Towards Full-Duplex Multihop Multiflow — A Study of Non-Layered Two Unicast Wireless Networks

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Abstract—Starting from the elemental $2 \times 2 \times 2$ interference channel, there has been much progress in the understanding of multihop multiflow wireless networks through degrees of freedom (DoF) studies that have produced important ideas such as (aligned) interference neutralization. However, much of this progress has been limited to layered connectivity models that are essentially motivated by the assumption that wireless networks can only operate in half-duplex mode. Motivated by recent breakthroughs in full-duplex radio technology, in this work we expand the $2 \times 2 \times 2$ interference channel model beyond layered connectivity in order to study the impact of full-duplex operation. In particular we study the impact of intra-layer connectivity between relays that are in the same layer, the impact of direct inter-layer connectivity between sources and destinations, and the impact of intermediate inter-layer connectivity that connects sources or destinations, not directly to each other, but to relay nodes in non-adjacent layers in a $2 \times 2 \times 2$ interference channel. We show that intra-layer links and intermediate inter-layer interference links do not cause a collapse of DoF while direct interference links do cause a collapse of DoF.

Index Terms—Capacity, degrees of freedom, full-duplex, interference neutralization, multihop multiflow

I. INTRODUCTION

Characterizing the capacity of wireless networks is one of the most important problems of network information theory. Recent years have seen rapid progress on mainly two settings: single flow multihop networks and single hop multiflow (interference) networks. In [1], it is shown that the capacity (within a constant gap) of the single source multihop multicast network is given by the network min-cut. In [2], the capacity of a wireless multihop broadcast network where a single source sends independent messages to multiple destinations over a multihop network is characterized within a constant gap from the cut-set bound. On the other hand, a variety of capacity approximations in the form of DoF, generalized degrees of freedom (GDoF), capacity within a constant gap and even the exact capacity have been obtained for various single hop interference networks [3], [4].

Spurred by the advances in single hop multiflow, and multihop single flow settings, there has been increasing interest in the setting that combines the two — the multihop multiflow setting, starting with the most elemental multihop multiflow network, the $2 \times 2 \times 2$ interference channel formed by concatenation of two 2-user interference channels. In [5], it is shown that the $2 \times 2 \times 2$ interference network achieves the cut-set outer bound value of 2 DoF for almost all channel coefficients. In [6], Shomorony and Avestimehr characterize the DoF for two-unicast layered wireless networks with arbitrary number of layers, arbitrary number of nodes per layer, and arbitrary connectivity between adjacent layers. It is shown that 2 unicast layered wireless networks can only have 1, 2 or 3/2 DoF. A related work on 2 unicast networks by Wang, Kamath and Tse in [7] characterizes the capacity region for two unicast information flows over a layered linear deterministic network. The DoF result for the layered two unicast problem is generalized by Wang, Gou and Jafar in [8] allowing all possible unicasts between 2 sources and 2 destinations. In this setting, known as 2-source 2-sink all-unicast layered wireless networks (also known as multihop X networks), each source has an independent message for each destination. Wang, Gou and Jafar show in [8] that with arbitrary number of nodes in each layer and with arbitrary connectivity between adjacent layers, the DoF belong to the set $\{1, 4/3, 2, 5/3, 2\}$ almost surely.

The $2 \times 2 \times 2$ setting has been successfully extended in several directions, including finite-field settings in [9], time-varying linear schemes in [10], [11], [12], [13] and to the $K \times K$ network in [14]. However, the extensions have been limited primarily to layered network models, in large part due to the traditional assumption that wireless networks must operate in a half-duplex mode. With the growing interest and remarkable recent progress in building practical full-duplex radios [15], [16], [17], [18], [19], [20], [21], it is clear that this traditional assumption is no longer valid. Consequently, there is the need to update the theoretical models to include the full-duplex assumption, and to understand the implications of full-duplex operation in challenging network settings such as the multihop multiflow problem. This work presents the outcomes of our efforts in this direction.

Layered network models are essentially motivated by the assumption of half-duplex communication where each layer, i.e., all nodes in a layer, are either transmitting or receiving at any time, but not both simultaneously. Thus, the nodes within each layer do not hear each other. Also, among all layers that
are within communication range of each other, only one layer is active at any time. However, in full-duplex communications, this is not the case. Since the nodes can receive and transmit simultaneously, all nodes within a layer can hear each other as well, resulting in intra-layer links. For example, consider the simplest setting of multihop interference networks consisting of 2 sources, 2 relays and 2 destinations as shown in Fig. 1. If relay nodes $R_1$ and $R_2$ are assumed to be half-duplex and they are always either both transmitting or both receiving simultaneously, then they cannot hear each other, i.e., the links between them do not exist. This is no longer the case if $R_1$ and $R_2$ can transmit and receive simultaneously. While a first order capacity characterization in the form of DoF for the layered case is known to meet the min-cut outer bound in [5], it remains unknown if this is still the case when there are interfering links between two relays due to full-duplex operations. It is this problem that we would like to explore first in this paper. As will be shown in Section II, the min-cut outer bound of 2 DoF is achieved almost surely.

The results summarized so far indicate that the intra-layer interference introduced by full-duplex relays causes no loss of DoF for the underlying layered $2 \times 2 \times 2$ IC, while the end-to-end inter-layer cross-links that connect a source directly to its unintended destination cause a collapse of DoF. It remains to understand the role of intermediate inter-layer links that connect sources or destinations, not directly to each other, but to intermediate relay nodes. To include inter-layer links to/from relay nodes, we further study the $2 \times 2 \times 2$ IC formed by concatenation of 3 interference channels with arbitrary inter-layer connectivity as shown in Fig. 3, where the black links indicate the underlying layered connectivity and the red dashed links represent all other possible inter-layer connectivities in the network. As will be shown in Section IV, even for this class of networks, the min-cut outer bound of 2 DoF can be almost surely achieved except when there is a direct interfering link from a source to its unintended destination. In this case, the DoF collapse to one.

II. THE $2 \times 2 \times 2$ INTERFERENCE CHANNEL WITH INTERFERING RELAYS

In this section, we explore the $2 \times 2 \times 2$ IC with interfering relays shown in Fig. 1.

A. System Model

The received signal at relay $R_k$, $k \in \{1,2\}$ in time slot $t$ is $Y_{R_k}(t) = F_{k1}(t)X_1(t) + F_{k2}(t)X_2(t) + H_{kk}(t)X_{R_k}(t) + Z_k(t)$ where $k = 1$ if $k = 2$ and $k = 2$ if $k = 1$. $F_{kj}(t)$, $k, j \in \{1,2\}$, is the complex channel coefficient from $S_j$ to $R_k$. $H_{kk}$ is the channel coefficient from $R_k$ to $R_k$. $X_j(t)$ is the input signal from $S_j$, $X_{R_k}(t)$ is the input signal from $R_k$ and $Z_k(t)$ is the independent identically distributed (i.i.d.) zero mean unit variance circularly symmetric complex Gaussian noise.

The received signal at $D_k$ in time slot $t$ is given by $Y_k(t) = G_{k1}(t)X_{R_1}(t) + G_{k2}(t)X_{R_2}(t) + N_k(t)$, where $G_{kj}(t)$, $k, j \in \{1,2\}$, is the complex channel coefficient from $R_j$ to $D_k$ and $N_k(t)$ is the i.i.d. Gaussian noise. We assume every node in the network has an average power constraint $P$. The relays are full-duplex. In addition, the relays are causal, i.e., the transmitted signals at relays only depend on the past received signals but not the current received signals. We assume that sources and relays know all channels while destinations only know channels from relays. To avoid degenerate conditions, we assume the absolute values of channel coefficients of the layered links are bounded between a nonzero minimum value and a finite maximum value. However, the channel coefficients of intra-layer links can be of any arbitrary value. We will consider two settings where channel coefficients are time-varying or
constant: 1) the channel coefficients change and are drawn i.i.d. from a continuous distribution for every channel use, and 2) the channel coefficients are drawn i.i.d. from a continuous distribution before the transmissions, and once they are drawn, they remain unchanged during the entire transmission.

Source $S_k$, $k \in \{1, 2\}$ has a message $W_k$ for destination $D_k$. We denote the size of message $W_k$ as $|W_k|$. For the codewords spanning $n$ channel uses, the rates $R_k = \frac{\log |W_k|}{n}$ are achievable if the probability of error for both messages can be simultaneously made arbitrarily small by choosing an appropriately large $n$. The sum-capacity $C_2(P)$ is the maximum achievable sum rate. The number of degrees of freedom is defined as $d = \lim_{P \to \infty} \frac{C_2(P)}{\log P}$.

**B. Main Result**

The main result is presented in the following theorem.

**Theorem 1:** For the $2 \times 2 \times 2$ IC with interfering relays defined in Section II-A, the total number of DoF is equal to 2 for both constant and time-varying channel coefficients, almost surely.

Since the min-cut DoF outer bound is 2, we only need to provide an achievable scheme. The achievable scheme is designed on top of that for the $2 \times 2 \times 2$ IC with additional effort to cancel interference caused by relays. Thus, before presenting the scheme for this channel, it is helpful to first review the achievable scheme for the $2 \times 2 \times 2$ IC proposed in [5].

**C. Review: Achievable Scheme for the $2 \times 2 \times 2$ IC**

Signaling is performed over $M$ dimensions regardless of whether it is time or rational dimensions. Over these $M$ dimensions, $S_1$ sends $M$ signals and $S_2$ sends $M-1$ signals so that a total of $2M-1$ DoF is achieved. Since $M$ can be chosen arbitrarily large, we can achieve arbitrarily close to the cut set bound of 2. Here we illustrate the scheme for the constant channel over rational dimensions. With a scaling factor that is needed to satisfy the power constraints, the transmitted signals are $X_1 = \sum_{k=1}^{M} v_1,k a_k$ and $X_2 = \sum_{k=1}^{M-1} v_2,k b_k$ where $v_{1,m}$ are rational “beamforming” directions, $a_k$, and $b_k$ are lattice symbols that will be specified later. Essentially each symbol carries $\frac{1}{M}$ DoF. We choose $v_{1,1} = (F_{11} F_{22})^{M-1}$. For $i = 1, \ldots, M-1$, $v_{1,i+1}$ and $v_{2,i}$ are chosen as

$$v_{1,i+1} = (F_{12} F_{21})^{i}(F_{11} F_{22})^{M-1-i}$$

$$v_{2,i} = (F_{11} F_{22})^{M-i} F_{12}^{i}(F_{21} F_{22})^{M-1-i}.$$  \hfill (1) \hfill (2)

With this choice, $2M-1$ symbols are aligned in the $M$ dimensional space as shown in Table I and II, so that they can be resolved. After resolving these symbols, they are transmitted over the second hop again in an aligned fashion similar to the first hop but with phases reversal such that interference is canceled at each destination. Specifically, $R_1$ sends the demodulated $a_1$, $a_2 + b_1$, $\ldots$, $a_M + b_{M-1}$ along rational “beamforming” directions $v_{R_1,1}$, $\ldots$, $v_{R_1,M}$. $R_2$ sends the demodulated $a_1 + b_1$, $\ldots$, $a_{M-1} + b_{M-1}$ along “beamforming” directions $v_{R_2,1}$, $\ldots$, $v_{R_2,M-1}$. The “beamforming” directions are chosen as $v_{R_1,1} = (G_{11} G_{22})^{M-1}$ and

$$v_{R_1,i+1} = (G_{12} G_{21})^{i}(G_{11} G_{22})^{M-1-i}$$

$$v_{R_2,i} = -G_{M-i}^{1} G_{12}^{i} G_{21}^{M-1-i}.$$  \hfill (3) \hfill (4)

**D. Outline for Achievable Schemes for Constant Channels**

Now consider the $2 \times 2 \times 2$ IC with interfering relays. In this section, we provide the intuition behind the achievable scheme and the rigorous description of the achievable scheme is deferred to Appendix A.

In time slot 1, $S_1$ sends symbols $a_1(1), \ldots, a_M(1)$ and $S_2$ sends symbols $b_1(1), \ldots, b_{M-1}(1)$ along “beamforming” directions $v_{1,1}, \ldots, v_{1,M}$ and $v_{2,1}, \ldots, v_{2,M-1}$ given in (1) and (2), respectively. Since relays are causal, relays do not transmit any signal in the first time slot. As a consequence, relays only receive signals sent by sources so that they can demodulate those aligned symbols as shown in Table I and II.

In time slot 2, $S_1$ and $S_2$ will transmit new symbols $a_1(2), \ldots, a_M(2)$ and $b_1(2), \ldots, b_{M-1}(2)$ again along $v_{1,1}, \ldots, v_{1,M}$ and $v_{2,1}, \ldots, v_{2,M-1}$, respectively. Relays will transmit the symbols that were already demodulated in the previous time slot using the scheme for the $2 \times 2 \times 2$ IC as described in Section II-C, so that they can be decoded at the destinations. Due to interfering links between the relays, these symbols are not only received at the destinations, but also at the relays. Next, we will illustrate how to cancel these interfering symbols.

The idea is based on the observation that interfering symbols were already received in an aligned fashion and demodulated in the previous time slot. Specifically, the interfering symbols and the demodulated symbols at $R_1$ and $R_2$ are listed in Table III and IV, respectively. The relay can exploit its memory to partially cancel interfering symbols using the previously demodulated symbols. The remaining interfering symbols can be canceled by utilizing the source nodes.

First, consider canceling $a_1(1)$. Note that at $R_1$, $a_1(1)$ was already demodulated in the first time slot. Thus $R_1$ can construct $H_{12} v_{R_1,1} a_1(1)$, which is the interfering signal as shown in Table III, and then subtract it from its received signal to cancel $a_1(1)$. Similarly, $R_2$ can cancel $a_1(1)$ by generating $-H_{21} v_{R_1,1} (a_1(1) + b_1(1))$ and then adding it to its received signal.

Second, consider canceling $b_1(1)$. Note that after adding $-H_{21} v_{R_1,1} (a_1(1) + b_1(1))$ to the received signal at $R_2$ to cancel $a_1(1)$, the effective direction of $b_1(1)$ becomes $H_{21}(v_{R_2,2} - v_{R_1,1})$. Then $S_2$ can resend $b_1(1)$ along $u_{2,1} = -F_{22} H_{21}(v_{R_2,2} - v_{R_1,1})$ so that it is received along $-H_{21}(v_{R_2,2} - v_{R_1,1})$, and thus is canceled at $R_2$. At $R_1$, the effective direction of $b_1(1)$ becomes the sum of the direction of interfering $b_1(1)$ from $R_2$ and that of the resent $b_1(1)$ from $S_2$, which is $H_{12} v_{R_2,1} + F_{12} u_{2,1}$. Therefore, $R_1$ will add $-(H_{12} v_{R_2,1} + F_{12} u_{2,1})(a_2(1) + b_1(1))$ to its received signal to cancel $b_1(1)$.

Third, consider canceling $a_2(1)$. Note that the effective direction of $a_2(1)$ at $R_1$ is $H_{12}(v_{R_2,2} - v_{R_2,1}) - F_{12} u_{2,1}$. Therefore, $S_1$ can resend $a_2(1)$ along the “beamforming” direction $u_{1,1} = -F_{11} (H_{12}(v_{R_2,2} - v_{R_2,1}) - F_{12} u_{2,1})$ to cancel it. Similarly, at $R_2$, the effective direction of $a_2(1)$
In this case, the DoF collapse to 1. This can be easily generalized to the complex case using Theorem 7 in [23].

### III. $2 \times 2 \times 2$ INTERFERENCE NETWORKS WITH ARBITRARY CONNECTIVITY

In this section, we explore the $2 \times 2 \times 2$ interference networks with arbitrary connectivity as shown in Fig. 2. Note that due to full-duplex communications, nodes are simultaneously capable of transmitting and receiving from each other if they are within communication range of each other. The main result is presented in the following theorem.

**Theorem 2:** For the $2 \times 2 \times 2$ interference networks with arbitrary connectivity shown in Fig. 2, the total number of DoF is equal to 2, almost surely, except the case in which there is a direct link between a source and its interfering destination. In this case, the DoF collapse to 1.

**Proof:** When there is a direct link between a source and its interfering destination regardless of the remaining connectivity, the DoF collapse to 1 as already shown in [24].

Since the min-cut DoF outer bound is 2, we only need to provide achievable schemes. Suppose the sources choose not to listen and the destinations choose not to transmit. Then except the red dashed links between the two relays, all other red dashed links do not contribute to the transmit and receive signals in the network, and thus are effectively removed from the network. As a result, the remaining network becomes the $2 \times 2 \times 2$ IC with interfering relays and with either direct links from the sources to their own destinations or not depending on the network connectivity. If such links do not exist, then as shown before, 2 DoF can be achieved. If such links exist, all nodes still use the same achievable scheme as if such links were absent so that interference can still be canceled. The only difference is that there is inter-symbol interference among desired signals. It can be easily seen that this can be removed by exploiting memory at destinations. Therefore, 2 DoF can be achieved.

### IV. THE $2 \times 2 \times 2$ INTERFERENCE NETWORKS WITH ARBITRARY INTER-LAYER CONNECTIVITY

In this section, we study the $2 \times 2 \times 2$ interference networks with arbitrary inter-layer connectivity shown in Fig. 3.

**A. System Model**

The received signal at relays $A_j, B_j$ and destination $D_j$, $j = 1, 2$, in time $t$, respectively, are

\[
Y_{A_j}(t) = H_{A_j,1}(t)X_1(t) + H_{A_j,2}(t)X_2(t) + N_{A_j}(t)
\]

\[
Y_{B_j}(t) = H_{B_j,1}(t)X_1(t) + H_{B_j,2}(t)X_2(t) + N_{B_j}(t)
\]

\[
Y_j(t) = H_{jA_j}(t)X_1(t) + H_{jB_j}(t)X_2(t) + \sum_{i=1}^{2} (H_{jA_i}(t)X_{A_i}(t) + H_{jB_i}(t)X_{B_i}(t)) + N_j(t)
\]
where \( X_i(t) \), \( X_{A_i}(t) \) and \( X_{B_i}(t) \) are the input signals at \( S_i \), \( A_i \) and \( B_i \), respectively. \( H_{A_i}(t), H_{B_i}(t), H_{j_i}(t), H_{j_i-A_i}(t), H_{j_i}(t) \) are the channels from \( S_i \) to \( A_j, B_j, D_j \), from \( A_i \) to \( B_j, D_j \) and from \( B_i \) to \( D_j \), respectively. Note that while the channels of the underlying layered network must be non-zero, depending on network connectivity, each inter-layer channel may or may not exist. \( N_{A_i}(t), N_{B_j}(t) \) and \( N_{j_i}(t) \) are the additive i.i.d. Gaussian noise terms.

We assume that the relays are causal. In addition, the channel coefficients follow a block fading model, i.e., they remain constant in a sufficiently long coherence block and switch to a value that is independently and identically drawn from a continuous distribution from one block to the next. It is assumed that global channel knowledge is available at every node.

**B. Main Result**

**Theorem 3:** For the \( 2 \times 2 \times 2 \times 2 \) IC with arbitrary inter-layer connectivity defined in Section IV-A, the total number of DoF is 2, almost surely, except the case in which there is a direct link from a source to its interfering destination. In this case, the DoF collapse to 1.

**Proof:** Let us consider the case in which there is a direct interfering link from one source to the other destination. Since 1 DoF can be achieved by simply sending only one message, we only need to provide the outer bound. Consider the case in which there is a direct link from \( S_1 \) to \( D_2 \). Suppose we let \( S_2, A_2 \) and \( B_2 \) fully cooperate so that they effectively become one transmitter with 3 antennas. Similarly, \( A_1, B_1 \) and \( D_1 \) are allowed to perfectly cooperate so that they form one receiver with 3 antennas. We end up with a two user interference channel with 1 antenna at transmitter 1 and receiver 2 while 3 antennas at transmitter 2 and receiver 1 and there are noisy links from transmitter 1 to transmitter 2, from receiver 1 to receiver 2 and from receiver 1 to transmitter 2. As proved in [24], the DoF of this channel cannot be more than 1. Since cooperation does not reduce the capacity, the original channel cannot have more than 1 DoF as well. Following the same argument, it can be shown that the networks cannot have more than 1 DoF when there is a direct link from \( S_2 \) to \( D_1 \). For the case in which there are no such direct interfering links, since the min-cut DoF outer bound is 2, we only need to provide achievable schemes.

**C. Representative Networks: Ideas for the Achievable Schemes**

In this section, we present several representative networks to illustrate various ideas behind the achievable schemes. The proofs for general cases are presented in Appendix B where the networks are divided into different classes and depending on the type, one or more ideas represented in this section will be used to construct the achievable schemes. In all cases, our goal is to cancel all interference at the destinations to achieve one DoF per message for a total of 2 DoF.

1) A transform approach: Consider the class of non-layered 2-source 2-sink multihop interference networks shown in Fig. 4. In this class of networks, we can identify one layer (the layer consisting of nodes \( R_1 \) and \( R_2 \) in the figure) such that it separates the network into two non-layered parts between which no link exists. Also no directed cycles are present in each part. Suppose in time \( t \), all relay nodes within each of these two non-layered parts only forward the signals received in time \( t - 1 \), then the channel input-output relationships between sources and relay nodes \( R_1 \) and \( R_2 \) become

\[
Y_{i1}(t) = \sum_{l=1}^{L_i} d_{ii}(t) X_i(t-l) + \sum_{l=1}^{L_i} d_{ii}(t) X_i(t-l) + N_i(t),
\]

where \( L_{ij} \), \( \forall i, j \in \{1, 2\} \), is the delay index defined as the number of nodes on a path from \( S_j \) to \( R_i \), \( L_{ij} \) is the maximum delay between \( S_j \) and \( R_i \), and \( \bar{Z}_i \) is the effective noise at \( R_i \). \( F_{ij}[^{(l)}] \) is the effective channel coefficient of all paths with delay \( l_{ij} \) between \( S_j \) and \( R_i \), which is a sum of the products of the channel coefficients of links along each path with delay \( l_{ij} \). Note that depending on the network connectivity, some of delays between 0 to \( L_{ij} \) may not exist, and thus the channel coefficients associated with those non-existing paths are zero. Similarly, the channel input-output relationships between relays and destinations become

\[
Y_{i2}(t) = \sum_{l=1}^{L_i} d_{ij}(t) X_i(t-l) + \sum_{l=1}^{L_i} d_{ij}(t) X_i(t-l) + N_i(t),
\]

where \( D_{ij} \) is the maximum delay between \( R_i \) and \( D_j \). \( d_{ij} \) is the delay index, and \( N_i \) is the effective noise. In addition, \( G_{ij}[^{(l)}] \) is the effective channel coefficient of the path with delay \( l_{ij} \) between \( R_j \) and \( D_i \). Since we assume channels follow a block fading model, they are time-invariant within every coherence block. In the following we first consider signaling in one coherence block and thus omit the time indices of channel coefficients.

It can be easily seen that the channels between \( S_1, S_2 \) and \( R_1, R_2 \), and channels between \( R_1, R_2 \) and \( D_1, D_2 \) become inter-symbol interference (ISI) channels. We can remove the ISI and convert the channel into a parallel \( 2 \times 2 \times 2 \) IC by introducing a cyclic prefix and using DFT/IDFT. Specifically, consider using the channel \( N_c + D \) times in one coherence block where \( D = \max\{L_{ij}, D_{ij}\} \). Let us first consider the channel between \( S_1 \), \( S_2 \) and \( R_1, R_2 \). For \( N_c \) transmitted symbols at \( S_i \), \( x_i = [x_i(1), \ldots, x_i(N_c)]^T \), we create a length \( N_c + D \) input block by adding \( D \) cyclic prefix to \( x_i \), which is given by \( X_i = [x_i(N_c-D+1), x_i(N_c), x_i(1), \ldots, x_i(N_c)]^T \). By discarding the first \( D \) received symbols, \( R_i \) obtains a block of \( N_c \) symbols, \( Y_{R_i} = [Y_{R_i}(D+1), \ldots, Y_{R_i}(D+N_c)]^T \), which is given by \( Y_{R_i} = F_{ij} x_i + F_{ij} x_i + \bar{Z}_i \) where \( F_{ij} \) is a circulant matrix with the first row given by \( [F_{ij}^{[0]} \ 0 \ \cdots \ 0 \ F_{ij}^{[D]} \ F_{ij}^{[D-1]} \ \cdots \ F_{ij}^{[1]}] \). It is well known that the circulant matrix can be decomposed as \( F_{ij} = U^{-1} \tilde{F}_{ij} U \) where \( U \) is the DFT matrix in which the \((k, n)\)th entry is equal to \( \frac{\sin(\pi k n)}{\pi k n} e^{-j\pi k n}, \ k, n = 0, \ldots, N_c-1, \) and \( \tilde{F}_{ij} \) is a diagonal matrix with the \( n \)th diagonal entry equal to the \( n \)th entry of the

![Fig. 4. A transform approach to a class of non-layered 2-source 2-sink non-layered multihop interference networks](image-url)
column vector \( \sqrt{N_c} U \mathbf{f}_i[0], \ldots, \mathbf{f}_i[D], 0, \ldots, 0 \)^T. By setting \( x_i = U^{-1} \bar{x}_i \) and multiplying the received signal with \( U \), the signal becomes \( \bar{Y}_{R_i} = U Y_{R_i} = \bar{F}_i \bar{x}_i + \bar{F}_i \bar{z}_i + \bar{Z}_{R_i} \). Thus, after these operations the original ISI channels between sources and relays are converted to parallel channels.

The same procedure can be applied to convert the ISI channels between the relays and destinations to parallel channels: \( \bar{Y}_i = \bar{G}_{i} \bar{x}_{R_i} + \bar{G}_{i} \bar{z}_{R_i} + \bar{N}_i \). After the transformation, the originally non-layered channel is converted to a parallel 2 \times 2 \times 2 IC with \( N_c \) sub-channels in each coherence block. By signaling over \( M \) coherence time blocks, each sub-channel becomes a 2 \times 2 \times 2 time-varying IC, over which the aligned interference neutralization schemes proposed in [5] can be applied to cancel interference. It should be noted that while the interference can be canceled, it remains to check if the desired signals are resolvable since the channels are no longer generic after transformation. As shown in [5], the desired signals are resolvable if \( G_{12}^n(t) / G_{11}^n(t) \) and \( F_{12}^n(t) / F_{22}^n(t) \), respectively, are all distinct over \( t \) to 1 \( M \) coherence time blocks, \( n \)th sub-channel in the 1th block. It will be shown later in the appendix that the condition is satisfied for all cases considered in the paper, so that \( 2M-1 \) DoF can be achieved on each sub-channel. Thus, the total DoF achieved is \( \frac{2M-1}{M} N_c \) where \( D \) is a constant depending on the network connectivity. As \( M \) and \( N_c \) go to infinity, 2 DoF are achieved.

2) Interference neutralization exploiting memory: Consider the network shown in Fig. 5(a). Let us first shut down \( B_1 \), resulting in the effective channel connectivity shown in Fig. 5(b). Now consider canceling interference from \( S_1 \) to \( D_2 \). Note that there are two paths from \( S_1 \) to \( D_2 \): \( S_1 \rightarrow A_1 \rightarrow B_2 \rightarrow D_2 \) and \( S_1 \rightarrow A_2 \rightarrow B_2 \rightarrow D_2 \). We will design the scaling factors at node \( A_1 \) and \( A_2 \) in a manner that allows interfering signals passing through these two paths to cancel each other before arriving at \( D_2 \). Specifically, \( A_1 \) simply forwards its received signal while \( A_2 \) uses a scaling factor \( \alpha(t) \). With this transmission scheme, the transmitted signal \( X_1(t) \) from \( S_1 \) is received at \( B_2 \) in time slot \( t \) as \( (H_{B2A1}(t) H_{A1}(t-1)+\alpha(t) H_{B2A2}(t) H_{A2}(t-1)) X_1(t-1) \). In order to cancel interference, \( \alpha(t) \) is chosen to force \( H_{B2A1}(t) H_{A1}(t-1)+\alpha(t) H_{B2A2}(t) H_{A2}(t-1) = 0 \), which leads to \( \alpha(t) = -\frac{H_{B2A1}(t) H_{A1}(t-1)}{H_{B2A2}(t) H_{A2}(t-1)} \). Note that while signal \( X_1(t) \) is canceled at \( D_2 \), it is not canceled at its desired destination \( D_1 \) almost surely since channels from \( A_1, A_2 \) to \( D_1 \) and \( B_2 \) are distinct almost surely.

Next consider canceling interference from \( S_2 \) to \( D_1 \). Note that there are two kinds of paths: delays one and two. In particular, paths \( S_2 \rightarrow A_1 \rightarrow D_1 \), \( S_2 \rightarrow A_2 \rightarrow D_1 \) and \( S_2 \rightarrow A_2 \rightarrow B_2 \rightarrow D_1 \) have delay one while paths \( S_2 \rightarrow A_1 \rightarrow B_2 \rightarrow D_1 \) and \( S_2 \rightarrow A_2 \rightarrow B_2 \rightarrow D_1 \) have delay two. We first cancel interference with delay two by exploiting the memory at \( B_2 \). Suppose the transmit signal from \( S_2 \) is \( X_2(t) \), which will be received at \( B_2 \) in time slot \( t \) as

\[
\hat{Y}_{B2}(t) = H_{B2A2}(t) X_2(t) + (H_{B2A1}(t) H_{A1}(t-1)) X_2(t-1) + \alpha(t) H_{B2A2}(t) H_{A2}(t-1) X_2(t-1).
\]

For \( t = 1 \), \( \hat{Y}_{B2}(1) = H_{B2}(2) X_2(1) \). For \( t = 2 \),

\[
\hat{Y}_{B2}(2) = H_{B2}(2) X_2(2) + (H_{B2A1}(2) H_{A1}(2)) X_2(1) + \alpha(2) H_{B2A2}(2) H_{A2}(2) X_2(1).
\]

To cancel \( X_2(1) \), we can subtract \( \frac{H_{B2A1}(1) H_{A1}(2)}{H_{B2A2}(1)} \hat{Y}_{B2}(1) \) from \( \hat{Y}_{B2}(2) \) to obtain

\[
\hat{Y}_{B2}(2) = H_{B2}(2) X_2(2) - \frac{H_{B2A1}(1) H_{A1}(2)}{H_{B2A2}(1)} \hat{Y}_{B2}(1)
\]

As a result, all interfering signals with delay two are canceled.

To cancel interference with delay one, \( B_2 \) will choose the scaling factor \( \beta(t) \) in a manner that allows the interfering signals passing through three paths with delay one to be canceled before arriving at the destination. Specifically, in time \( t \), the transmit signal at \( B_2 \) is \( X_{B2}(t) = \beta(t) \hat{Y}_{B2}(1) \). With this transmission scheme, the transmit signal \( X_{B2}(t) \) from \( S_2 \) will be received at \( D_1 \) after passing through paths \( S_2 \rightarrow A_1 \rightarrow D_1 \), \( S_2 \rightarrow A_2 \rightarrow D_1 \) and \( S_2 \rightarrow B_2 \rightarrow D_1 \) as

\[
(H_{1A1}(t) H_{A1}(t-1) + \alpha(t) H_{1A2}(t) H_{A2}(t-1) + \beta(t) H_{B2}(t) H_{B2}(t-1)) X_2(t-1),
\]

which is forced to be zero by setting \( \beta(t) = -\frac{H_{1A1}(t) H_{A1}(t-1) + \alpha(t) H_{1A2}(t) H_{A2}(t-1)}{H_{B2}(t) H_{B2}(t-1)} \). Thus all interference from \( S_2 \) is canceled at \( D_1 \).

So far we have shown that all interfering signals are canceled at both destinations. Next, we will show that 2 DoF can be achieved. Consider an \( M+2 \) symbol-extended channel, i.e., signaling is performed over a block of \( M+2 \) time slots. Note that \( 2 \) corresponds to the maximum delay between any two nodes in the network. In each block, \( M \) symbols are sent followed by two zeros. The zero-padding is used to avoid inter-block interference and ensure that noise accumulation due to successively interference cancelation at \( B_2 \) does not build up across blocks. Within each block, the transmission scheme described above is used to cancel inter-user interference. By channel coding across blocks, \( \frac{2M}{M+2} \) DoF can be achieved, which goes to 2 as \( M \to \infty \). Note that symbol-extension with zero padding equal to the maximum delay of the network applies to all memory related schemes proposed in the paper. Due to limited space, we will omit such argument in the remaining part of the paper and focus on how to cancel inter-user interference.

3) Combining aligned interference neutralization with interference neutralization: Consider the network shown in Fig. 6. The transmission is performed over \( M \) dimensions which are obtained by coding across \( M \) time slots, each from one
independent coherence block. The effective channel after symbol extension becomes an $M \times M$ diagonal matrix $H(t)$ with independent diagonal entries, $H(M(t) - 1 + 1), \ldots, H(M(t))$.

The aligned interference neutralization scheme proposed in [5] is applied to the $2 \times 2 \times 2$ IC comprised of the first three layers to cancel interfering signals from $S_1$ to $S_2$ and from $S_2$ to $S_1$. In time $t$, $S_1$ and $S_2$ use $V_1(t)$ and $V_2(t)$ as beamforming matrices to send $M$ and $M - 1$ symbols denoted as the $M \times 1$ vector $a(t)$ and $(M - 1) \times 1$ vector $b(t)$, respectively: $X_1(t) = V_1(t)a(t)$ and $X_2(t) = V_2(t)b(t)$, where $V_1(t) = [v_{1,1}(t), \ldots, v_{1,M(t)}]$ and $V_2(t) = [v_{2,1}(t), \ldots, v_{2,M-1(t)}]$ are chosen in the same manner as in [5]: $v_{1,1}(t) = [1 \ 1 \ 1 \ \ldots \ 1]^T$, and for $i = 1, \ldots, M - 1$,

$$
\begin{align*}
\v_1, i + 1(t) &= (H_{A,11}(t)H_{A,12}(t)H_{B,21}(t)H_{A,21}(t))^{-1}v_{1,1}(t) \\
\v_2, i(t) &= (H_{A,21}(t)H_{A,22}(t)H_{B,11}(t)H_{A,12}(t))^{-1}
\quad H_{A,12}(t)H_{A,21}(t)v_{1,1}(t) \\
&= (H_{A,21}(t)H_{A,12}(t))^{-1}v_{1,1}(t)
\end{align*}
$$

Relays $A_1$ and $A_2$ will send the received signals in the next time slot by multiplying them with $A_1(t)$ and $A_2(t)$, respectively: $X_{A,i}(t) = A_i(t)Y_{A,i}(t - 1)$. With this transmission scheme, the received signals at $B_1$ and $B_2$ are given by (6) and (7), respectively, where $N_{B_i}(t)$ is the effective noise. As shown in [5], the following choices of $A_1(t)$ and $A_2(t)$ force $I_1(t) = I_2(t) = 0$:

$$
\begin{align*}
A_1(t) &= U_1(t)(H_{A,11}(t - 1)V_1(t - 1))^{-1} \\
A_2(t) &= U_2(t)(H_{A,22}(t - 1)V_1(t - 1))^{-1}
\end{align*}
$$

where $U_1(t) = [u_{1,1}(t) \ldots u_{1,M(t)}]$ and $U_2(t) = [u_{2,1}(t) \ldots u_{2,M(t)}]$ and for $i = 1, \ldots, M - 1$

$$
\begin{align*}
u_{1, i + 1}(t) &= (H_{B,11}(t)H_{B,12}(t)H_{B,21}(t)H_{B,22}(t))^{-1}
\quad u_{1,1}(t) \\
u_{2, i}(t) &= -(H_{B,11}(t)H_{B,12}(t)H_{B,21}(t)H_{B,22}(t))^{-1}
\quad H_{B,12}(t)H_{B,21}(t)u_{1,1}(t) \\
u_{2, M}(t) &= -(H_{B,11}(t)H_{B,12}(t)H_{B,21}(t)H_{B,22}(t))^{-1}
\quad u_{1, M}(t)
\end{align*}
$$

where $u_{1,1}(t) = [1 \ 1 \ 1 \ \ldots \ 1]^T$. Thus transmitted signals from $S_1$ and $S_2$ are neutralized at $B_2$ and $B_1$, respectively.

Our next goal is to cancel the transmitted signals from $S_1$ and $S_2$ that arrive at $B_1$ and $B_2$, respectively, through $A_1$ and $A_2$. Mathematically, we want to cancel $X_i(t - 1)$ from the received signal at $B_1$ as given in (6) and $X_2(t - 1)$ from the received signal at $B_2$ as given in (7). In time slot $t = 1$, the transmitted signals passing through $A_1$ and $A_2$ have not arrived at node $B_i$, $i = 1, 2$ and the received signals are given by $Y_{B_i}(1) = H_{B,i}(1)X_i(1) + N_{B_i}(1)$. In time slot $t = 2$, $B_i$ cancels $X_i(1)$ from its received signals as follows.

$$
\begin{align*}
\tilde{Y}_{B_i}(2) &= Y_{B_i}(2) - E_i(2)H_{B,i}^{-1}(1)Y_{B_i}(1) \\
&= H_{B,i}(2)X_i(2) + \tilde{N}_{B_i}(2)
\end{align*}
$$

where $\tilde{N}_{B_i}(2)$ is the effective noise. Similarly, in time $t > 2$, $B_i$ cancels $X_i(t - 1)$ as follows.

$$
\begin{align*}
\tilde{Y}_{B_i}(t) &= Y_{B_i}(t) - E_i(t)H_{B,i}(t - 1)\tilde{Y}_{B_i}(t - 1) \\
&= H_{B,i}(t)X_i(t) + \tilde{N}_{B_i}(t)
\end{align*}
$$

So far, we have canceled all interfering signals with delay two. It can be seen that now all interfering signals will be received with delay one. To cancel it, we design scaling factors at nodes $B_1$ and $B_2$. The transmitted signal at $B_i$ in time slot $t$ is given by $X_{B_i}(t) = B_i(t)\tilde{Y}_{B_i}(t - 1), i = 1, 2$. With this transmitted signal, the received signal at $D_i$ becomes

$$
\begin{align*}
Y_i(t) &= H_{i,i}(t)A_i(t)Y_{A,i}(t - 1)X_i(t - 1) \\
&+ H_{i,A_i}(t)A_i(t)H_{A,i}(t - 1)X_i(t - 1) \\
&+ \sum_{j=1}^{M} H_{B,j}(t)B_j(t)\tilde{Y}_{B_j}(t - 1) + N_i(t)
\end{align*}
$$

where (a) is obtained by using $\tilde{Y}_{B_i}(t)$ given in (11). To cancel the interfering signal $X_i(t - 1)$ from $Y_i(t)$, it is required that $H_{i,A_i}(t)A_i(t)H_{A,i}(t - 1) + H_{B,i}(t)B_j(t)H_{B,i}(t - 1) = 0$, which leads to $B_i(t) = -H_{B,i}(t)H_{i,A_i}(t)A_i(t)H_{A,i}(t - 1)H_{B,i}(t - 1)$.

After canceling all interference at the destination, it can be easily checked that the desired signals are received along full rank matrices and thus can be decoded.

V. Conclusion

We study the effects of non-layered inter-layer and intra-layer links, which are mainly motivated by full-duplex communications, on the fully connected layered interference networks. By characterizing the DoF of the $2 \times 2 \times 2$ IC with arbitrary connectivity and those of $2 \times 2 \times 2$ interference networks with arbitrary inter-layer connectivity, we conclude that direct interference links cause collapse of DoF while indirect interference links do not reduce DoF. While only a class of non-layered 2-source 2-sink two unicast interference networks are considered, we believe the ideas that emerged out of studying this class of non-layered networks will be useful to deal with the interference arriving along paths with different lengths in many non-layered multihop multilayer networks. In addition, the small networks studied in this work are useful to characterize a much larger class of non-layered networks which can be condensed to these small networks.
\[ Y_{B_1}(t) = H_{B_1}(t)X_1(t) + (H_{B_1A_1}(t)A_1(t)H_{A_1}(t-1) + H_{B_1A_2}(t)A_2(t)H_{A_2}(t-1))X_1(t-1) + E_{B_1}(t) \]

\[ Y_{B_2}(t) = H_{B_2}(t)X_2(t) + (H_{B_2A_1}(t)A_1(t)H_{A_1}(t-1) + H_{B_2A_2}(t)A_2(t)H_{A_2}(t-1))X_2(t-1) + E_{B_2}(t) \]

\[ C = \frac{\xi}{\gamma} P^{M-1+2\epsilon} \] where \( \xi = \min\left(\frac{1}{\xi_1}, \frac{1}{\xi_2}\right) \) and \( \xi_j^2 \) is given in (12), and \( B = \frac{\xi}{\gamma} P^{M-1+2\epsilon} \) where \( \xi = \min\left(\frac{1}{\xi_1}, \frac{1}{\xi_2}\right) \) and \( \xi_j^2 = \sum_{m,n=1}^{M} |v_{r_{m},n}v_{r_{j},n}|^2 \) as given in [5].

Relays: Consider the first time slot. As mentioned before, since relays do not transmit in the first time slot, the received signals are the same as the 2 × 2 × 2 IC [5], which is omitted here. The relays will demodulate the aligned symbols as described in [5]. Specifically, \( R_1 \) will demodulate \( a_1(1), a_2(1) + b_1(1), \ldots, a_M(1) + b_{M-1}(1) \) as \( \hat{x}_{R_1,1}(1), \hat{x}_{R_1,2}(1), \ldots, \hat{x}_{R_1,M}(1) \), respectively. \( R_2 \) will demodulate \( a_1(1) + b_1(1), \ldots, a_1(1) + b_1(1), \ldots, a_M(1) + b_{M-1}(1), a_M(1) \) as \( \hat{x}_{R_2,1}(1), \hat{x}_{R_2,2}(1), \ldots, \hat{x}_{R_2,M-1}(1), \hat{x}_{R_2,M}(1) \), respectively.

Consider time slot \( t = 2 \). Let us first consider the transmitted signal. Relays will send the demodulated signals in time slot 1. Specifically, the transmitted signals are \( X_{R_1}(2) = A\sum_{k=1}^{M} v_{r_{1},k} \hat{x}_{R_1,k}(1) \) and \( X_{R_2}(2) = A\sum_{k=1}^{M} v_{r_{2},k} \hat{x}_{R_2,k}(2) \), where \( A \) is chosen to satisfy the power constraint. Now consider the received signals: \( Y_{R_1}(2) = F_{R_1}X_{R_1}(2) + F_{ii}X_{i}(2) + H_{R_1}X_{R_1}(2) + Z_{i}(2) \). \( R_1 \) will first locally generate the following signal and add it to its received signal: \( \hat{Y}_{R_1}(2) = A'(-2H_{12}v_{r_{1},1} \hat{x}_{R_1,1}(1) - \sum_{i=1}^{M-1} (H_{12}v_{r_{1},i} - F_{12}u_{i,1})) \). By writing \( \hat{x}_{R_1,1}(1) = a_1(1) + e_{R_1,1}(1), \hat{x}_{R_2,1}(1) = a_1(1) + b_1(1) + e_{R_2,1}(1), e_{R_1,1}(1) = b_1(1) + e_{R_2,1}(1), e_{R_2,2}(1) = e_{R_1,2}(1) + b_1(1) + e_{R_1,2}(1) \), and \( \hat{x}_{R_2,1}(1) = a_1(1) + e_{R_1,1}(1) \) where \( e \) represents the demodulation error, \( Y_{R_1}(2) = \hat{Y}_{R_1}(2) \) can be easily calculated as

\[ Y_{R_1}(2) = A' \left( F_{11}v_{1,1}a_1(2) + \sum_{i=2}^{M} F_{11}v_{1,i}(a_i(2) + b_{i-1}(2)) \right) \]

APPENDIX A

ACHIEVABILITY FOR THE 2 × 2 × 2 IC WITH INTERFERING RELAYS

We first provide the achievable scheme for the constant channels.

Sources: Message \( W_1 \) is split into \( M \) sub-messages. Sub-message \( W_{1,k_1}, k_1 \in \{1, \ldots, M\} \), is encoded using a codebook with the codeword of length \( n \) denoted as \( a_{k_1}(1), \ldots, a_{k_1}(n) \). Similarly, message \( W_2 \) is split into \( M - 1 \) sub-messages. Sub-message \( W_{2,k_2}, k_2 \in \{1, \ldots, M - 1\} \), is encoded using a codebook with the codeword of length \( n \) denoted as \( b_{k_2}(1), \ldots, b_{k_2}(n) \). For any \( \epsilon > 0 \) and a constant \( \gamma \), let \( C \) denote all integers in the interval \( \left[ -\gamma P^{M-1+2\epsilon}, \gamma P^{M-1+2\epsilon} \right] \), i.e., \( C = \{ x : x \in \mathbb{Z}, x \left[ \gamma P^{M-1+2\epsilon}, \gamma P^{M-1+2\epsilon} \right] \} \). \( a_{k_1}(t) \) and \( b_{k_2}(t) \) are obtained by uniform i.i.d. sampling on \( C \). Essentially, each sub-message carries \( \frac{1}{\gamma} \) DoF. Then in slot 1, the transmitted signals are \( X_1(1) = A \sum_{k_1=1}^{M} v_{1,k_1} a_{k_1}(1) \) and \( X_2(1) = A \sum_{k_2=1}^{M-1} u_{2,k_2} b_{k_2}(1) \), where \( v_{1,k_1} \) and \( u_{2,k_2} \) are chosen as in (1) and (2), respectively. To satisfy the power constraints, the constant scaling factor \( A \) is chosen to be \( \frac{\xi}{\gamma} P^{M-1+2\epsilon} \) where \( \xi = \min\left(\frac{1}{\xi_1}, \frac{1}{\xi_2}\right) \) as in [5].

Then in time \( t = 2, \ldots, n \), the transmitted signals are

\[ X_1(t) = A' \left( \sum_{k_1=1}^{M} v_{1,k_1} a_{k_1}(t) + \sum_{i=1}^{M-1} u_{1,i} a_{i+1}(t-1) \right) \]

\[ X_2(t) = A' \left( \sum_{k_2=1}^{M-1} v_{2,k_2} b_{k_2}(t) + \sum_{i=1}^{M-1} u_{2,i} b_{i+1}(t-1) \right) \]

where \( u_{2,i} = -F_{2i}^{-1}(H_{21}(v_{r_{2},i+1} - v_{r_{1},i}) - F_{21}u_{i,1}) \) and \( u_{1,i} = -F_{1i}^{-1}(H_{12}(v_{r_{1},i+1} - v_{r_{2},i}) - F_{12}u_{i,1}) \). Note that \( u_{1,0} \) is defined to be 0. And \( A' \) should be chosen to satisfy the power constraints, i.e.,

\[ E[X_t^2(t)] \leq \gamma^2 A'^2 \left( \sum_{k=1}^{M} v_{j,k}^2 + \sum_{i=1}^{M-1} u_{j,i}^2 \right) P^{M-1+2\epsilon} \leq P \]

where \( M_1 = M \) and \( M_2 = M - 1 \). To satisfy power constraints at both transmitters, we choose \( A' = \min\{C, B\} \) where \( C \) and \( B \) are scaling factors that can ensure sources and relays satisfy the power constraints, respectively. Specifically,
\(Y_{R_1}(2) + \tilde{Y}_{R_1}(2)\) as follows. Define the following constellation set \(C_{R_1} = \{A' | F_{11}v_{1,1}x_{R_1,1} + \cdots + F_{11}v_{1,M}x_{R_1,M}\}\), where \(x_{R_1,1}\) is an integer in the interval \([-\gamma P^{\frac{1}{M-I}}, \gamma P^{\frac{1}{M-I}}]\) and \(x_{R_1,i+1}\) is also an integer but in the interval \([-2\gamma P^{\frac{1}{M-I}}, 2\gamma P^{\frac{1}{M-I}}]\). Notice that \(v_{1,1}, \cdots, v_{1,M}\) are distinct monomial functions of channel coefficients and thus rationaly independent almost surely. Thus, there is a one-to-one mapping from \(C_{R_1}\) to \(x_{R_1,k_1}, k_1 \in \{1, \cdots, M\}\). Then Relay 2 will find the point in \(C_{R_2}\) which has the minimal distance between \(Y_{R_1}(2) + \tilde{Y}_{R_1}(2)\) and received signals at relays. The achievable scheme is exactly the same as that without such words and demodulation are replaced with time dimensions, to infinity, time indices 2 and 1 in the above analysis with \(t\) to \(t\) are generic and thus satisfy the condition almost surely. We only need to show that the \(F_{11}v(t)F_{21}(t)F_{22}(t)F_{11}(t)\) are all distinct almost surely. As shown in Section IV-C1, in time slot \(t\) \(F_{11}(t)F_{21}(t)F_{22}(t)F_{11}(t)\neq 0\) almost surely. Note that this is a polynomial in variables \(H(t)\) and \(H(t)\). To prove such polynomial is non-zero almost surely, it suffices to show it is not a zero polynomial. By setting \(H_{B_1,A_1}(t)\), \(H_{B_1,B_2}(t)\) and \(H_{B_2,B_2}(t)\) to be zero and \(H_{A,j}(t-1) = H_{A,j}(t-1) = H_{B_2,B_2}(t)\) to \(H_{B_2,B_2}(t)\) \(= H_{A,j}(t-1) = H_{A,j}(t-1) = H_{B_2,B_2}(t)\) \(= H_{B_2,B_2}(t)\) \(= 1\), \(j \in \{1,2\}\) and \(H_{A,2}(t-1) = 1, H_{A,2}(t-1) = 2, H_{B_2,B_2}(t) = 3, H_{B_2,B_2}(t) = 4, F_{11}(t)F_{21}(t)F_{22}(t)F_{11}(t)\neq \frac{1}{18\pi^2} \neq 0\). Therefore, it is not a zero polynomial and the polynomial is non-zero almost surely.

If none of \(N_A\) and \(N_B\) is equal to zero, then one must be equal to 1. Due to symmetry, let \(N_A\) to be 1 and there are inter-layer links through \(A_1\). In this case, we can shut down node \(A_1\) so that again layer \(B\) separates the network into two parts and there is no link between these two layers. Using the transform approach 2 DoF can be achieved almost surely.

Next, we will consider the case where \((N_A, N_B) = (2,2)\). Within this class, we further categorize the network based on the number of inter-layer links. Let us denote the number of inter-layer links on a path from \(S_1\) to \(D_2\) and \(S_2\) to \(D_1\) as \(L_1\) and \(L_2\), respectively. Note that \(0 \leq L_1, L_2 \leq 4\). Since \((N_A, N_B) = (2,2)\), the total number of inter-layer links must be at least four, i.e.,
When $(L_1, L_2) = (1, 3)$, there are four possible cases as shown in Fig. 8. The achievable schemes for these four cases are essentially the same. We will only describe the achievable scheme for the network shown in Fig. 8(a). In this case, we first cancel interference with delay one from $S_2$ to $D_1$ through paths $S_2 \rightarrow A_1 \rightarrow D_1$ and $S_2 \rightarrow A_2 \rightarrow D_1$. $A_1$ will forward its received signal in the next time slot and $A_2$ will amplify its received signal by $\alpha(t)$ and then forward it. $\alpha(t)$ is chosen to cancel interference along these two paths, i.e., $H_{D_1A_1}(t)H_{A_1S_2}(t-1) + \alpha(t)H_{D_2A_2}(t)H_{A_2S_2}(t-1) = 0$, leading to $\alpha(t) = -H_{D_2A_1}(t)H_{A_1S_2}(t-1)/H_{D_2A_2}(t)H_{A_2S_2}(t-1)$. After canceling this interfering signal, from $D_1$’s perspective, inter-layer links from $A_1$ to $D_1$ and from $A_2$ to $D_1$ are effectively removed from the network. The remaining network without these two links falls into the category of $(N_A, N_B) \neq (2, 2)$, for which using the transform approach can cancel all interference. What remains to be shown is that $F_{12}^n(t)F_{21}^n(t)/F_{22}^n(t)F_{11}^n(t)$ are distinct for all $t$ almost surely. Similar to previous arguments, it is sufficient to construct a set of channel coefficients such that $F_{12}^n(t)F_{21}^n(t)F_{22}^n(t)F_{11}^n(t) = F_{12}^n(t)F_{21}^n(t)F_{22}^n(t)F_{11}^n(t) \neq 0$. This is done by setting inter-layer links in the non-layered parts to be zero, i.e., $H_{B_1A_1}(t) = H_{B_2A_1}(t) = 0$, setting the remaining layered links as the same values chosen for the case shown in Fig. 7, and then setting remaining inter-layer links in such a manner that $\alpha(t) = 1$. With this choice, it is easily seen that $F_{12}^n(t)F_{21}^n(t)F_{22}^n(t)F_{11}^n(t) = F_{12}^n(t)F_{21}^n(t)F_{22}^n(t)F_{11}^n(t) = 4\alpha e^{-\frac{2\pi i}{N}}$. This completes the proof. Note that similar arguments can be applied to the cases where transform approach is used and will be omitted due to limited space.

When $(L_1, L_2) = (1, 4)$, there are four possible connectivities as shown in Fig. 9. The achievable schemes are very similar to the case where $(L_1, L_2) = (1, 3)$. For example, for the network shown in Fig. 9(a), using $A_1$ and $A_2$, interference along paths $S_2 \rightarrow A_1 \rightarrow D_1$ and $S_2 \rightarrow A_2 \rightarrow D_1$ can be canceled. The remaining interference can be canceled using the transform approach so that 2 DoF can be achieved.

When $(L_1, L_2) = (2, 2)$, there are a total of 6 possible connectivities. Two basic connectivities are shown in Fig. 10. By switching labels of the nodes $A_1$ and $A_2$ or $B_1$ and $B_2$ or both in the network shown in Fig. 10(a), we obtain another three connectivities. By switching labels of $S_1$ and $S_2$ as well as $D_1$ and $D_2$, we can obtain the last connectivity in this case. The achievability for Fig. 10(a) was already presented in Section IV-C3. For the network shown in Fig. 10(b), $A_1$ and $A_2$ can be used to cancel interference arriving at $D_1$ along paths $S_2 \rightarrow A_1 \rightarrow D_1$ and $S_2 \rightarrow A_2 \rightarrow D_1$. Once this interference is neutralized, the transform approach can be applied by using $B_1$ and $B_2$ to cancel the remaining interference to achieve 2 DoF.

When $(L_1, L_2) = (2, 3)$, there are a total of 12 possible connectivities in this case, which can be obtained by adding one inter-layer link from $S_2$ to $D_1$ in each of the connectivities of $(L_1, L_2) = (2, 3)$. Since there are two possible ways to add such link in each case and there are a total of 6 possible cases, the total number of connectivities is 12. We will consider adding links for three basic connectivities with $(L_1, L_2) = (2, 3)$ as shown in Fig. 11. The remaining cases can be obtained by switching the labels of some nodes. The networks obtained by adding one link based on the network given in Fig. 10(a) are shown in Fig. 11(a). Each of the two dashed lines represents one possible way to add the link. Similarly, Fig. 11(b) shows two possible connectivities by adding one inter-layer link based on the network shown in Fig.10(b). Finally, by adding links to the network obtained by switching labels $S_1$, $S_2$ and $D_1$, $D_2$, we obtain the last two cases as shown in Fig. 11(c).

Let us first consider the achievable scheme for the network shown in Fig.11(a). Suppose the link connecting $S_2$ and $B_1$ is added. The achievable scheme is similar to that of the case without this link presented in Section IV-C3. $S_1$, $S_2$, $A_1$ and $A_2$ will use the same transmission scheme as described in Section IV-C3. Then the received signal at $B_1$ is given by (13). The received signal at $B_2$ is given by (7). Note that (13) has
only one additional term $H_{B,2}X_2(t)$ compared to (6). These two nodes will use the same procedure to cancel $X_1(t-1)$ and $X_2(t-1)$ at $B_1$ and $B_2$, respectively. In time slot $t = 1$, there is no inter-symbol interference and the received signals are given by $Y_{B,1}(1) = H_{B,1}(1)X_1(1) + H_{B,2}(1)X_2(1) + \tilde{N}_{B,1}(1)$ and $Y_{B,2}(1) = H_{B,2}(1)X_2(1) + \tilde{N}_{B,2}(1)$. In time slot $t = 2$, $B_1$ cancels $X_1(1)$ as follows:

$$\tilde{Y}_{B,1}(2) = Y_{B,2}(2) - E_1(2)H_{B,1}^{-1}(1)Y_{B,1}(1)$$
$$= H_{B,1}(2)X_1(2) + H_{B,2}(2)X_2(2) - E_1(2)H_{B,1}^{-1}(1)H_{B,2}(1)X_2(1) + \tilde{N}_{B,1}(2)$$

where $\tilde{N}_{B,1}(2)$ is the effective noise. Similarly, in time $t > 2$, $B_1$ cancels $X_1(t-1)$ as shown in (14). For $B_2$, it cancels $X_2(1)$ in time $t = 2$ as follows: $Y'_{B,2}(2) = Y_{B,2}(2) = E_2(2)H_{B,1}^{-1}(1)Y_{B,1}(1) = H_{B,2}(2)X_2(2) + \tilde{N}'_{B,2}(2)$. In time $t > 2$, $B_2$ cancels $X_2(t-1)$ as follows:

$$Y'_{B,2}(t) = Y_{B,2}(t) - E_2(t)H_{B,2}^{-1}(t-1)Y'_{B,2}(t-1)$$
$$= H_{B,2}(t)X_2(t) + \tilde{N}'_{B,2}(t). \quad (15)$$

From $Y'_{B,2}(t)$, we can resolve $X_2(t)$ by inverting the effective channel, i.e., $Y'_{B,2}(t) = H_{B,2}^{-1}(t)Y'_{B,2}(t) = X_2(t) + H_{B,2}(t)\tilde{N}'_{B,2}(t)$.

So far, interference with delay two has been canceled. We still need to cancel interference with delay one. First consider interfering signals from $S_1$ to $D_1$, which are along paths $S_1 \rightarrow B_1 \rightarrow D_1$ and $S_1 \rightarrow A_2 \rightarrow D_2$. $B_1$ will send $\tilde{Y}_{B,1}(t)$ in the next block by multiplying a matrix $B_1(t)$, i.e., $X_{B,1}(t) = B_1(t)\tilde{Y}_{B,1}(t-1)$. With this transmission scheme, the interfering signal $X_{B,1}(t)$ is received at $D_2$ as $H_{B,2}(t)X_{B,1}(t-1) + H_{B,2}(t)A_2(t)H_{A,2}(t-1)X_1(t-1)$, which can be canceled by setting $B_1(t) = -H_{B,2}(t)A_2(t)H_{A,2}(t-1)H_{B,1}^{-1}(t-1)$. After canceling the interference from $S_1$, $D_2$ can cancel the SI of its desired signal using memory and then decode its message.

Next consider canceling interference from $S_2$ to $D_1$. The interference arriving at $D_1$ along paths $S_2 \rightarrow A_1 \rightarrow D_1$ and $S_2 \rightarrow B_1 \rightarrow D_1$ is given by

$$I_2(t) = (H_{A,1}(t)A_1(t)H_{A,2}(t-1) + H_{B,1}(t)B_1(t)H_{B,2}(t-1))X_2(t-1) + H_{B,1}(t)B_1(t)X_2(t-1).$$

To cancel it, $B_2$ will generate its transmitted signal in such a manner that it is arrived at $D_1$ as $-I_2(t)$. It can be seen that from $\tilde{Y}_{B,2}(1)$, $\tilde{Y}_{B,2}(2), \ldots, \tilde{Y}_{B,2}(t-1), B_2$ can construct the signal given in (16). Then the transmitted signal from $B_2$ is $X_{B,2}(t) = H_{B,2}^{-1}(t)\tilde{Y}_{B,2}(t)$. With this transmission scheme, all interference is canceled at $D_1$ and the received signal at $D_1$ becomes $Y_1(t) = (H_{A,1}(t)A_1(t)H_{A,2}(t-1) + H_{B,1}(t)B_1(t)H_{B,2}(t-1))X_2(t-1) + N'_1(t)$. After canceling all interference, $D_1$ can decode its message.

Next consider the case in which the dashed link from $A_2$ to $D_1$ in Fig. 11(a) is added. The achievable scheme is very similar to the case without this link. Specifically, all nodes except $B_2$ use the same scheme as the case in which that link is absent. The only difference is that the amplifying matrix $B_2(t)$ should be changed to account for the additional interference due to the additional link. In this network, the interference from $S_2$ to $D_1$ is the sum of signals arriving along three paths $S_2 \rightarrow A_1 \rightarrow D_1$, $S_2 \rightarrow A_2 \rightarrow D_1$ and $S_2 \rightarrow B_2 \rightarrow D_1$: 

$$H_{A,1}(t)A_1(t)H_{A,2}(t-1) + H_{B,1}(t)B_1(t)H_{B,2}(t-1) + H_{1,2}(t)B_2(t)H_{B,2}(t-1)X_2(t-1) - E_1(t)\tilde{Y}_{B,1}(1)H_{B,1}(1)X_1(1) + \tilde{N}'_{B,2}(t)$$

Next consider the networks shown in Fig. 11(b). Let us first use $A_1$ and $A_2$ to cancel interfering signals along paths $S_2 \rightarrow A_1 \rightarrow D_1$ and $S_2 \rightarrow A_2 \rightarrow D_1$. Once this interfering signal is canceled, links $A_1 \rightarrow D_1$ and $A_2 \rightarrow D_1$ are effectively removed from the network. The remaining interference can be canceled using the transform approach by $B_1$ and $B_2$.

Finally, for the networks shown in Fig. 11(c), $B_1$ and $B_2$ will be used to cancel interference along paths $S_2 \rightarrow B_1 \rightarrow D_1$ and $S_2 \rightarrow B_2 \rightarrow D_1$. The remaining interference can be canceled using $A_1$ and $A_2$ by exploiting the transform approach.

When $(L_1,L_2) = (2,4)$, there are a total of 6 possible cases. Three basic connectivities are shown in Fig. 12. The remaining three possible connectivities can be obtained by switching labels $A_1$ and $A_2$, or $B_1$ and $B_2$, or both in the network shown in Fig. 12(c). For the case shown in Fig. 12(a), by shutting down nodes $A_1$ and $A_2$, the network becomes the $2 \times 2 \times 2$ IC for which 2 DoF can be achieved [5]. Similarly, by shutting down nodes $B_1$ and $B_2$ in the network shown in Fig. 12(b), it becomes the $2 \times 2 \times 2$ IC as well. For the network shown in Fig. 12(c), by shutting down nodes $A_2$ and $B_2$, the network becomes the $2 \times 2 \times 2$ IC with interfering relays, for
\[
\begin{align*}
\tilde{Y}_{B_1}(t) &= Y_{B_1}(t) - E_1(t)H_{B_1}^{-1}(t-1)\tilde{Y}_{B_1}(t-1) \\
&= H_{B_1}(t)X_1(t) + H_{B_1}(t)X_2(t) + \sum_{n=1}^{t-1} (-1)^{t-n} \left( \prod_{m=n}^{t-1} E_1(m+1)H_{B_1}^{-1}(m) \right) H_{B_1}(n)\tilde{Y}_{B_1}(n) + \tilde{N}_{B_1}(t) 
\end{align*}
\]

which 2 DoF can be achieved as shown before.

\[
\begin{align*}
-\tilde{Y}_{B_2}(t) &= (H_{A_1}(t)A_1(t)H_{A_1}^{-1}(t-1) + H_{B_1}(t)B_1(t)H_{B_1}(t-1))\tilde{Y}_{B_2}(t-1) \\
&= H_{B_1}(t)B_1(t) \left( \sum_{n=1}^{t-2} (-1)^{t-n-1} \prod_{m=n}^{t-2} E_1(m+1)H_{B_1}^{-1}(m) \right) H_{B_1}(n)\tilde{Y}_{B_1}(n) = I_2(t) + \tilde{N}_{B_2}(t)
\end{align*}
\]

A_2 in Fig. 14(a) and 14(d), and shutting down B_1 and B_2 in Fig. 14(b) and 14(c), we obtain the 2 \times 2 \times 2 IC for which 2 DoF can be achieved.

Fig. 12. Networks with \((L_1, L_2) = (2, 4)\)

When \((L_1, L_2) = (3, 3)\), the total number of possible connectivities is 12. Four basic networks are shown in Fig. 13. By switching the labels of \(B_1\) and \(B_2\) in Fig. 13(b) and labels of \(A_1\) and \(A_2\) in Fig. 13(c), two more connectivities are obtained. Switching \(A_1\) and \(A_2\), or \(B_1\) and \(B_2\) or both in Fig. 13(b) and 13(d) will give us three more connectivities for each figure for a total of six possible connectivities. By shutting down \(B_1\) and \(B_2\) in Fig. 13(a) and shutting down \(A_1\) and \(A_2\) in Fig. 13(c), we obtain the 2 \times 2 IC for which 2 DoF can be achieved [5]. By shutting down \(A_1\) and \(B_2\) in Fig. 13(c) and shutting down \(A_1\) and \(B_1\) in Fig. 13(d), we obtain the 2 \times 2 IC with interfering relays for which 2 DoF can be achieved. Achievable schemes for other cases follow directly.

Fig. 13. Networks with \((L_1, L_2) = (3, 3)\)

When \((L_1, L_2) = (3, 4)\), there are a total of four possible connectivities as shown in Fig. 14. By shutting down \(A_1\) and \(A_2\) in Fig. 14(a) and 14(d), and shutting down \(B_1\) and \(B_2\) in Fig. 14(b) and 14(c), we obtain the 2 \times 2 \times 2 IC for which 2 DoF can be achieved.

Fig. 14. Networks with \((L_1, L_2) = (3, 4)\)

When \((L_1, L_2) = (4, 4)\), simply shutting down node \(A_1\) and \(A_2\) gives us the 2 \times 2 \times 2 IC for which 2 DoF can be achieved.

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