Adaptive Resource Allocation in Composite Fading Environments

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Abstract— **We obtain optimal resource allocation policies for a single user single-carrier system and a multiple-access multi-carrier-CDMA system when the transmitter adapts to the variations in the short-term mean (slow fade) in a combined slow and fast fading (composite fading) environment. For the single user system, we maximize the average throughput achieved by the user, while in the uplink MC-CDMA system, we maximize the sum of average rates of the users in the system. For each system, we find the optimal resource allocation policies for two scenarios. The first is when is system is designed for voice transmission, where the bit error rate (BER) of each user, averaged over the fast fade, is maintained at a desired value. The second is when the system is designed for data transmission, where the BER of each user is maintained *below* a desired value for a given percentage of time. We find that, for the single-user system, the solution for both the voice and data transmission cases is *waterfilling*, and that waterfilling is the asymptotically optimal solution to multi-user problems in both scenarios, i.e is nearly optimal for a large number of users. We also find that, when dealing with a voice system, the solution is independent of the distributions of the slow and fast fades and similar to the solutions obtained for noncomposite fading environments (fast or slow fading).**

I. Introduction

The requirements and expectations of wireless systems are increasing as rapidly as their popularity. To compete with wired systems, adaptive schemes have been recognized as pivotal and are an integral part of future wireless systems [1]. In this context, there has been a great focus on adaptive modulation for single user systems [2][3], and to some extent for multiuser code-division multiple access (CDMA) systems [4]. Most of the literature in this area focuses on maximizing the average throughput achieved by the system, defined as the average rate achieved by a single user, or the sum of rates of users in a multiuser system, with constraints on the bit error rate (BER) and power of each user.

Adaptive modulation algorithms typically assume perfect and instantaneous knowledge of the channel gain at the transmitter, which is unrealistic. Specifically, channel estimation at the receiver and feedback to the transmitter have inherent delays, and thus the transmitter cannot rely on obtaining instantaneous estimates to determine its power policy. This is especially true in systems where the fade decorrelates over a time period that is of the same order as the estimation and feedback delay. Also, in most practical systems [1], the bandwidth of the feedback channel is very small, leading to small feedback channel capacity and hence imperfect estimates at the transmitter.

Wireless channels typically exhibit multipath fading and/or shadowing where the multipath fading changes much faster than the shadowing. Hence, the former is referred to as fast fading and the latter as slow fading. We will refer to the case when both are present as composite fading and the case when only one is present as noncomposite fading. In this paper, we consider a channel model with composite fading and assume that the transmitter adapts only to the slow fading where this slow fading has been estimated and fed back by the receiver. This allows for delay in estimation and feedback, since the short-term mean remains constant for a much larger duration than the typical delays involved in these processes. This model has been previously applied to study truncated power control in CDMA systems in [5].

We study adaptive modulation for both voice and data systems in this paper. The requirements of a voice system translate into a constraint that the short-term bit error rate (BER) (i.e the BER averaged over the fast fade) be maintained below some desired value for each user whenever that user is transmitting. We shall refer to this as the short-term BER constraint case. For data systems, we require that BER at every time instant (as a function of the total fade) of each user be maintained below a desired maximum. Since the transmitter has knowledge of only the slow fade, and hence can vary its power and rate only with the slow fade, it is clear that this requirement cannot be met *all* the time for all types of fast fading distributions. Hence, we allow for a percentage of time (called outage) when the system cannot achieve a BER lower than the maximum. We shall refer to this as the instantaneous BER with outage constraint case.

We develop the system model for the single user case in Section II-A, and find the optimum resource allocation strategies for the single user system in II-B and II-C. In Section III-A, we introduce the system model for an uplink single cell multi-carrier-CDMA (MC-CDMA) system and find the corresponding asymptotically optimal resource allocation strategies in III-B and III-C. In Section IV, we show that the results obtained when the short-term BER of the system is constrained to be constant is independent of the distributions of the fast and slow fades. We conclude with Section V.

II. Resource Allocation in a Single User System

We use boldface for vectors, with $\mathbf{h} = [h_1 h_2 \dots h_N]$, and $\mathrm{E}_f(x)$ for the expectation of x with respect to f. We use $f = [g]^+$ to mean $f = g$ for $g > 0$ and $f = 0$ otherwise.

A. System Model

In the single user case, we consider uncoded transmission in flat fading where the received symbol at time n is

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given by

$$y(n) = \sqrt{\bar{h}}(n)x(n) + w(n), \quad (1)$$

with $h(n)$ denoting the fade, $x(n)$ the data symbol, and $w(n)$ AWGN noise with variance one. $h(n)$ is modeled as $r(n)\bar{h}(n)$, where $r(n)$ is fast (Rayleigh) fading of unit mean, and $\bar{h}(n)$ is the slow fading, and also the short term mean of $h(n)$. We assume that both the transmitter and the receiver have perfect knowledge of $\mathbf{h}_n$ at time n. We shall henceforth drop n for notational convenience. The short term mean $\bar{h}(n)$ is fed back to the transmitter. The user has an average power constraint of $\overline{P}$.

To adapt its resources, the transmitter may change its transmission bandwidth (hence the symbol time) or the size of its (complex) constellation (M) from which symbols are picked for transmission. Clearly, the former scheme is highly complex to implement in real systems. Many modems and third generation systems use the latter idea, that of variable constellation size. Thus, the transmitter changes its transmission rate and its transmit power by varying the constellation size M and the average power of this constellation.

Although realistically M can take only discrete values, we assume that it can take on all non-negative real values. There are many reasons why this assumption is useful. First, it helps us understand the maximum limits that such a scheme can achieve, i.e the added constraint of discrete values can only degrade system performance. Secondly, it transforms problems that are very hard to solve (often NP-Complete, see [6]) into simpler problems, many of which are convex. In fact, this method of transforming discrete variables into continuous ones is often used in optimization literature [7].

To proceed with the problem definition, we need an expression for the performance measure (the BER) in terms of the remaining system parameters (the power and rate). These have been obtained for additive white Gaussian noise (AWGN) channels in [2][3]. The instantaneous BER (BER at any time instant n) can be obtained for MQAM or MPSK modulation as [3]:

$$\mathfrak{E}_{inst}(h) \approx c_1 \exp\left(-\frac{c_2 h P}{M-1}\right) \quad (2)$$

where c_1, c_2, are constants and P is the transmit power used when the channel state equals h. It is found [3] that these exponential approximations are tight to within a dB of simulation results. Finding the average of $\mathfrak{E}_{inst}(h)$ over the distribution of r gives us the short-term BER ($\mathfrak{E}_{st}(\bar{h})$).

B. Short-term average constraints on the BER

Averaging $\mathfrak{E}_{inst}(h)$ over the fast fade r (assumed Rayleigh) we obtain

$$\mathfrak{E}_{st}(\bar{h}) \approx \frac{c_1}{\frac{c_2 \bar{h} P(\bar{h})}{M(\bar{h})-1} + 1}. \quad (3)$$

Next, we shall use the expression developed above to formulate an optimization problem that maximizes the average throughput of the system.

We desire that the short-term average BER given by Equation (3) be held constant at $\overline{\mathfrak{E}}$ whenever the user transmits. Note that this is equivalent to saying that:

$$\frac{\bar{h}P(\bar{h})}{M(\bar{h})-1} = K, \quad (4)$$

where $K = (c_1/\overline{\mathfrak{E}} - 1)/c_2$. We can use Equation (4) to write M in terms of P or vice versa. Noting that we can obtain the instantaneous rate of the user from the constellation size M as $\log(M)$, we formulate the throughput maximization problem as:

Problem Definition 1:

$$\max \mathbb{E}_{\bar{h}} \log\left(1 + \tfrac{\bar{h}P(\bar{h})}{K}\right)$$

such that

$$\mathbb{E}_{\bar{h}} P(\bar{h}) = \bar{P}.$$

Observation 1: Problem 1 is a convex problem in its variable $P(\bar{h})$.

Proof: The objective to be maximized is concave in $P(\bar{h})$ and the constraint is linear, hence it is a convex optimization problem.

Hence, a unique solution for this problem can be obtained by framing the dual (Lagrangian) problem. We can now perform an unconstrained optimization of the dual problem to obtain the optimal power policy as:

$$P(\bar{h}) = \left[\frac{1}{\lambda} - \frac{K}{\bar{h}}\right]^+, \quad (5)$$
$$M(\bar{h}) = 1 + \frac{\bar{h}P(\bar{h})}{K}.$$

where λ is a (Lagrangian) constant that can be obtained by using the power policy expression (5) in the power constraint. The solution in (5) is *waterfilling* [2] relative to the short term channel average $\bar{h}$, i.e. the power is increased as the average channel gain $\bar{h}$ increases above a given cutoff value. Note that the optimal power adaptation for adaptive modulation in [2] and [3] and for channel capacity in [8] for noncomposite fading channels, with perfect and instantaneous channel information at transmitter and receiver is also waterfilling relative to the instantaneous channel. Thus, there is a similarity between the two solutions.

We can prove a much more general result that the waterfilling nature of the optimum power policy is independent of the fast and slow fading distributions. This is formally stated and proved in Section IV.

C. Instantaneous Constraints on BER with outage

The requirement that the instantaneous BER given by Equation (2) be upper bounded by $\overline{\mathfrak{E}}$ can be rewritten as:

$$\frac{hP(\bar{h})}{M(\bar{h})-1} \geq \frac{\log(c_1) - \log(\overline{\mathfrak{E}})}{c_2}. \quad (6)$$

Since the transmit power and rate are functions of $\bar{h}$ and not of h, this requirement cannot be met for fast fading distributions that can take on values that are arbitrarily close to zero. Note that the Rayleigh distribution is one such distribution. Hence, we meet this requirement a fixed percentage $100\bar{\mathcal{X}}$ of the time. We say that the system is in *outage*, i.e cannot meet its BER requirement, with probability $1-\bar{\mathcal{X}}$. For Rayleigh fast fading, these requirements can be equivalently written as

$$\frac{h_0 P(\bar{h})}{M(\bar{h})-1} = \frac{\log(c_1)-\log(\overline{\mathfrak{E}})}{c_2} \tag{7}$$

for some h_0 such that

$$\int_{h_0}^{\infty} \frac{1}{\bar{h}} e^{h/\bar{h}} = \bar{x}. \tag{8}$$

Equations (7) and (8) can be combined and written as

$$\frac{\bar{h}P(\bar{h})}{M(\bar{h})-1} = K, \tag{9}$$

where $K=-(\log(c_1)-\log(\overline{\mathfrak{E}}))/(c_2\log(\bar{\mathcal{X}}))$. Note that the constraint imposed by (9) is identical to (4), except for a different value for the constant K. Since the objective and the remaining constraints on the system are the same as in Problem 1, the solution obtained is identical in form to (5) and hence can be written as

$$P(\bar{h}) = [1/\lambda - K/h]^+ \tag{10}$$

This concludes our analysis of single user throughput maximization problems. We can also consider an analogous problem of "power minimization" which is of the form:

Problem Definition 2:

$$\min \mathbb{E}_{\bar{h}} P(\bar{h})$$

such that

$$\mathbb{E}_{\bar{h}} \log\left(1+\tfrac{c_2\bar{h}P(\bar{h})}{c_1/\overline{\mathfrak{E}}-1}\right) = \overline{R}$$

Here, we desire that the system achieve an average data rate $\overline{R}$, and wish to minimize the power consumed by this process while meeting a short-term average BER requirement at the receiver. Such a problem is interesting from the point of view of increasing battery lifetime in the system. Problem 2 is the dual optimization problem [7] to Problem 1. It is also convex, and hence its Lagrangian formulation provides a unique solution. Moreover, this unique solution is the same as that obtained for the primal problem (5) and (10), except that the constant λ is now calculated using the throughput constraint instead of the power constraint.

III. Resource Allocation in uplink Multi-Carrier CDMA systems

A. System Model

We consider uncoded transmission over a synchronous flat fading multiple-access (uplink) MC-CDMA discrete-time system with N users and L sub-carriers (referred to as sub-bands) for each user. The signal received at time n is given by

$$\mathbf{y}(n) = \sum_{i=1}^{N}\sum_{j=1}^{L} \sqrt{h_{i,j}}(n)\mathbf{s}_{i,j}(n)x_{i,j}(n) + w_{i,j}(n), \tag{11}$$

where $x_{i,j}(n)$ denotes the transmitted symbol, $\mathbf{s}_{i,j}$ the spreading sequence, $w_{i,j}(n)$ the additive Gaussian noise and $h_{i,j}(n)$ the stationary and ergodic channel gain corresponding to User i in sub-band j. The $h_{i,j}(n)$ are assumed to be i.i.d., and to result from a combination of fast (assumed Rayleigh) fading and slow fading. Equivalently,

$$h_{i,j}(n) = r_{i,j}(n)\bar{h}_{i,j}(n) \tag{12}$$

i.e, $h_{i,j}$ results from the product of a fast fade $r_{i,j}(n)$ and a slow fade $\bar{h}_{i,j}(n)$. We assume $r_{i,j}(n)$ to be unit mean. Equivalently, $\bar{h}_{i,j}(n)$ is the short-term mean of $h_{i,j}(n)$ (hence the symbol $\bar{h}_{i,j}$ for the slow fade). We assume that $\bar{h}_{i,j}(n)$ is known perfectly at the transmitter and receiver at time n.

Note that, with appropriate scaling, we may assume that the noise variance is unity.The short-term means $\bar{h}_{i,j}(n)$ for each user and each sub-band are fed back from the base-station to all the users in the system. For notational convenience, we shall henceforth drop the dependence of the system parameters on n.

Since most CDMA systems in use today use the conventional matched filter receiver, we assume the same for our MC-CDMA system, with one matched filter for every spreading sequence $s_{i,j}$. We assume that any two spreading sequences have a constant cross correlation given by ρ.

We now characterize the constraints imposed on this system by practical requirements. The power of each User i is required not to exceed $\bar{P}$ on average, where the average is over the composite fading distribution $h_{i,j}(n)$. Constructing a matrix $\mathbf{H}(n)$ whose (i,j)th element is $h_{i,j}(n)$ and denoting the power of User i in sub-channel j by $P_{i,j}$, we have $\mathbb{E}_{\mathbf{H}}\sum_{j=1}^{L} P_{i,j} = \bar{P}$ for $1 \le i \le N$.

First, we analyze the MC-CDMA system given by Equation (11) with one sub-band per user, i.e with $L=1$. This is equivalent to a synchronous CDMA system without inter-symbol interference (ISI). There are three well known techniques for dynamic rate adaptation CDMA systems: multi-code, multi-bit-rate and variable-constellation size methods, which are explained in [9]. To maintain continuity with previous sections, we focus on variable-constellation size schemes in this paper.

In a variable-constellation size scheme, each user is assigned a single spreading sequence, but can vary his constellation size $M_i(\bar{\mathbf{H}})$ (and hence his rate) and his transmit power $P_i(\bar{\mathbf{H}})$ with the channel $\bar{\mathbf{H}}$. In this scenario, we wish to obtain the power and rate allocation policies that maximize the sum of throughputs of the users in the system (called the sum rate). As in Section II, we desire to relate the performance measure (BER) of each user with the power vector $\mathbf{P}(\bar{\mathbf{H}})$ and rate policy vector $\mathbf{M}(\bar{\mathbf{H}})$ of all the users in the system. For this, we use the BER expressions for non-adaptive transmission in AWGN and fading channels in [10] and modify them suitably. We find the AWGN BER for User i to be approximated by:

$$\mathfrak{E}(\mathbf{H}) \approx c_1 \exp\left(\frac{-c_2\bar{h}_i P_i}{(M-1)(1+\rho\sum_{j\neq i}\bar{h}_j P_j)}\right). \tag{13}$$

We find that the BER averaged over fast Rayleigh fading for User i (called the short-term averaged BER) can be approximated by

$$\mathfrak{E}(\bar{\mathbf{H}}) \approx \frac{c_1}{1+\frac{c_2 \bar{h}_i P_i}{(M-1)(1+\rho\sum_{j\neq i}\bar{h}_j P_j)}}. \qquad (14)$$

In the expression above, we make a Gaussian approximation on the interference and replace the instantaneous interference by its average over the fast fade as done in [10]. As discussed in [11], these assumptions hold when dealing with a large number of users with long codes.

B. Short-term constraints on the BER

As in the single user case, we set the short-term averaged BER of User i to be constant at $\overline{\mathfrak{E}}_i$ whenever User i is transmitting. This constraint allows us to write the instantaneous rate of User i ($\log(M_i)$) in terms of the transmit powers of the users as

$$\log\left(1+\frac{c_2\bar{h}_i P_i}{(1+\sum_{j\neq i}\bar{h}_j P_j)(c_1/\overline{\mathfrak{E}}_i - 1)}\right). \qquad (15)$$

Calling $(c_1/\overline{\mathfrak{E}}_i - 1)/c_2$ as K_i, the sum rate objective, power constraint and the short-term BER requirements give the problem definition as

Problem Definition 3:

$$\max \quad \mathbb{E}_{\bar{\mathbf{h}}} \sum_{i=1}^{N} \log(1+\frac{\bar{h}_i P_i}{1+\rho\sum_{j\neq i}\bar{h}_j P_j}\frac{1}{K_i})$$

subject to

$$\mathbb{E}_{\bar{\mathbf{h}}} P_i = \bar{P}. \qquad 1 \leq i \leq N$$

This problem is *not convex* in the variables P_i. Thus, solving for the global optimum explicitly is in general a very difficult problem. Hence, numerical techniques like steepest descent, simulated annealing etc. must be used to obtain the optimum solution. These algorithms, however, are computationally intensive, and do not provide any intuition about the final solution. Next, we shall present a simple algorithm termed "iterative waterfilling" that is *asymptotically optimal*, i.e that is near-optimal when the number of users N is large. This algorithm is as follows:

Iterative Waterfilling Algorithm:

1. Initialize $\mathbf{P} = \mathbf{0}$.
2. Repeat for i running from 1 to N: Find the maximum rate User i can achieve, given the power policies of Users $1, \ldots, i-1, i+1, \ldots N$.
3. Repeat Step 2 until the powers P_i of all the users converge.

This algorithm is essentially the greedy algorithm in competitive equilibrium [12], where each user maximizes his rate individually. Since the rate of each user is maximized when he "waterfills" to the noise and interference he observes, this algorithm is termed "iterative waterfilling". This algorithm is asymptotically optimal because when the number of users are large, the choice of power policy P_i of User i does not significantly affect the interference seen by the other users and hence their power policy. Thus, it is optimal for each user to choose the greedy policy of maximizing his own rate.

This greedy policy is also near-optimal if $K_1 \approx K_2 \approx \ldots K_N \approx K \approx 1/\rho$ for any value of N. For this case, the iterative waterfilling algorithm finds that only the user with the best channel $\bar{h}_i$ should transmit at any given time. The conditions for optimality of a solution to Problem 3 can be obtained by forming the Lagrangian and differentiating it. We get

$$\frac{Nh_i}{K+\sum_j \bar{h}_j P_j} - \sum_{k\neq i}\frac{h_i}{K+\sum_{j\neq k}\bar{h}_j P_j} = \lambda_i \quad 1 \leq i \leq N$$

where λ_i is the Lagrangian constant. Note that the user with the best channel transmitting alone and waterfilling to the noise satisfies the condition for optimality above.

This iterative waterfilling algorithm has been used to calculate the optimum power and rate policies to achieve sum rate capacities for multi-antenna multiple access channels (MIMO MAC) in [13]. Note that a multiple access system with multiple receive antennas and a multiple access CDMA system are analogs of one another [14]. Also note that waterfilling has been found to be asymptotically optimal for achieving sum rate capacity for MAC CDMA [14]. Thus, some of the results obtained for single user and multiuser capacity problems are also true for practical adaptive communication systems.

C. Instantaneous constraints on the BER with outage

As observed in the single user case, the instantaneous BER cannot be bounded above when the transmitter has no knowledge of the instantaneous channel. Thus, we allow an outage with probability $1-\bar{\mathcal{X}}$ when this requirement cannot be met. This translates into an equivalent constraint of the form

$$\frac{\bar{h}_i P_i}{(M_i-1)(1+\rho\sum_{j\neq i}\bar{h}_j P_j)} = K \qquad (16)$$

where $K = -\frac{\log(c_1/\bar{\mathfrak{E}})}{c_2\log(\bar{\mathcal{X}})}$. Note that this constraint is identical to the BER constraint in the short-term average constraint case, except with a different constant K. Since the objective and power constraints are identical, this system has a similar solution as that for Problem 3.

We also point out that we can choose to minimize the sum of the average powers of the users, given that the users desire average rates $\bar{\mathbf{R}}$. As observed in the single user case (Section II), this problem has the similar solution for the power and rate policy as the sum-rate maximization problem solved above.

The model we chose for the CDMA system was that of a single sub-band of an MC-CDMA system. This models CDMA poorly, since it neglects the frequency-selective nature of wide-band CDMA, which is one of the key features of modern day CDMA systems. We did so for two reasons. Firstly, the analysis of adaptive CDMA with ISI is almost impossible, and secondly, because we wish to use these results to analyze an MC-CDMA system with L sub-bands, where the assumptions of negligible ISI are justified. (Please see [15] for further details)

First, we point out that an MC-CDMA system with L sub-bands is similar to a single sub-band (CDMA) system described above, but with NL users and with the power constraints no longer over individual users, but across blocks of L users in the NL user system. Again, we limit analysis in this paper to the case when variable-constellation size schemes are used in each sub-band at each transmitter on the uplink.

As in the single sub-band case (Problem 3), the problem can be formulated as:

Problem Definition 4:

$$\max \mathbb{E}_{\bar{\mathbf{H}}} \quad \sum_{i=1}^{N} \sum_{j=1}^{L} \log(1 + \frac{c_2 \bar{h}_i P_i}{1+\sum_{j\neq i} \bar{h}_j P_j} \frac{1}{c_1/\bar{\mathfrak{E}}_i - 1})$$

subject to

$$\mathbb{E}_{\bar{\mathbf{H}}} \sum_{j=1}^{L} P_{i,j} = \bar{P}_i.$$

Thus, the asymptotic solution for this problem is two dimensional waterfilling, where the transmitter waterfills to the noise and interference he observes in each sub-band and across his L sub-bands. If all the sub-bands have the same channel distribution, then this is equivalent to assigning an average power of $\bar{P}/L$ to each sub-band of User i, and waterfilling to the noise and interference in each sub-band.

IV. Independence of resource allocation policies from the fading distribution

Now we show the most important result of this paper. **Main Theorem**: The optimum power (and rate) policy solutions to all problems where the short-term average BER is constrained is independent of the actual distributions of the fast and slow fades, except for the constants involved in these expressions.

Proof: Let $h = \bar{h}r$ be the composite fading, where $\bar{h}$ is the slow fade and r is the unit mean fast fade. Let r have a probability distribution function $f(r)$. Then the short-term probability of bit error can be obtained by averaging the instantaneous BER given in Equation (2) as follows:

$$\mathfrak{E}_{st} = \int_0^\infty c_1 \exp(-c_2 h g(\bar{h}) f(r) dr, \qquad (17)$$

where $g(\bar{h} = P(\bar{h})/(M(\bar{h}) - 1)$. Substituting $h = \bar{h}r$ we find that this is equivalent to

$$\int_0^\infty c_1 \exp(-c_2 r \bar{h} g(\bar{h})) f(r) dr = \overline{\mathfrak{E}} \qquad (18)$$

For any distribution $f(r)$, the equality given in (18) will hold only when $\bar{h}g(\bar{h})$ is constant. In other words, irrespective of the distributions of the fast and slow fades, the short-term BER constraint boils down to $\bar{h}g(\bar{h})$ being constant.

Thus the solution to the short-term mean feedback case is as if *the fast fade were absent.* Note that the single fading environment handled in [3] is a special case where either the fast fade $r = 1$ or slow fade $\bar{h}$ is constant.

Another important corollary of this theorem is that all the resource allocation policies obtained by researchers for the single fading environment with instantaneous perfect feedback can be translated to the composite fading environment with short-term mean feedback.

V. Conclusions

In this paper, we identify the need for a more realistic model for the information that is fed back to the transmitter from the receiver, and propose one where the short-term mean of a composite fading channel is fed back to the transmitter. To study the requirements of different applications, we study both a short-term average and an instantaneous constraint with outage on the BER. We show that, for both types of constraints, the solution is waterfilling in the single user case, and that waterfilling is asymptotically optimal in the multiuser multi-carrier CDMA case. We show that for a short-term constraint on the BER, the optimum power policy is independent of the fast and slow fade distributions and is similar in form to the case of a non-composite fading channel with instantaneous and perfect channel information at the transmitter and receiver [2][3].

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