1}_{\tilde{\mathbf{y}}_1} = \mathbf{W}_1\mathcal{H}_1\mathbf{c} + \underbrace{\mathbf{W}_1\mathbf{n}_1}_{\tilde{\mathbf{n}}_1},$$

i.e.

$$\tilde{\mathbf{y}}_1 = \tilde{\mathcal{H}}_{11}\mathbf{c}_1 + \tilde{\mathbf{n}}_1$$

rendering the received signal at destination $d = 1$ free of interference, where $[\tilde{\mathcal{H}}_{11}]$ is of dimension $[\tilde{\mathcal{H}}_{11}] = M_R T - \min(m_T T\eta, N(K-1)) \times m_T$. Thus, effectively we have a single-user system with m_T transmit and $M_R - \min(m_T\eta, r(K-1))$ receive antennas. Now, a space-time code can be employed which achieves the diversity-rate tradeoff described in [29].

IV. Effects of channel estimation errors

The results obtained so far are based on the fact that channel knowledge is perfect. In practical scenarios, this is usually not the case. In this section, channel estimation errors are taken into account by modeling the channel between source s and destination d as

$$\mathbf{H}_{sd} = \mathbf{C}_{sd} + \sqrt{\mathsf{SNR}^{-\frac{1}{\delta}}}\mathbf{E}_{sd}.$$

Here, $\mathbf{C}_{sd}$ is the estimated channel known to the nodes, $\mathbf{H}_{sd}$ is the actual channel and $\mathbf{E}_{sd}$ describes the error in estimating the channel. The impact of the channel estimation error is described by the parameter δ. For $\delta = 0$ we have perfect channel estimation, while $\delta \to \infty$ means that the channel estimation error is independent of the SNR. Furthermore, the channel estimation error $\mathbf{E}_{sd}$ and the estimated channel $\mathbf{C}_{sd}$ have equal variance for $\delta \to \infty$, which will result in an error floor.

The analysis is similar to the case with perfect channel state information. Without loss of generality, we consider again only the first user-pair. After some manipulations, we arrive at

$$\tilde{\mathbf{y}}_1 = \tilde{\mathcal{C}}_{11}\mathbf{c}_1 + \sqrt{\mathsf{SNR}^{-\frac{1}{\delta}}}\sum_{s=1}^{K}\tilde{\mathcal{E}}_{s1}\mathbf{c}_s + \tilde{\mathbf{n}}_1,$$

where the first term describes the desired signal, the second term describes the self-interference and the crosstalk from the other sources due to imperfect interference alignment.

By using a linear MMSE detector, given by

$$\mathbf{D} = \left[\tilde{\mathcal{C}}_{11}^H\tilde{\mathcal{C}}_{11} + \left(K\mathsf{SNR}^{-\frac{1}{\delta}} + \frac{1}{\mathsf{SNR}}\right)\mathbf{I}\right]^{-1}\tilde{\mathcal{C}}_{11}^H,$$

the SINR of the n-th component of the desired siganl $\mathbf{c}_1$ can be written as [35]

$$\mathsf{SINR}_n^{(1)} = \frac{1}{[\mathbf{R_I}]_{n,n}} - 1, \quad n = 1\dots N,$$

where the -1 term is to account for the bias of the MMSE receiver and

$$\mathbf{R_I} = \mathbb{E}\left[(\mathbf{D}\tilde{\mathbf{y}}_1 - \mathbf{c}_1)(\mathbf{D}\tilde{\mathbf{y}}_1 - \mathbf{c}_1)^H\right].$$

It follows for $n = 1,\dots,N$ that

$$\mathsf{SINR}_n^{(1)} = \frac{1}{\left[\left(\mathbf{I} + \frac{\mathsf{SNR}}{1+K\mathsf{SNR}^{1-\frac{1}{\delta}}}\tilde{\mathcal{C}}_{11}^H\tilde{\mathcal{C}}_{11}\right)^{-1}\right]_{n,n}} - 1.$$

We are interested in the diversity behavior. Thus by considering the high SNR case, which results in the zero-forcing solution, it follows that

$$\mathsf{SINR}_n^{(1)} \geq \frac{\mathsf{SNR}}{1+K\mathsf{SNR}^{1-\frac{1}{\delta}}}\frac{1}{\left[\left(\tilde{\mathcal{C}}_{11}^H\tilde{\mathcal{C}}_{11}\right)^{-1}\right]_{n,n}}, \quad n = 1\dots N$$

which can be again lower bounded by

$$\mathsf{SINR}_n^{(1)} \geq \frac{\mathsf{SNR}}{2\max\left(1, K\mathsf{SNR}^{1-\frac{1}{\delta}}\right)}\frac{1}{\left[\left(\tilde{\mathcal{C}}_{11}^H\tilde{\mathcal{C}}_{11}\right)^{-1}\right]_{n,n}}.$$

Ignoring the constant terms gives

$$\mathsf{SINR}_n^{(1)} \sim \mathsf{SNR}^{\min\left(1,\frac{1}{\delta}\right)},$$

which shows that the overall diversity is now given by

$$D^{\delta}_{M_T,M_R,K}(\eta, r, m_T) = \min\left(1, \frac{1}{\delta}\right) d_{M_T,M_R,K}(\eta, r, m_T).$$

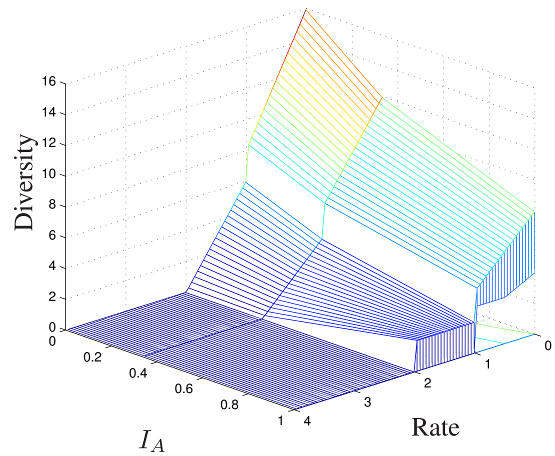

Fig. 4. Diversity-Rate-Interference Alignment Tradeoff for $K = 3$ user interference network with $M_T = M_R = 4$ antennas at each node. For $I_A = 1$, perfect interference alignment is achieved, while $I_A = 0$ refers to the case without any alignment.

We observe that small errors in the estimation of the channels are tolerated. However, once $\delta > 1$, the diversity decreases due to self-interference and imperfect alignment. Furthermore, $\delta \to \infty$ results in an error floor.

V. Illustration

The tradeoff between diversity, interference alignment and rate is illustrated for the $K = 3$ user interference channel in Fig. 4. Here, all nodes are equipped with $M_T = M_R = 4$ antennas. Let us introduce the interference alignment indicator I_A, with $0 \leq I_A \leq 1$, which is a measure on the amount of interference alignment obtained. For the $K = 3$ user case it is given by

$$I_A = \frac{\frac{K}{2} - \eta}{\frac{K}{2} - 1} = 3 - 2\eta.$$

Note that the figure is obtained by maximizing (6) over all possible m_T for a given rate and I_A. From the figure, we observe that for low rate aligning the interference is rather harmful, since all antennas can be used to increase diversity. As the rate increases, though interference alignment becomes necessary in order to achieve diversity gains. This can be observed in particular at $r = 2$, where interference alignment is essential to achieve diversity gains. Let r_{th} be the threshold rate above which interference alignment is useful. In Fig. 5 the threshold rate is shown as a function of I_A. From $I_A = 1/3$ on the threshold rates are realizable ($r_{th} \leq 2$) and alignment is helpful.

VI. Conclusion

In this work, we analyzed the tradeoff between diversity, rate and interference alignment in the context of a multi-user multi-antenna interference network. We have shown that interference alignment is needed if the system is operated at or close to the maximum achievable rate. For low rates the preferred strategy is to utilize all antennas in order to achieve high

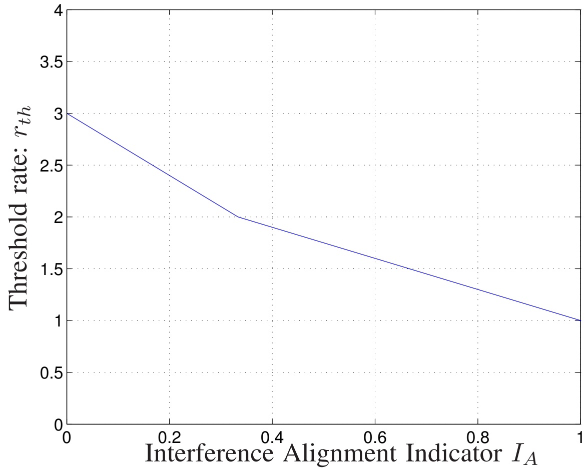

Fig. 5. Threshold rate r_{th} for $K = 3$ user interference network with $M_T = M_R = 4$ antennas at each node.

diversity gains, rather than using some of the antennas to align the interference. In this case, using interference cancelation methods, e.g. in [16], will even further improve the diversity. We also investigated the impact of channel estimation errors on the diversity of the system. It turns out that small channel estimation errors can be tolerated, while larger errors reduce the diversity gain significantly.

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