Towards Full-Duplex Multihop Multiflow — A Study of Non-Layered Two Unicast Wireless Networks

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***Abstract*—Starting from the elemental $2 \times 2 \times 2$ interference channel, there has been much progress in the understanding of multihop multiflow wireless networks through degrees of freedom (DoF) studies that have produced important ideas such as (aligned) interference neutralization. However, much of this progress has been limited to layered connectivity models that are essentially motivated by the assumption that wireless networks can only operate in half-duplex mode. Motivated by recent breakthroughs in full-duplex radio technology, in this work we expand the $2 \times 2 \times 2$ interference channel model beyond layered connectivity in order to study the impact of full-duplex operation. In particular we study the impact of intra-layer connectivity between relays that are in the same layer, the impact of direct inter-layer connectivity between sources and destinations, and the impact of intermediate inter-layer connectivity that connects sources or destinations, not directly to each other, but to relay nodes in non-adjacent layers in a $2 \times 2 \times 2 \times 2$ interference channel. We show that intra-layer links and intermediate inter-layer interference links do not cause a collapse of DoF while direct interference links do cause a collapse of DoF.**

***Index Terms*—Capacity, degrees of freedom, full-duplex, interference neutralization, multihop multiflow**

I. Introduction

Characterizing the capacity of wireless networks is one of the most important problems of network information theory. Recent years have seen rapid progress on mainly two settings: single flow multihop networks and single hop multiflow (interference) networks. In [1], it is shown that the capacity (within a constant gap) of the single source multihop multicast network is given by the network min-cut. In [2], the capacity of a wireless multihop broadcast network where a single source sends independent messages to multiple destinations over a multihop network is characterized within a constant gap from the cut-set bound. On the other hand, a variety of capacity approximations in the form of DoF, generalized degrees of freedom (GDoF), capacity within a constant gap and even the exact capacity have been obtained for various single hop interference networks [3], [4].

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Spurred by the advances in single hop multiflow, and multihop single flow settings, there has been increasing interest in the setting that combines the two — the *multihop multiflow* setting, starting with the most elemental multihop multiflow network, the $2 \times 2 \times 2$ interference channel formed by concatenation of two 2-user interference channels. In [5], it is shown that the $2 \times 2 \times 2$ interference network achieves the cut-set outer bound value of 2 DoF for almost all channel coefficients. In [6], Shomorony and Avestimehr characterize the DoF for two-unicast layered wireless networks with arbitrary number of layers, arbitrary number of nodes per layer, and arbitrary connectivity between adjacent layers. It is shown that 2 unicast layered wireless networks can only have 1, 2 or $3/2$ DoF. A related work on 2 unicast networks by Wang, Kamath and Tse in [7] characterizes the capacity region for two unicast information flows over a layered linear deterministic network. The DoF result for the layered two unicast problem is generalized by Wang, Gou and Jafar in [8] allowing all possible unicasts between 2 sources and 2 destinations. In this setting, known as 2-source 2-sink all-unicast layered wireless networks (also known as multihop X networks), each source has an independent message for each destination. Wang, Gou and Jafar show in [8] that with arbitrary number of nodes in each layer and with arbitrary connectivity between adjacent layers, the DoF belong to the set $\{1, 4/3, 3/2, 5/3, 2\}$ almost surely.

The $2 \times 2 \times 2$ setting has been successfully extended in several directions, including finite-field settings in [9], time-varying linear schemes in [10], [11], [12], [13] and to the $K \times K \times K$ network in [14]. However, the extensions have been limited primarily to layered network models, in large part due to the traditional assumption that *wireless networks must operate in a half-duplex mode*. With the growing interest and remarkable recent progress in building practical full-duplex radios [15], [16], [17], [18], [19], [20], [21], it is clear that this traditional assumption is no longer valid. Consequently, there is the need to update the theoretical models to include the full-duplex assumption, and to understand the implications of full-duplex operation in challenging network settings such as the multihop multiflow problem. This work presents the outcomes of our efforts in this direction.

Layered network models are essentially motivated by the assumption of *half-duplex* communication where each layer, i.e., all nodes in a layer, are either transmitting or receiving at any time, but not both simultaneously. Thus, the nodes within each layer do not hear each other. Also, among all layers that

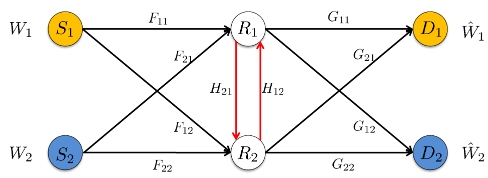


Fig. 1. $2 \times 2 \times 2$ IC with interfering relays

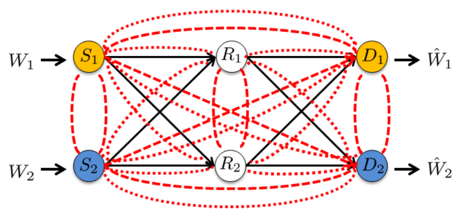


Fig. 2. The $2 \times 2 \times 2$ IC with arbitrary connectivity

are within communication range of each other, only one layer is active at any time. However, in full-duplex communications, this is not the case. Since the nodes can receive and transmit simultaneously, all nodes within a layer can hear each other as well, resulting in intra-layer links. For example, consider the simplest setting of multihop interference networks consisting of 2 sources, 2 relays and 2 destinations as shown in Fig. 1. If relay nodes R_1 and R_2 are assumed to be half-duplex and they are always either both transmitting or both receiving simultaneously, then they cannot hear each other, i.e., the links between them do not exist. This is no longer the case if R_1 and R_2 can transmit and receive simultaneously. While a first order capacity characterization in the form of DoF for the layered case is known to meet the min-cut outer bound in [5], it remains unknown if this is still the case when there are interfering links between two relays due to full-duplex operations. It is this problem that we would like to explore first in this paper. As will be shown in Section II, the min-cut outer bound of 2 DoF is still achieved in this case. The achievable scheme is built upon aligned interference neutralization, originally introduced in [5] for the case without interfering relays, with interesting modifications needed to cancel intra-layer interference by exploiting the memory of sources and relays.

With the understanding of how to deal with the intra-layer interference between relays, we further allow the sources and destinations to operate in the full-duplex mode. In this case, since every node can transmit and receive simultaneously, there may exist a two-way link between any two nodes depending on the communication ranges, leading to all possible connectivities as shown in the red dashed links in Fig. 2. Note that two links between any two nodes in the figure represent the forward link (due to transmitting) and the backward link (due to receiving), respectively. For this network, it has already been shown in [24] that if there is a direct interfering link from a source to the other destination, the DoF collapse to one. However, it remains unknown if such DoF collapse still happens without direct interfering links. This is the problem that we explore in Section III. What we show in this work is that for all cases without direct interfering links, the min-cut outer bound of 2 DoF is achieved almost surely.

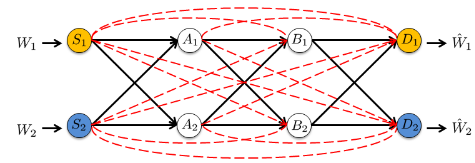


Fig. 3. $2 \times 2 \times 2 \times 2$ IC with arbitrary inter-layer connectivity

The results summarized so far indicate that the intra-layer interference introduced by full-duplex relays causes no loss of DoF for the underlying layered $2 \times 2 \times 2$ IC, while the end-to-end inter-layer cross-links that connect a source directly to its unintended destination cause a collapse of DoF. It remains to understand the role of intermediate inter-layer links that connect sources or destinations, not directly to each other, but to intermediate relay nodes. To include inter-layer links to/from relay nodes, we further study the $2 \times 2 \times 2 \times 2$ IC formed by concatenation of 3 interference channels with arbitrary inter-layer connectivity as shown in Fig. 3, where the black links indicate the underlying layered connectivity and the red dashed links represent all other possible inter-layer connectivities in the network. As will be shown in Section IV, even for this class of networks, the min-cut outer bound of 2 DoF can be almost surely achieved except when there is a direct interfering link from a source to its unintended destination. In this case, the DoF collapse to one.

II. The $2 \times 2 \times 2$ Interference Channel with Interfering Relays

In this section, we explore the $2 \times 2 \times 2$ IC with interfering relays shown in Fig. 1.

A. System Model

The received signal at relay R_k, $k \in \{1, 2\}$ in time slot t is $Y_{R_k}(t) = F_{k1}(t)X_1(t)+F_{k2}(t)X_2(t)+H_{k\bar{k}}(t)X_{R_{\bar{k}}}(t)+Z_k(t)$ where $\bar{k} = 1$ if $k = 2$ and $\bar{k} = 2$ if $k = 1$. $F_{kj}(t)$, $\forall k, j \in \{1, 2\}$, is the complex channel coefficient from S_j to R_k, $H_{k\bar{k}}$ is the channel coefficient from $R_{\bar{k}}$ to R_k, $X_j(t)$ is the input signal from S_j, $X_{R_{\bar{k}}}(t)$ is the input signal from $R_{\bar{k}}$ and $Z_k(t)$ is the independent identically distributed (i.i.d.) zero mean unit variance circularly symmetric complex Gaussian noise. The received signal at D_k in time slot t is given by $Y_k(t) = G_{k1}(t)X_{R_1}(t)+G_{k2}(t)X_{R_2}(t)+N_k(t)$, where $G_{kj}(t)$, $\forall k, j \in \{1, 2\}$, is the complex channel coefficient from R_j to D_k and $N_k(t)$ is the i.i.d. Gaussian noise. We assume every node in the network has an average power constraint P. The relays are *full-duplex*. In addition, the relays are causal, i.e., the transmitted signals at relays only depend on the *past* received signals but not the current received signals. We assume that sources and relays know all channels while destinations only know channels from relays. To avoid degenerate conditions, we assume the absolute values of channel coefficients of the layered links are bounded between a nonzero minimum value and a finite maximum value. However, the channel coefficients of intra-layer links can be any arbitrary value. We will consider two settings where channel coefficients are time-varying or constant: 1) the channel coefficients change and are drawn i.i.d. from a continuous distribution for every channel use, and 2) the channel coefficients are drawn i.i.d. from a continuous distribution before the transmissions, and once they are drawn, they remain unchanged during the entire transmission.

Source S_k, $k \in \{1,2\}$ has a message W_k for destination D_k. We denote the size of message W_k as $|W_k|$. For the codewords spanning n channel uses, the rates $\mathcal{R}_k = \frac{\log(|W_k|)}{n}$ are achievable if the probability of error for both messages can be simultaneously made arbitrarily small by choosing an appropriately large n. The sum-capacity $\mathcal{C}_\Sigma(P)$ is the maximum achievable sum rate. The number of degrees of freedom is defined as $d = \lim_{P\to\infty} \frac{\mathcal{C}_\Sigma(P)}{\log P}$.

B. Main Result

The main result is presented in the following theorem.

Theorem 1: For the $2\times2\times2$ IC with interfering relays defined in Section II-A, the total number of DoF is equal to 2 for both constant and time-varying channel coefficients, almost surely.

Since the min-cut DoF outer bound is 2, we only need to provide an achievable scheme. The achievable scheme is designed on top of that for the $2\times2\times2$ IC with additional effort to cancel interference caused by relays. Thus, before presenting the scheme for this channel, it is helpful to first review the achievable scheme for the $2\times2\times2$ IC proposed in [5].

C. Review: Achievable Scheme for the $2\times2\times2$ IC

Signaling is performed over M dimensions regardless of whether it is time or rational dimensions. Over these M dimensions, S_1 sends M signals and S_2 sends $M-1$ signals so that a total of $\frac{2M-1}{M}$ DoF is achieved. Since M can be chosen arbitrarily large, we can achieve arbitrarily close to the cut set bound of 2. Here we illustrate the scheme for the constant channel over rational dimensions. With a scaling factor that is needed to satisfy the power constraints, the transmitted signals are $X_1 = \sum_{k_1=1}^{M} v_{1,k_1} a_{k_1}$ and $X_2 = \sum_{k_2=1}^{M-1} v_{2,k_2} b_{k_2}$ where $v_{i,m}$ are rational "beamforming" directions. a_{k_1} and b_{k_2} are lattice symbols that will be specified later. Essentially each symbol carries $\frac{1}{M}$ DoF. We choose $v_{1,1} = (F_{11}F_{22})^{M-1}$. For $i = 1, \cdots, M-1$, $v_{1,i+1}$ and $v_{2,i}$ are chosen as

$$v_{1,i+1} = (F_{12}F_{21})^i (F_{11}F_{22})^{M-i-1} \quad (1)$$
$$v_{2,i} = F_{11}^{M-i} F_{12}^{i-1} F_{21}^{i} F_{22}^{M-1-i}. \quad (2)$$

With this choice, $2M-1$ symbols are aligned in the M dimensional space as shown in Table I and II, so that they can be resolved. After resolving these symbols, they are transmitted over the second hop again in an aligned fashion similar to the first hop but with phases reversal such that interference is canceled at each destination. Specifically, R_1 sends the demodulated a_1, a_2+b_1, $\cdots$, a_M+b_{M-1} along rational "beamforming" directions $v_{R_1,1}, \cdots, v_{R_1,M}$. R_2 sends the demodulated $a_1+b_1, \cdots, a_{M-1}+b_{M-1}$ along "beamforming" directions $v_{R_2,1}, \cdots, v_{R_2,M-1}$. The "beamforming" directions are chosen as $v_{R_1,1} = (G_{11}G_{22})^{M-1}$ and

$$v_{R_1,i+1} = (G_{12}G_{21})^i (G_{11}G_{22})^{M-i-1} \quad (3)$$
$$v_{R_2,i} = -G_{11}^{M-i} G_{12}^{i-1} G_{21}^{i} G_{22}^{M-1-i}. \quad (4)$$

D. Outline for Achievable Schemes for Constant Channels

Now consider the $2\times2\times2$ IC with interfering relays. In this section, we provide the intuition behind the achievable scheme and the rigorous description of the achievable scheme is deferred to Appendix A.

In time slot 1, S_1 sends symbols $a_1(1), \cdots, a_M(1)$ and S_2 sends symbols $b_1(1), \cdots, b_{M-1}(1)$ along "beamforming" directions $v_{1,1}, \cdots, v_{1,M}$ and $v_{2,1}, \cdots, v_{2,M-1}$ given in (1) and (2), respectively. Since relays are causal, relays do not transmit any signal in the first time slot. As a consequence, relays only receive signals sent by sources so that they can demodulate those aligned symbols as shown in Table I and II.

In time slot 2, S_1 and S_2 will transmit new symbols $a_1(2), \cdots, a_M(2)$ and $b_1(2), \cdots, b_{M-1}(2)$ again along $v_{1,1}, \cdots, v_{1,M}$ and $v_{2,1}, \cdots, v_{2,M-1}$, respectively. Relays will transmit the symbols that were already demodulated in the previous time slot using the scheme for the $2\times2\times2$ IC as described in Section II-C, so that they can be decoded at the destinations. Due to interfering links between the relays, these symbols are not only received at the destinations, but also at the relays. Next, we will illustrate how to cancel these interfering symbols.

The idea is based on the observation that interfering symbols were already received in an aligned fashion and demodulated in the previous time slot. Specifically, the interfering symbols and the demodulated symbols at R_1 and R_2 are listed in Table III and IV, respectively. The relay can exploit its memory to partially cancel interfering symbols using the previously demodulated symbols. The remaining interfering symbols can be canceled by utilizing the source nodes.

First, consider canceling $a_1(1)$. Note that at R_1, $a_1(1)$ was already demodulated in the first time slot. Thus R_1 can construct $H_{12}v_{R_2,1}a_1(1)$, which is the interfering signal as shown in Table III, and then subtract it from its received signal to cancel $a_1(1)$. Similarly, R_2 can cancel $a_1(1)$ by generating $-H_{21}v_{R_1,1}(a_1(1)+b_1(1))$ and then adding it to its received signal.

Second, consider canceling $b_1(1)$. Note that after adding $-H_{21}v_{R_1,1}(a_1(1)+b_1(1))$ to the received signal at R_2 to cancel $a_1(1)$, the effective direction of $b_1(1)$ becomes $H_{21}(v_{R_1,2} - v_{R_1,1})$. Then S_2 can resend $b_1(1)$ along $u_{2,1} = -F_{22}^{-1}H_{21}(v_{R_1,2} - v_{R_1,1})$ so that it is received along $-H_{21}(v_{R_1,2} - v_{R_1,1})$, and thus is canceled at R_2. At R_1, the effective direction of $b_1(1)$ becomes the sum of the direction of interfering $b_1(1)$ from R_2 and that of the resent $b_1(1)$ from S_2, which is $H_{12}v_{R_2,1} + F_{12}u_{2,1}$. Therefore, R_1 will add $-(H_{12}v_{R_2,1} + F_{12}u_{2,1})(a_2(1)+b_1(1))$ to its received signal to cancel $b_1(1)$.

Third, consider canceling $a_2(1)$. Note that the effective direction of $a_2(1)$ at R_1 is $H_{12}(v_{R_2,2} - v_{R_2,1}) - F_{12}u_{2,1}$. Therefore, S_1 can resend $a_2(1)$ along the "beamforming" direction $u_{1,1} = -F_{11}^{-1}(H_{12}(v_{R_2,2} - v_{R_2,1}) - F_{12}u_{2,1})$ to cancel it. Similarly, at R_2, the effective direction of $a_2(1)$

TABLE I
ALIGNMENT AT RELAY 1

dimension	$F_{11}v_{1,1}$	$F_{11}v_{1,2}$	$F_{11}v_{1,3}$	$\cdots$	$F_{11}v_{1,i+1}$	$\cdots$	$F_{11}v_{1,M}$
dimension		$F_{12}v_{2,1}$	$F_{12}v_{2,2}$	$\cdots$	$F_{12}v_{2,i}$	$\cdots$	$F_{12}v_{2,M-1}$
symbol	a_1	a_2+b_1	a_3+b_2		$a_{i+1}+b_i$		a_M+b_{M-1}

TABLE II
ALIGNMENT AT RELAY 2

dimension	$F_{21}v_{1,1}$	$F_{21}v_{1,2}$	$\cdots$	$F_{21}v_{1,i}$	$\cdots$	$F_{21}v_{1,M-1}$	$F_{21}v_{1,M}$
dimension	$F_{22}v_{2,1}$	$F_{22}v_{2,2}$	$\cdots$	$F_{22}v_{2,i}$	$\cdots$	$F_{22}v_{2,M-1}$	
symbol	a_1+b_1	a_2+b_2		a_i+b_i		$a_{M-1}+b_{M-1}$	a_M

TABLE III
INTERFERING SYMBOLS AND DEMODULATED SYMBOLS AT RELAY 1

Dimension of interfering symbols	$H_{12}v_{R_2,1}$	$H_{12}v_{R_2,2}$	$\cdots$	$H_{12}v_{R_2,i}$	$\cdots$	$H_{12}v_{R_2,M-1}$	
Interfering symbols	a_1+b_1	a_2+b_2	$\cdots$	a_i+b_i	$\cdots$	$a_{M-1}+b_{M-1}$	
Demodulated symbols	a_1	a_2+b_1	$\cdots$	$a_{i+1}+b_i$	$\cdots$	$a_{M-1}+b_{M-2}$	a_M+b_{M-1}

TABLE IV
INTERFERING SYMBOLS AND DEMODULATED SYMBOLS AT RELAY 2

Dimension of interfering symbols	$H_{21}v_{R_1,1}$	$H_{21}v_{R_1,2}$	$\cdots$	$H_{21}v_{R_1,i+1}$	$\cdots$	$H_{21}v_{R_1,M-1}$	$H_{21}v_{R_1,M}$
Interfering symbols	a_1	a_2+b_1	$\cdots$	$a_{i+1}+b_i$	$\cdots$	$a_{M-1}+b_{M-2}$	a_M+b_{M-1}
Demodulated symbols	a_1+b_1	a_2+b_2	$\cdots$	a_i+b_i	$\cdots$	$a_{M-1}+b_{M-1}$	a_M

becomes $H_{21}v_{R_1,2}+F_{21}u_{1,1}$. R_2 can add $-(H_{21}v_{R_1,2}+F_{21}u_{1,1})(a_2(1)+b_2(1))$ to its received signal to cancel $a_2(1)$.

Continuing this process, it can be seen that all interfering symbols can be canceled.

Remark: Note that the scheme presented above works for *arbitrary* H_{12} and H_{21}[1]. Also note that while the scheme is presented for real channels where all signals, channel coefficients and noises are real values, it can be easily generalized to the complex case using Theorem 7 in [23].

III. $2\times2\times2$ INTERFERENCE NETWORKS WITH ARBITRARY CONNECTIVITY

In this section, we explore the $2\times2\times2$ interference networks with arbitrary connectivity as shown in Fig. 2. Note that due to full-duplex communications, nodes are simultaneously capable of transmitting and receiving from each other if they are within communication range of each other. The main result is presented in the following theorem.

Theorem 2: For the $2\times2\times2$ interference networks with arbitrary connectivity shown in Fig. 2, the total number of DoF is equal to 2, almost surely, except the case in which there is a direct link between a source and its interfering destination. In this case, the DoF collapse to 1.

Proof: When there is a direct link between a source and its interfering destination regardless of the remaining connectivity, the DoF collapse to 1 as already shown in [24]. Now let us consider the cases where such links do not exist. Since the min-cut DoF outer bound is 2, we only need to provide achievable schemes. Suppose the sources choose not to listen and the destinations choose not to transmit. Then except the red dashed links between the two relays, all other red dashed links do not contribute to the transmit and receive signals in the network, and thus are effectively removed from the network. As a result, the remaining network becomes the $2\times2\times2$ IC with interfering relays and with either direct links from the sources to their own destinations or not depending on the network connectivity. If such links do not exist, then as shown before, 2 DoF can be achieved. If such links exist, all nodes still use the same achievable scheme as if such links were absent so that interference can still be canceled. The only difference is that there is inter-symbol interference among desired signals. It can be easily seen that this can be removed by exploiting memory at destinations. Therefore, 2 DoF can be achieved. ■

[1]Note that the requirement on the non-zero channel coefficients of the underlying layered network is a sufficient but not necessary condition to achieve 2 DoF. This is also true for the networks studied in the remaining parts of the paper. Allowing zero-channels takes the problem in a different direction by adding the element of topological complexity. It significantly changes the character of the problem, requiring topological notions such as condensed networks and manageable interference, which are fairly involved even in layered settings, as noted by [6].

IV. THE $2\times2\times2\times2$ INTERFERENCE NETWORKS WITH ARBITRARY INTER-LAYER CONNECTIVITY

In this section, we study the $2\times2\times2\times2$ interference networks with arbitrary inter-layer connectivity shown in Fig. 3.

A. System Model

The received signal at relays A_j, B_j and destination D_j, $j=1,2$, in time t, respectively, are

$$\begin{aligned}
Y_{A_j}(t) &= H_{A_j1}(t)X_1(t)+H_{A_j2}(t)X_2(t)+N_{A_j}(t)\\
Y_{B_j}(t) &= H_{B_j1}(t)X_1(t)+H_{B_j2}(t)X_2(t)\\
&\quad +H_{B_jA_1}(t)X_{A_1}(t)+H_{B_jA_2}(t)X_{A_2}(t)+N_{B_j}(t)\\
Y_j(t) &= H_{j1}(t)X_1(t)+H_{j2}(t)X_2(t)+\\
&\quad \sum_{i=1}^{2}(H_{jA_i}(t)X_{A_i}(t)+H_{jB_i}(t)X_{B_i}(t))+N_j(t)
\end{aligned}$$

where $X_i(t)$, $X_{A_i}(t)$ and $X_{B_i}(t)$ are the input signals at S_i, A_i and B_i, respectively. $H_{A_ji}(t)$, $H_{B_ji}(t)$, $H_{ji}(t)$, $H_{B_jA_i}(t)$, $H_{jA_i}(t)$, $H_{jB_i}(t)$ are the channels from S_i to A_j, B_j, D_j, from A_i to B_j, D_j and from B_i to D_j, respectively. Note that while the channels of the underlying layered network must be non-zero, depending on network connectivity, each inter-layer channel may or may not exist. $N_{A_j}(t)$, $N_{B_j}(t)$ and $N_j(t)$ are the additive i.i.d. Gaussian noise terms.

We assume that the relays are causal. In addition, the channel coefficients follow a block fading model, i.e., they remain constant in a sufficiently long coherence block and switch to a value that is independently and identically drawn from a continuous distribution from one block to the next. It is assumed that global channel knowledge is available at every node.

B. Main Result

Theorem 3: For the $2 \times 2 \times 2 \times 2$ IC with arbitrary inter-layer connectivity defined in Section IV-A, the total number of DoF is 2, almost surely, except the case in which there is a direct link from a source to its interfering destination. In this case, the DoF collapse to 1.

Proof: Let us consider the case in which there is a direct interfering link from one source to the other destination. Since 1 DoF can be achieved by simply sending only one message, we only need to provide the outer bound. Consider the case in which there is a direct link from S_1 to D_2. Suppose we let S_2, A_2 and B_2 fully cooperate so that they effectively become one transmitter with 3 antennas. Similarly, A_1, B_1 and D_1 are allowed to perfectly cooperate so that they form one receiver with 3 antennas. We end up with a two user interference channel with 1 antenna at transmitter 1 and receiver 2 while 3 antennas at transmitter 2 and receiver 1 and there are noisy links from transmitter 1 to transmitter 2, from receiver 1 to receiver 2 and from receiver 1 to transmitter 2. As proved in [24], the DoF of this channel cannot be more than 1. Since cooperation does not reduce the capacity, the original channel cannot have more than 1 DoF as well. Following the same argument, it can be shown that the networks cannot have more than 1 DoF when there is a direct link from S_2 to D_1. For the case in which there are no such direct interfering links, since the min-cut DoF outer bound is 2, we only need to provide achievable schemes. ∎

C. Representative Networks: Ideas for the Achievable Schemes

In this section, we present several representative networks to illustrate various ideas behind the achievable schemes. The proofs for general cases are presented in Appendix B where the networks are divided into different classes and depending on the type, one or more ideas represented in this section will be used to construct the achievable schemes. In all cases, our goal is to cancel all interference at the destinations to achieve one DoF per message for a total of 2 DoF.

1) A transform approach: Consider the class of non-layered 2-source 2-sink multihop interference networks shown in Fig. 4. In this class of networks, we can identify one layer (the

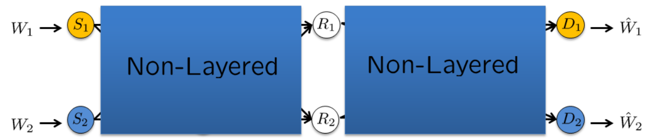
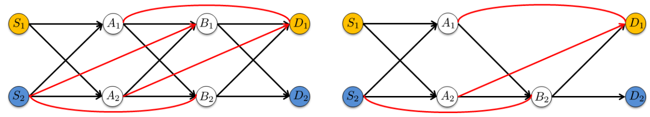


Fig. 4. A transform approach to a class of non-layered 2-source 2-sink non-layered multihop interference networks

layer consisting of nodes R_1 and R_2 in the figure) such that it separates the network into two non-layered parts between which no link exists. Also no directed cycles are present in each part. Suppose in time t, all relay nodes within each of these two non-layered parts only forward the signals received in time $t-1$, then the channel input-output relationships between sources and relay nodes R_1 and R_2 become $Y_{R_i}(t) = \sum_{l_{ii}=0}^{L_{ii}} F_{ii}^{[l_{ii}]}(t)X_i(t-l_{ii})+\sum_{l_{i\bar{i}}=0}^{L_{i\bar{i}}} F_{i\bar{i}}^{[l_{i\bar{i}}]}(t)X_{\bar{i}}(t-l_{i\bar{i}})+\hat{Z}_i(t)$, where l_{ij}, $\forall i,j \in \{1,2\}$, is the delay index defined as the number of nodes on a path from S_j to R_i, L_{ij} is the maximum delay between S_j and R_i, and $\hat{Z}_i$ is the effective noise at R_i. $F_{ij}^{[l_{ij}]}$ is the effective channel coefficient of all paths with delay l_{ij} between S_j and R_i, which is a sum of the products of the channel coefficients of links along each path with delay l_{ij}. Note that depending on the network connectivity, some of delays between 0 to L_{ij} may not exist, and thus the channel coefficients associated with those non-existing paths are zero. Similarly, the channel input-output relationships between relays and destinations become $Y_i(t) = \sum_{d_{ii}=0}^{D_{ii}} G_{ii}^{[d_{ii}]}(t)X_{R_i}(t-d_{ii})+\sum_{d_{i\bar{i}}=0}^{D_{i\bar{i}}} G_{i\bar{i}}^{[d_{i\bar{i}}]}(t)X_{R_{\bar{i}}}(t-d_{i\bar{i}})+\hat{N}_i(t)$, where D_{ij} is the maximum delay between R_j and D_i, d_{ij} is the delay index, and $\hat{N}_i$ is the effective noise. In addition, $G_{ij}^{[d_{ij}]}$ is the effective channel coefficient of the path with delay d_{ij} between R_j and D_i. Since we assume channels follow a block fading model, they are time-invariant within every coherence block. In the following we first consider signaling in one coherence block and thus omit the time indices of channel coefficients.

It can be easily seen that the channels between S_1, S_2 and R_1, R_2, and channels between R_1, R_2 and D_1, D_2 become inter-symbol interference (ISI) channels. We can remove the ISI and convert the channel into a parallel $2 \times 2 \times 2$ IC by introducing a cyclic prefix and using DFT/IDFT. Specifically, consider using the channel $N_c + D$ times in one coherence block where $D = \max\{L_{ij}, D_{ij}\}$. Let us first consider the channel between S_1, S_2 and R_1, R_2. For N_c transmitted symbols at S_i, $\mathbf{x}_i = [x_i(1), \cdots, x_i(N_c)]^T$, we create a length $N_c + D$ input block by adding D cyclic prefix to $\mathbf{x}_i$, which is given by $\mathbf{X}_i = [x_i(N_c-D+1), x_i(N_c), x_i(1), \cdots, x_i(N_c)]^T$. By discarding the first D received symbols, R_i obtains a block of N_c symbols, $\mathbf{Y}_{R_i} = [Y_{R_i}(D+1), \cdots, Y_{R_i}(D+N_c)]^T$, which is given by $\mathbf{Y}_{R_i} = \mathbf{F}_{ii}\mathbf{x}_i + \mathbf{F}_{i\bar{i}}\mathbf{x}_{\bar{i}} + \hat{\mathbf{Z}}_i$ where $\mathbf{F}_{ij}$ is a circulant matrix with the first row given by $[F_{ij}^{[0]}\ 0\ \cdots\ 0\ F_{ij}^{[D]}\ F_{ij}^{[D-1]}\ \cdots\ F_{ij}^{[1]}]$. It is well known that the circulant matrix can be decomposed as $\mathbf{F}_{ij} = \mathbf{U}^{-1}\tilde{\mathbf{F}}_{ij}\mathbf{U}$ where $\mathbf{U}$ is the DFT matrix in which the (k,n)th entry is equal to $\frac{1}{\sqrt{N_c}}e^{\frac{-j2\pi kn}{N_c}}$, $k, n = 0, \ldots, N_c-1$, and $\tilde{\mathbf{F}}_{ij}$ is a diagonal matrix with the nth diagonal entry equal to the nth entry of the



(a) A three-hop non-layered network (b) Effective channel after shutting down node B_1

Fig. 5. Interference neutralization exploiting memory

column vector $\sqrt{N_c}\mathbf{U}[F_{ij}^{[0]}, \cdots, F_{ij}^{[D]}, 0, \cdots, 0]^T$. By setting $\mathbf{x}_i = \mathbf{U}^{-1}\tilde{\mathbf{x}}_i$ and multiplying the received signal with $\mathbf{U}$, the signal becomes $\tilde{\mathbf{Y}}_{R_i} = \mathbf{U}\mathbf{Y}_{R_i} = \tilde{\mathbf{F}}_{ii}\tilde{\mathbf{x}}_i + \tilde{\mathbf{F}}_{i\bar{i}}\tilde{\mathbf{x}}_{\bar{i}} + \tilde{\mathbf{Z}}_{R_i}$. Thus, after these operations the original ISI channels between sources and relays are converted to parallel channels.

The same procedure can be applied to convert the ISI channels between the relays and destinations to parallel channels: $\tilde{\mathbf{Y}}_i = \tilde{\mathbf{G}}_{ii}\tilde{\mathbf{x}}_{R_i} + \tilde{\mathbf{G}}_{i\bar{i}}\tilde{\mathbf{x}}_{R_{\bar{i}}} + \tilde{\mathbf{N}}_i$. After the transformation, the originally non-layered channel is converted to a parallel $2 \times 2 \times 2$ IC with N_c sub-channels in each coherence block. By signaling over M coherence time blocks, each sub-channel becomes a $2 \times 2 \times 2$ time-varying IC, over which the aligned interference neutralization schemes proposed in [5] can be applied to cancel interference. It should be noted that while the interference can be canceled, it remains to check if the desired signals are resolvable since the channels are no longer generic after transformation. As shown in [5], the desired signals are resolvable if $\tilde{G}_{12}^n(t)\tilde{G}_{21}^n(t)/(\tilde{G}_{11}^n(t)\tilde{G}_{22}^n(t))$ and $\tilde{F}_{12}^n(t)\tilde{F}_{21}^n(t)/(\tilde{F}_{22}^n(t)\tilde{F}_{11}^n(t))$, respectively, are all distinct over $t = 1$ to M, where $\tilde{G}_{ij}^n(t)$ and $\tilde{F}_{ij}^n(t)$ are the channels of the nth sub-channel in the tth block. It will be shown later in the appendix that the condition is satisfied for all cases considered in the paper, so that $\frac{2M-1}{M}$ DoF can be achieved on each sub-channel. Thus, the total DoF achieved is $\frac{2M-1}{M} \cdot \frac{N_c}{N_c+D}$ where D is a constant depending on the network connectivity. As M and N_c go to infinity, 2 DoF are achieved.

2) Interference neutralization exploiting memory: Consider the network shown in Fig. 5(a). Let us first shut down B_1, resulting in the effective channel connectivity shown in Fig. 5(b). Now consider canceling interference from S_1 to D_2. Note that there are two paths from S_1 to D_2: $S_1 \to A_1 \to B_2 \to D_2$ and $S_1 \to A_2 \to B_2 \to D_2$. We will design the scaling factors at node A_1 and A_2 in a manner that allows interfering signals passing through these two paths to cancel each other before arriving at D_2. Specifically, A_1 simply forwards its received signal while A_2 uses a scaling factor $\alpha(t)$. With this transmission scheme, the transmitted signal $X_1(t)$ from S_1 is received at B_2 in time slot t as $(H_{B_2A_1}(t)H_{A_11}(t-1)+\alpha(t)H_{B_2A_2}(t)H_{A_21}(t-1))X_1(t-1)$. In order to cancel interference, $\alpha(t)$ is chosen to force $H_{B_2A_1}(t)H_{A_11}(t-1)+\alpha(t)H_{B_2A_2}(t)H_{A_21}(t-1) = 0$, which leads to $\alpha(t) = -\frac{H_{B_2A_1}(t)H_{A_11}(t-1)}{H_{B_2A_2}(t)H_{A_21}(t-1)}$. Note that while signal $X_1(t)$ is canceled at D_2, it is not canceled at its desired destination D_1 almost surely since channels from A_1, A_2 to D_1 and B_2 are distinct almost surely.

Next consider canceling interference from S_2 to D_1. Note that there are two kinds of paths: delays one and two. In particular, paths $S_2 \to A_1 \to D_1$, $S_2 \to A_2 \to D_1$ and $S_2 \to B_2 \to D_1$ have delay one while paths $S_2 \to A_1 \to B_2 \to D_1$ and $S_2 \to A_2 \to B_2 \to D_1$ have delay two. We first cancel interference with delay two by exploiting the memory at B_2. Suppose the transmit signal from S_2 is $X_2(t)$, which will be received at B_2 in time slot t as

$$\begin{aligned}\bar{Y}_{B_2}(t) &= H_{B_22}(t)X_2(t) + (H_{B_2A_1}(t)H_{A_12}(t-1) \\ &\quad +\alpha(t)H_{B_2A_2}(t)H_{A_22}(t-1))X_2(t-1).\end{aligned}$$

For $t = 1$, $\bar{Y}_{B_2}(1) = H_{B_22}(1)X_2(1)$. For $t = 2$,

$$\begin{aligned}\bar{Y}_{B_2}(2) &= H_{B_22}(2)X_2(2) + (H_{B_2A_1}(2)H_{A_12}(1) \\ &\quad +\alpha(2)H_{B_2A_2}(2)H_{A_22}(1))X_2(1).\end{aligned}$$

To cancel $X_2(1)$, we can subtract $\frac{H_{B_2A_1}(2)H_{A_12}(1)+\alpha(2)H_{B_2A_2}(2)H_{A_22}(1)}{H_{B_22}(1)}\bar{Y}_{B_2}(1)$ from $\bar{Y}_{B_2}(2)$ to obtain $Y'_{B_2}(2) = H_{B_22}(2)X_2(2)$. We can continue this procedure to cancel $X_2(t-1)$ in every $t > 2$ at B_2 as follows: $Y'_{B_2}(t) = \bar{Y}_{B_2}(t) - \frac{H_{B_2A_1}(t)H_{A_12}(t-1)+\alpha(t)H_{B_2A_2}(t)H_{A_22}(t-1)}{H_{B_22}(t-1)}Y'_{B_2}(t-1) = H_{B_22}(t)X_2(t)$. As a result, all interfering signals with delay two are canceled.

To cancel interference with delay one, B_2 will choose the scaling factor $\beta(t)$ in a manner that allows the interfering signals passing through three paths with delay one to be canceled before arriving at the destination. Specifically, in time t, the transmit signal at B_2 is $X_{B_2}(t) = \beta(t)Y'_{B_2}(t-1)$. With this transmission scheme, the transmit signal $X_2(t)$ from S_2 will be received at D_1 after passing through paths $S_2 \to A_1 \to D_1$, $S_2 \to A_2 \to D_1$ and $S_2 \to B_2 \to D_1$ as

$$\begin{aligned}&(H_{1A_1}(t)H_{A_12}(t-1) + \alpha(t)H_{1A_2}(t)H_{A_22}(t-1) \\ &\quad +\beta(t)H_{1B_2}(t)H_{B_22}(t-1))X_2(t-1),\end{aligned}$$

which is forced to be zero by setting $\beta(t) = -\frac{H_{1A_1}(t)H_{A_12}(t-1)+\alpha(t)H_{1A_2}(t)H_{A_22}(t-1)}{H_{1B_2}(t)H_{B_22}(t-1)}$. Thus all interference from S_2 is canceled at D_1.

So far we have shown that all interfering signals are canceled at both destinations. Next, we will show that 2 DoF can be achieved. Consider an $M+2$ symbol-extended channel, i.e., signaling is performed over a block of $M+2$ time slots. Note that 2 corresponds to the maximum delay between any two nodes in the network. In each block, M symbols are sent followed by two zeros. The zero-padding is used to avoid inter-block interference and ensure that noise accumulation due to successively interference cancelation at B_2 does not build up across blocks. Within each block, the transmission scheme described above is used to cancel inter-user interference. By channel coding across blocks, $\frac{2M}{M+2}$ DoF can be achieved, which goes to 2 as $M \to \infty$. Note that symbol-extension with zero padding equal to the maximum delay of the network applies to all memory related schemes proposed in the paper. Due to limited space, we will omit such argument in the remaining part of the paper and focus on how to cancel inter-user interference.

3) Combining aligned interference neutralization with interference neutralization: Consider the network shown in Fig. 6. The transmission is performed over M dimensions which are obtained by coding across M time slots, each from one

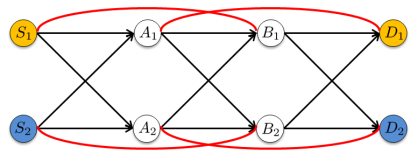


Fig. 6. $2 \times 2 \times 2 \times 2$ with 4 inter-layer links

independent coherence block. The effective channel after symbol extension becomes an $M \times M$ diagonal matrix $\mathbf{H}(t)$ with independent diagonal entries, $H(M(t-1)+1), \cdots, H(Mt)$.

The aligned interference neutralization scheme proposed in [5] is applied to the $2 \times 2 \times 2$ IC comprised of the first three layers to cancel interfering signals from S_1 to B_2 and from S_2 to B_1. In time t, S_1 and S_2 use $\mathbf{V}_1(t)$ and $\mathbf{V}_2(t)$ as beamforming matrices to send M and $M-1$ symbols denoted as the $M \times 1$ vector $\mathbf{a}(t)$ and $(M-1) \times 1$ vector $\mathbf{b}(t)$, respectively: $\mathbf{X}_1(t) = \mathbf{V}_1(t)\mathbf{a}(t)$ and $\mathbf{X}_2(t) = \mathbf{V}_2(t)\mathbf{b}(t)$, where $\mathbf{V}_1(t) = [\mathbf{v}_{1,1}(t), \cdots, \mathbf{v}_{1,M}(t)]$ and $\mathbf{V}_2(t) = [\mathbf{v}_{2,1}(t), \cdots, \mathbf{v}_{2,M-1}(t)]$ are chosen in the same manner as in [5]: $\mathbf{v}_{1,1}(t) = [1\,1 \cdots 1]^T$, and for $i = 1, \cdots, M-1$,

$$\begin{aligned} \mathbf{v}_{1,i+1}(t) &= \left(\mathbf{H}^{-1}_{A_1 1}(t)\mathbf{H}_{A_1 2}(t)\mathbf{H}^{-1}_{A_2 2}(t)\mathbf{H}_{A_2 1}(t)\right)^i \mathbf{v}_{1,1}(t) \\ \mathbf{v}_{2,i}(t) &= \left(\mathbf{H}^{-1}_{A_2 2}(t)\mathbf{H}_{A_2 1}(t)\mathbf{H}^{-1}_{A_1 1}(t)\mathbf{H}_{A_1 2}(t)\right)^{i-1} \\ &\quad \mathbf{H}^{-1}_{A_2 2}(t)\mathbf{H}_{A_2 1}(t)\mathbf{v}_{1,1}(t) \end{aligned} \tag{5}$$

Relays A_1 and A_2 will send the received signals in the next time slot by multiplying them with $\mathbf{A}_1(t)$ and $\mathbf{A}_2(t)$, respectively: $\mathbf{X}_{A_i}(t) = \mathbf{A}_i(t)\mathbf{Y}_{A_i}(t-1)$. With this transmission scheme, the received signals at B_1 and B_2 are given by (6) and (7), respectively, where $\mathbf{N}'_{B_i}(t)$ is the effective noise. As shown in [5], the following choices of $\mathbf{A}_1(t)$ and $\mathbf{A}_2(t)$ force $\mathbf{I}_1(t) = \mathbf{I}_2(t) = \mathbf{0}$:

$$\mathbf{A}_1(t) = \mathbf{U}_1(t)(\mathbf{H}_{A_1 1}(t-1)\mathbf{V}_1(t-1))^{-1} \tag{8}$$
$$\mathbf{A}_2(t) = \mathbf{U}_2(t)(\mathbf{H}_{A_2 1}(t-1)\mathbf{V}_1(t-1))^{-1} \tag{9}$$

where $\mathbf{U}_1(t) = [\mathbf{u}_{1,1}(t) \cdots \mathbf{u}_{1,M}(t)]$ and $\mathbf{U}_2(t) = [\mathbf{u}_{2,1}(t) \cdots \mathbf{u}_{2,M}(t)]$ and for $i = 1, \cdots, M-1$

$$\begin{aligned} \mathbf{u}_{1,i+1}(t) &= \left(\mathbf{H}^{-1}_{B_1 A_1}(t)\mathbf{H}_{B_2 A_1}(t)\mathbf{H}^{-1}_{B_2 A_2}(t)\mathbf{H}_{B_1 A_2}(t)\right)^i \\ &\quad \mathbf{u}_{1,1}(t) \\ \mathbf{u}_{2,i}(t) &= -\left(\mathbf{H}^{-1}_{B_2 A_2}(t)\mathbf{H}_{B_1 A_2}(t)\mathbf{H}^{-1}_{B_1 A_1}(t)\mathbf{H}_{B_2 A_1}(t)\right)^{i-1} \\ &\quad \mathbf{H}^{-1}_{B_2 A_2}(t)\mathbf{H}_{B_2 A_1}(t)\mathbf{u}_{1,1}(t) \\ \mathbf{u}_{2,M}(t) &= -\mathbf{H}^{-1}_{B_2 A_2}(t)\mathbf{H}_{B_2 A_1}(t)\mathbf{u}_{1,M}(t) \end{aligned}$$

where $\mathbf{u}_{1,1}(t) = [1\ 1 \cdots 1]^T$. Thus transmitted signals from S_1 and S_2 are neutralized at B_2 and B_1, respectively.

Our next goal is to cancel the transmitted signals from S_1 and S_2 that arrive at B_1 and B_2, respectively, through A_1 and A_2. Mathematically, we want to cancel $\mathbf{X}_1(t-1)$ from the received signal at B_1 as given in (6) and $\mathbf{X}_2(t-1)$ from the received signal at B_2 as given in (7). In time slot $t = 1$, the transmitted signals passing through A_1 and A_2 have not arrived at node B_i, $i = 1, 2$ and the received signals are given by $\mathbf{Y}_{B_i}(1) = \mathbf{H}_{B_i i}(1)\mathbf{X}_i(1) + \mathbf{N}'_{B_i}(1)$. In time slot $t = 2$, B_i cancels $\mathbf{X}_i(1)$ from its received signals as follows.

$$\begin{aligned} \tilde{\mathbf{Y}}_{B_i}(2) &= \mathbf{Y}_{B_i}(2) - \mathbf{E}_i(2)\mathbf{H}^{-1}_{B_i i}(1)\mathbf{Y}_{B_i}(1) \\ &= \mathbf{H}_{B_i i}(2)\mathbf{X}_i(2) + \tilde{\mathbf{N}}_{B_i}(2) \end{aligned} \tag{10}$$

where $\tilde{\mathbf{N}}_{B_i}(2)$ is the effective noise. Similarly, in time $t > 2$, B_i cancels $\mathbf{X}_i(t-1)$ as follows.

$$\begin{aligned} \tilde{\mathbf{Y}}_{B_i}(t) &= \mathbf{Y}_{B_i}(t) - \mathbf{E}_i(t)\mathbf{H}^{-1}_{B_i i}(t-1)\tilde{\mathbf{Y}}_{B_i}(t-1) \\ &= \mathbf{H}_{B_i i}(t)\mathbf{X}_i(t) + \tilde{\mathbf{N}}_{B_i}(t) \end{aligned} \tag{11}$$

So far, we have canceled all interfering signals with delay two. It can be seen that now all interfering signals will be received with delay one. To cancel it, we design scaling factors at nodes B_1 and B_2. The transmitted signal at B_i in time slot t is given by $\mathbf{X}_{B_i}(t) = \mathbf{B}_i(t)\tilde{\mathbf{Y}}_{B_i}(t-1), i = 1, 2$. With this transmitted signal, the received signal at D_i becomes

$$\begin{aligned} \mathbf{Y}_i(t) &= \mathbf{H}_{iA_i}(t)\mathbf{A}_i(t)\mathbf{H}_{A_i \bar{i}}(t-1)\mathbf{X}_{\bar{i}}(t-1) \\ &\quad + \mathbf{H}_{iA_i}(t)\mathbf{A}_i(t)\mathbf{H}_{A_i i}(t-1)\mathbf{X}_i(t-1) \\ &\quad + \sum_{j=1}^{2} \mathbf{H}_{iB_j}(t)\mathbf{B}_j(t)\tilde{\mathbf{Y}}_{B_j}(t-1) + \mathbf{N}'_i(t) \\ &\overset{(a)}{=} (\mathbf{H}_{iA_i}(t)\mathbf{A}_i(t)\mathbf{H}_{A_i \bar{i}}(t-1) \\ &\quad + \mathbf{H}_{iB_{\bar{i}}}(t)\mathbf{B}_{\bar{i}}(t)\mathbf{H}_{B_{\bar{i}} \bar{i}}(t-1))\mathbf{X}_{\bar{i}}(t-1) \\ &\quad + (\mathbf{H}_{iA_i}(t)\mathbf{A}_i(t)\mathbf{H}_{A_i i}(t-1) \\ &\quad + \mathbf{H}_{iB_i}(t)\mathbf{B}_i(t)\mathbf{H}_{B_i i}(t-1))\mathbf{X}_i(t-1) + \mathbf{N}''_i(t) \end{aligned}$$

where (a) is obtained by using $\tilde{\mathbf{Y}}_{B_i}(t)$ given in (11). To cancel the interfering signal $\mathbf{X}_{\bar{i}}(t-1)$ from $\mathbf{Y}_i(t)$, it is required that $\mathbf{H}_{iA_i}(t)\mathbf{A}_i(t)\mathbf{H}_{A_i \bar{i}}(t-1) + \mathbf{H}_{iB_{\bar{i}}}(t)\mathbf{B}_{\bar{i}}(t)\mathbf{H}_{B_{\bar{i}} \bar{i}}(t-1) = \mathbf{0}$, which leads to $\mathbf{B}_{\bar{i}}(t) = -\mathbf{H}^{-1}_{iB_{\bar{i}}}(t)\left(\mathbf{H}_{iA_i}(t)\mathbf{A}_i(t)\mathbf{H}_{A_i \bar{i}}(t-1)\right)\mathbf{H}^{-1}_{B_{\bar{i}} \bar{i}}(t-1)$.

After canceling all interference at the destination, it can be easily checked that the desired signals are received along full rank matrices and thus can be decoded.

V. Conclusion

We study the effects of non-layered inter-layer and intra-layer links, which are mainly motivated by full-duplex communications, on the fully connected layered interference networks. By characterizing the DoF of the $2 \times 2 \times 2$ IC with arbitrary connectivity and those of $2 \times 2 \times 2 \times 2$ interference networks with arbitrary inter-layer connectivity, we conclude that direct interference links cause collapse of DoF while indirect interference links do not reduce DoF. While only a class of non-layered 2-source 2-sink two unicast interference networks are considered, we believe the ideas that emerged out of studying this class of non-layered networks will be useful to deal with the interference arriving along paths with different lengths in many non-layered multihop multiflow networks. In addition, the small networks studied in this work are useful to characterize a much larger class of non-layered networks which can be condensed to these small networks.

$$\begin{aligned}\mathbf{Y}_{B_1}(t) &= \mathbf{H}_{B_11}(t)\mathbf{X}_1(t) + \underbrace{(\mathbf{H}_{B_1A_1}(t)\mathbf{A}_1(t)\mathbf{H}_{A_11}(t-1) + \mathbf{H}_{B_1A_2}(t)\mathbf{A}_2(t)\mathbf{H}_{A_21}(t-1))}_{\mathbf{E}_1(t)}\mathbf{X}_1(t-1)\\ &\quad + \underbrace{(\mathbf{H}_{B_1A_1}(t)\mathbf{A}_1(t)\mathbf{H}_{A_12}(t-1) + \mathbf{H}_{B_1A_2}(t)\mathbf{A}_2(t)\mathbf{H}_{A_22}(t-1))\mathbf{X}_2(t-1)}_{\mathbf{I}_1(t)=\mathbf{0}} + \mathbf{N}'_{B_1}(t) \qquad (6)\\ \mathbf{Y}_{B_2}(t) &= \mathbf{H}_{B_22}(t)\mathbf{X}_2(t) + \underbrace{(\mathbf{H}_{B_2A_1}(t)\mathbf{A}_1(t)\mathbf{H}_{A_12}(t-1) + \mathbf{H}_{B_2A_2}(t)\mathbf{A}_2(t)\mathbf{H}_{A_22}(t-1))}_{\mathbf{E}_2(t)}\mathbf{X}_2(t-1)\\ &\quad + \underbrace{(\mathbf{H}_{B_2A_1}(t)\mathbf{A}_1(t)\mathbf{H}_{A_11}(t-1) + \mathbf{H}_{B_2A_2}(t)\mathbf{A}_2(t)\mathbf{H}_{A_21}(t-1))\mathbf{X}_1(t-1)}_{\mathbf{I}_2(t)=\mathbf{0}} + \mathbf{N}'_{B_2}(t) \qquad (7)\end{aligned}$$

APPENDIX A
ACHIEVABILITY FOR THE $2\times 2\times 2$ IC WITH INTERFERING RELAYS

We first provide the achievable scheme for the constant channels.

Sources: Message W_1 is split into M sub-messages. Sub-message W_{1,k_1}, $k_1 \in \{1,\ldots,M\}$, is encoded using a codebook with the codeword of length n denoted as $a_{k_1}(1),\cdots,a_{k_1}(n)$. Similarly, message W_2 is split into $M-1$ sub-messages. Sub-message W_{2,k_2}, $k_2 \in \{1,\ldots,M-1\}$, is encoded using a codebook with the codeword of length n denoted as $b_{k_2}(1),\cdots,b_{k_2}(n)$. For any $\epsilon > 0$ and a constant γ, let $\mathcal{C}$ denote all integers in the interval $\left[-\gamma P^{\frac{1-\epsilon}{2(M+\epsilon)}}, \gamma P^{\frac{1-\epsilon}{2(M+\epsilon)}}\right]$, i.e., $\mathcal{C} = \left\{x : x\in\mathbb{Z}\cap\left[-\gamma P^{\frac{1-\epsilon}{2(M+\epsilon)}}, \gamma P^{\frac{1-\epsilon}{2(M+\epsilon)}}\right]\right\}$. $a_{k_1}(t)$ and $b_{k_2}(t)$ are obtained by uniform i.i.d. sampling on $\mathcal{C}$. Essentially, each sub-message carries $\frac{1-\epsilon}{M+\epsilon}$ DoF. Then in time slot 1, the transmitted signals are $X_1(1) = A\sum_{k_1=1}^{M} v_{1,k_1}a_{k_1}(1)$ and $X_2(1) = A\sum_{k_2=1}^{M-1} v_{2,k_2}b_{k_2}(1)$ where v_{1,k_1} and v_{2,k_2} are chosen as in (1) and (2), respectively. To satisfy the power constraints, the constant scaling factor A is chosen to be $\frac{\xi}{\gamma}P^{\frac{M-1+2\epsilon}{2(M+\epsilon)}}$ where $\xi = \min\left(\frac{1}{\sum_{k_1=1}^{M} v_{1,k_1}^2}, \frac{1}{\sum_{k_2=1}^{M-1} v_{2,k_2}^2}\right)$ as in [5].

Then in time $t = 2,\ldots,n$, the transmitted signals are

$$\begin{aligned} X_1(t) &= A'\left(\sum_{k_1=1}^{M} v_{1,k_1}a_{k_1}(t) + \sum_{i=1}^{M-1} u_{1,i}a_{i+1}(t-1)\right)\\ X_2(t) &= A'\left(\sum_{k_2=1}^{M-1} v_{2,k_2}b_{k_2}(t) + \sum_{i=1}^{M-1} u_{2,i}b_i(t-1)\right)\end{aligned}$$

where $u_{2,i} = -F_{22}^{-1}(H_{21}(v_{R_1,i+1} - v_{R_1,i}) - F_{21}u_{1,i-1})$ and $u_{1,i} = -F_{11}^{-1}(H_{12}(v_{R_2,i+1} - v_{R_2,i}) - F_{12}u_{2,i})$. Note that $u_{1,0}$ is defined to be 0. And A' should be chosen to satisfy the power constraints, i.e.,

$$E\left[X_j^2(t)\right] \le \gamma^2 A'^2 \underbrace{\left(\sum_{k_j=1}^{M_j} v_{j,k_j}^2 + \sum_{i=1}^{M-1} u_{j,i}^2\right)}_{\xi_j^2} P^{\frac{1-\epsilon}{(M+\epsilon)}} \le P \quad (12)$$

where $M_1 = M$ and $M_2 = M-1$. To satisfy power constraints at both transmitters, we choose $A' = \min\{C, B\}$ where C and B are scaling factors that can ensure sources and relays satisfy the power constraints, respectively. Specifically, $C = \frac{\xi}{\gamma}P^{\frac{M-1+2\epsilon}{2(M+\epsilon)}}$ where $\xi = \min(\frac{1}{\xi_1}, \frac{1}{\xi_2})$ and ξ_j^2 is given in (12), and $B = \frac{\xi}{2\gamma}P^{\frac{M-1+2\epsilon}{2(M+\epsilon)}}$ where $\xi = \min(\frac{1}{\xi_1}, \frac{1}{\xi_2})$ and $\xi_j^2 = \sum_{m,n=1}^{M_j} |v_{R_j,m}v_{R_j,n}|$ as given in [5].

Relays: Consider the first time slot. As mentioned before, since relays do not transmit in the first time slot, the received signals are the same as the $2\times 2\times 2$ IC [5], which is omitted here. The relays will demodulate the aligned symbols as described in [5]. Specifically, R_1 will demodulate $a_1(1)$, $a_2(1)+b_1(1)$, $\cdots$, $a_M(1)+b_{M-1}(1)$ as $\hat{x}_{R_1,1}(1), \hat{x}_{R_1,2}(1), \ldots, \hat{x}_{R_1,M}(1)$, respectively. R_2 will demodulate $a_1(1)+b_1(1)$, $\cdots$, $a_i(1)+b_i(1)$, $\cdots$, $a_{M-1}(1)+b_{M-1}(1)$, $a_M(1)$ as $\hat{x}_{R_2,1}(1), \hat{x}_{R_2,2}(1), \ldots, \hat{x}_{R_2,M-1}(1), \hat{x}_{R_2,M}(1)$, respectively.

Consider time slot $t = 2$. Let us first consider the transmitted signal. Relays will send the demodulated signals in time slot 1. Specifically, the transmitted signals are $X_{R_1}(2) = A'\sum_{k_1=1}^{M} v_{R_1,k_1}\hat{x}_{R_1,k_1}(1)$ and $X_{R_2}(2) = A'\sum_{k_2=1}^{M-1} v_{R_2,k_2}\hat{x}_{R_2,k_2}(1)$, where A' is chosen to satisfy the power constraint and v_{R_1,k_1} and v_{R_2,k_2} are given in (3) and (4), respectively.

Now consider the received signals: $Y_{R_i}(2) = F_{ii}X_i(2) + F_{i\bar{i}}X_{\bar{i}}(2) + H_{i\bar{i}}X_{R_{\bar{i}}}(2) + Z_i(2)$. R_1 will first locally generate the following signal and add it to its received signal: $\bar{Y}_{R_1}(2) = A'(-H_{12}v_{R_2,1}\hat{x}_{R_1,1}(1) - \sum_{i=1}^{M-1}(H_{12}v_{R_2,i} + F_{12}u_{2,i})\hat{x}_{R_1,i+1}(1))$. $\bar{Y}_{R_1}(2) = A'(-H_{12}v_{R_2,1}\hat{x}_{R_1,1}(1) - \sum_{i=1}^{M-1}(H_{12}v_{R_2,i} + F_{12}u_{2,i})\hat{x}_{R_1,i+1}(1))$. By writing $\hat{x}_{R_1,1}(1) = a_1(1) + e_{R_1,1}(1)$, $\hat{x}_{R_1,i}(1) = a_i(1) + b_{i-1}(1) + e_{R_1,i}(1)$, $i = 2,\cdots,M$, and $\hat{x}_{R_2,j}(1) = a_j(1) + b_j(1) + e_{R_2,j}(1)$ and $\hat{x}_{R_2,M}(1) = a_M(1) + e_{R_2,M}(1)$ where e represents the demodulation error, $Y_{R_1}(2) + \bar{Y}_{R_1}(2)$ can be easily calculated as

$$\begin{aligned} & Y_{R_1}(2) + \bar{Y}_{R_1}(2)\\ =\ & A'\left(F_{11}v_{1,1}a_1(2) + \sum_{i=2}^{M} F_{11}v_{1,i}(a_i(2) + b_{i-1}(2))\right)\\ & + A'H_{12}\sum_{j=1}^{M-1} v_{R_2,j}e_{R_2,j}(1) - A'H_{12}v_{R_2,1}e_{R_1,1}(1)\\ & - A'\sum_{j=1}^{M-1}(H_{12}v_{R_2,j} + F_{12}u_{2,j})e_{R_1,j+1}(1) + Z_1(2)\end{aligned}$$

Next, R_1 will demodulate the aligned symbols using the signal

$Y_{R_1}(2)+\bar{Y}_{R_1}(2)$ as follows. Define the following constellation set $\mathcal{C}_{R_1} = \{A'(F_{11}v_{1,1}x_{R_1,1}+\cdots+F_{11}v_{1,M}x_{R_1,M})\}$, where $x_{R_1,1}$ is an integer in the interval $\left[-\gamma P^{\frac{1-\epsilon}{2(M+\epsilon)}}, \gamma P^{\frac{1-\epsilon}{2(M+\epsilon)}}\right]$ and $x_{R_1,i+1}$ is also an integer but in the interval $\left[-2\gamma P^{\frac{1-\epsilon}{2(M+\epsilon)}}, 2\gamma P^{\frac{1-\epsilon}{2(M+\epsilon)}}\right]$. Notice that $v_{1,1},\cdots,v_{1,M}$ are distinct monomial functions of channel coefficients and thus rationally independent almost surely. Thus, there is a one-to-one mapping from $\mathcal{C}_{R_1}$ to x_{R_1,k_1}, $k_1 \in \{1,\cdots,M\}$. Then Relay R_1 will find the point in $\mathcal{C}_{R_1}$ which has the minimal distance between $Y_{R_1}(2)+\bar{Y}_{R_1}(2)$, and then make a hard decision on x_{R_1,k_1} by mapping the point to $\hat{x}_{R_1,k_1}(2)$. Using Theorem 5 in [22], it can be shown that the minimum distance between constellation points increases with P. Thus, the demodulation error $e(1)$ in the first time slot goes to zero as $P \to \infty$, which allows the demodulation error in this time slot to go to zero as $P \to \infty$ as well.

Similarly, R_2 will generate and then add the following signal to its received signal: $\bar{Y}_{R_2}(2) = A'(-H_{21}v_{R_1,1}\hat{x}_{R_2,1}(1) - \sum_{i=1}^{M-1}(H_{21}v_{R_1,i+1} + F_{21}u_{1,i})\hat{x}_{R_2,i+1}(1))$. Then, R_2 will demodulate the aligned symbols from $\bar{Y}_{R_2}(2)+\bar{Y}_{R_2}(2)$ as $\hat{x}_{R_2,1}(2),\ldots,\hat{x}_{R_2,M}(2)$ in the same manner.

After relays demodulate the aligned symbols, they will transmit them in the next time slot. In time slot $t=3,\ldots,n+1$, relays will follow the same procedure. By replacing the time indices 2 and 1 in the above analysis with t and $t-1$, respectively, we can obtain the equations for the transmitted and received signals at relays.

Destinations: The destinations will use the same decoding method as the $2\times 2\times 2$ IC in [5]. The probability of error of all the demodulations will go to zero as SNR tends to infinity. Then, the coded rate for each symbol approaches $\frac{1}{M}$ DoF according to the rational dimension framework [22]. Therefore, a total of $\frac{2M}{M-1}$ DoF can be achieved. As M goes to infinity, $\frac{2M}{M-1} \to 2$.

For the time-varying channels, the scheme follows from that of the constant channel in a straightforward manner. In particular, in this case, the rational dimensions, lattice codewords and demodulation are replaced with time dimensions, Gaussian codewords and linear transformation, respectively. Due to limited space, the detailed description is omitted.

Appendix B
Achievability proof for the $2\times 2\times 2\times 2$ IC with arbitrary inter-layer connectivity

In this section, we provide the achievable schemes for the $2\times 2\times 2\times 2$ IC with arbitrary inter-layer connectivity and without the direct links from sources to destinations. If there is a direct link between a source to its desired destination, then the achievable scheme is exactly the same as that without such link. In this case, the destination only needs to use memory to cancel ISI of its desired signals.

We categorize all cases based on the number of nodes in each layer that participate in inter-layer links. Denote the number of nodes that participate in inter-layer links in layer A and layer B as N_A, N_B, respectively, where $0 \le N_A, N_B \le 2$. Then we divide networks with arbitrary inter-layer connectivity into two categories: $(N_A, N_B) = (2,2)$ and $(N_A, N_B) \neq (2,2)$. If $(N_A, N_B) \neq (2,2)$, then it implies that at least one of N_A and N_B is one or zero. If one of them is zero, say N_A is zero, then there is no inter-layer links from layer A to the destinations. Such networks are shown in Fig. 7. In this case, layer B separates the network into two parts and there is no link between these two parts. This belongs to the class of networks considered in Section IV-C1. To show that such networks can achieve 2 DoF, the only thing remains to be checked is whether the channels satisfy the sufficient conditions that $\tilde{G}^n_{12}(t)\tilde{G}^n_{21}(t)/(\tilde{G}^n_{11}(t)\tilde{G}^n_{22}(t))$ and $\tilde{F}^n_{12}(t)\tilde{F}^n_{21}(t)/(\tilde{F}^n_{22}(t)\tilde{F}^n_{11}(t))$, respectively, are all distinct over $t=1$ to M, where G^n_{ij} and F^n_{ij} are the channels of the nth sub-channel. In this case, channels from layer B to destinations are generic and thus satisfy the condition almost surely. We only need to show that the $\tilde{F}^n_{12}(t)\tilde{F}^n_{21}(t)/(\tilde{F}^n_{22}(t)\tilde{F}^n_{11}(t))$ are all distinct almost surely. As shown in Section IV-C1, in time slot t, $\tilde{F}^n_{11}(t) = (H_{B_1A_1}(t)H_{A_11}(t-1)+H_{B_1A_2}(t)H_{A_21}(t-1))e^{-\frac{j2\pi n}{N_c}} + H_{B_11}(t)$, $\tilde{F}^n_{21}(t) = (H_{B_2A_1}(t)H_{A_11}(t-1) + H_{B_2A_2}(t)H_{A_21}(t-1))e^{-\frac{j2\pi n}{N_c}} + H_{B_21}(t)$, $\tilde{F}^n_{12}(t) = (H_{B_1A_1}(t)H_{A_12}(t-1) + H_{B_1A_2}(t)H_{A_22}(t-1))e^{-\frac{j2\pi n}{N_c}} + H_{B_12}(t)$, $\tilde{F}^n_{22}(t) = (H_{B_2A_1}(t)H_{A_11}(t-1) + H_{B_2A_2}(t)H_{A_21}(t-1))e^{-\frac{j2\pi n}{N_c}} + H_{B_22}(t)$. We will show $\tilde{F}^n_{12}(t)\tilde{F}^n_{21}(t)\tilde{F}^n_{22}(t')\tilde{F}^n_{11}(t') - \tilde{F}^n_{12}(t')\tilde{F}^n_{21}(t')\tilde{F}^n_{22}(t)\tilde{F}^n_{11}(t) \neq 0$ almost surely. Note that this is a polynomial in variables $H(t)$ and $H(t')$. To prove such polynomial is non-zero almost surely, it suffices to show it is not a zero polynomial. By setting $H_{B_11}(t)$, $H_{B_21}(t)$, $H_{B_12}(t)$ and $H_{B_12}(t)$ to be zero and $H_{A_jj}(t-1) = H_{A_jj}(t'-1) = H_{B_jA_j}(t) = H_{B_jA_j}(t') = H_{A_12}(t-1) = H_{A_12}(t'-1) = H_{B_1A_2}(t) = H_{B_1A_2}(t') = 1$, $j \in \{1,2\}$ and $H_{A_21}(t-1) = 1$, $H_{A_21}(t'-1) = 2$, $H_{B_2A_1}(t) = 3$, $H_{B_2A_1}(t') = 4$, $\tilde{F}^n_{12}(t)\tilde{F}^n_{21}(t)\tilde{F}^n_{22}(t')\tilde{F}^n_{11}(t') - \tilde{F}^n_{12}(t')\tilde{F}^n_{21}(t')\tilde{F}^n_{22}(t)\tilde{F}^n_{11}(t) = 48e^{-\frac{j8\pi n}{N_c}} \neq 0$. Therefore, it is not a zero polynomial and the polynomial is non-zero almost surely.

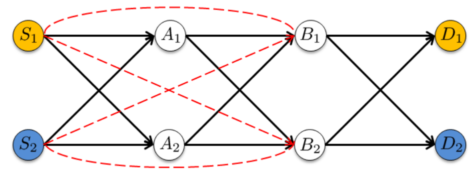


Fig. 7. Networks with $N_A = 0$

If none of N_A and N_B is equal to zero, then one must be equal to 1. Due to symmetry, let N_A to be 1 and there are inter-layer links through A_1. In this case, we can shut down node A_1 so that again layer B separates the network into two parts and there is no link between these two layers. Using the transform approach 2 DoF can be achieved almost surely.

Next, we will consider the case where $(N_A, N_B) = (2,2)$. Within this class, we further categorize the network based on the number of inter-layer links. Let us denote the number of inter-layer links on a path from S_1 to D_2 and S_2 to D_1 as L_1 and L_2, respectively. Note that $0 \le L_1, L_2 \le 4$. Since $(N_A, N_B) = (2,2)$, the total number of inter-layer links must be at least four, i.e.,

$L_1 + L_2 \geq 4$, including all possible combinations of (L_1, L_2): $(0,4), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)$ and $(4,0), (3,1), (4,1), (3,2), (4,2), (4,3)$. Due to symmetry, we only consider the case where $L_1 \leq L_2$. We consider all cases one by one.

When $(L_1, L_2) = (0,4)$, the network is shown in Fig. 5(a). And the achievable scheme is presented in Section IV-C2.

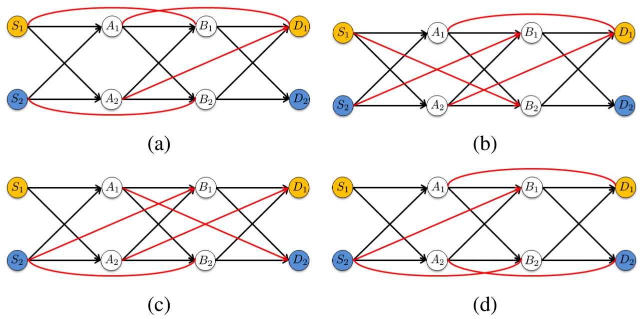


Fig. 8. Networks with $(L_1, L_2) = (1,3)$

When $(L_1, L_2) = (1,3)$, there are four possible cases as shown in Fig. 8. The achievable schemes for these four cases are essentially the same. We will only describe the achievable scheme for the network shown in Fig. 8(a). In this case, we first cancel interference with delay one from S_2 to D_1 through paths $S_2 \to A_1 \to D_1$ and $S_2 \to A_2 \to D_1$. A_1 will forward its received signal in the next time slot and A_2 will amplify its received signal by $\alpha(t)$ and then forward it. $\alpha(t)$ is chosen to cancel interference along these two paths, i.e., $H_{D_1A_1}(t)H_{A_1S_2}(t-1) + \alpha(t)H_{D_1A_2}(t)H_{A_2S_2}(t-1) = 0$, leading to $\alpha(t) = -H_{D_1A_1}(t)H_{A_1S_2}(t-1)/H_{D_1A_2}(t)H_{A_2S_2}(t-1)$. After canceling this interfering signal, from D_1's perspective, inter-layer links from A_1 to D_1 and from A_2 to D_1 are effectively removed from the network. The remaining network without these two links falls into the category of $(N_A, N_B) \neq (2,2)$, for which using the transform approach can cancel all interference. What remains to be shown is that $\tilde{F}^n_{12}(t)\tilde{F}^n_{21}(t)/(\tilde{F}^n_{22}(t)\tilde{F}^n_{11}(t))$ are distinct for all t almost surely. Similar to previous arguments, it is sufficient to construct a set of channel coefficients such that $\tilde{F}^n_{12}(t)\tilde{F}^n_{21}(t)\tilde{F}^n_{22}(t')\tilde{F}^n_{11}(t') - \tilde{F}^n_{12}(t')\tilde{F}^n_{21}(t')\tilde{F}^n_{22}(t)\tilde{F}^n_{11}(t) \neq 0$. This is done by setting inter-layer links in the non-layered parts to be zero, i.e., $H_{B_1 1}(t) = H_{B_2 2}(t) = 0$, setting the remaining layered links as the same values chosen for the case shown in Fig. 7, and then setting remaining inter-layer links in such a manner that $\alpha(t) = 1$. With this choice, it is easily seen that $\tilde{F}^n_{12}(t)\tilde{F}^n_{21}(t)\tilde{F}^n_{22}(t')\tilde{F}^n_{11}(t') - \tilde{F}^n_{12}(t')\tilde{F}^n_{21}(t')\tilde{F}^n_{22}(t)\tilde{F}^n_{11}(t) = 48e^{-\frac{j8\pi n}{N_c}}$. This completes the proof. Note that similar arguments can be applied to the cases where transform approach is used and will be omitted due to limited space.

When $(L_1, L_2) = (1,4)$, there are four possible connectivities as shown in Fig. 9. The achievable schemes are very similar to the case where $(L_1, L_2) = (1,3)$. For example, for the network shown in Fig. 9(a), using A_1 and A_2, interference along paths $S_2 \to A_1 \to D_1$ and $S_2 \to A_2 \to D_1$ can be canceled. The remaining interference can be canceled using the transform approach so that 2 DoF can be achieved.

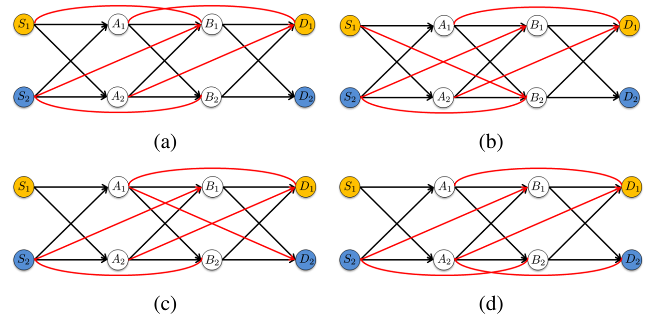


Fig. 9. Networks with $(L_1, L_2) = (1,4)$

When $(L_1, L_2) = (2,2)$, there are a total of 6 possible connectivities. Two basic connectivities are shown in Fig. 10. By switching labels of the nodes A_1 and A_2 or B_1 and B_2 or both in the network shown in Fig. 10(a), we obtain another three connectivities. By switching labels of S_1 and S_2 as well as D_1 and D_2, we can obtain the last connectivity in this case. The achievablility for Fig. 10(a) was already presented in Section IV-C3. For the network shown in Fig. 10(b), A_1 and A_2 can be used to cancel interference arriving at D_1 along paths $S_2 \to A_1 \to D_1$ and $S_2 \to A_2 \to D_1$. Once this interference is neutralized, the transform approach can be applied by using B_1 and B_2 to cancel the remaining interference to achieve 2 DoF.

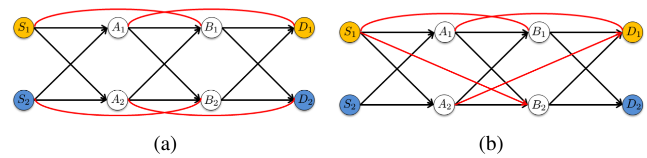


Fig. 10. Networks with $(L_1, L_2) = (2,2)$

When $(L_1, L_2) = (2,3)$, there are a total of 12 possible connectivities in this case, which can be obtained by adding one inter-layer link from S_2 to D_1 in each of the connectivities of $(L_1, L_2) = (2,3)$. Since there are two possible ways to add such link in each case and there are a total of 6 possible cases, the total number of connectivities is 12. We will consider adding links for three basic connectivities with $(L_1, L_2) = (2,3)$ as shown in Fig. 11. The remaining cases can be obtained by switching the labels of some nodes. The networks obtained by adding one link based on the network given in Fig. 10(a) are shown in Fig. 11(a). Each of the two dashed lines represents one possible way to add the link. Similarly, Fig. 11(b) shows two possible connectivities by adding one inter-layer link based on the network shown in Fig.10(b). Finally, by adding links to the network obtained by switching labels S_1, S_2 and D_1, D_2, we obtain the last two cases as shown in Fig. 11(c).

Let us first consider the achievable scheme for the network shown in Fig.11(a). Suppose the link connecting S_2 and B_1 is added. The achievable scheme is similar to that of the case without this link presented in Section IV-C3. S_1, S_2, A_1 and A_2 will use the same transmission scheme as described in Section IV-C3. Then the received signal at B_1 is given by (13). The received signal at B_2 is given by (7). Note that (13) has

$$\mathbf{Y}_{B_1}(t) = \mathbf{H}_{B_11}(t)\mathbf{X}_1(t) + \mathbf{H}_{B_12}(t)\mathbf{X}_2(t) + \underbrace{(\mathbf{H}_{B_1A_1}(t)\mathbf{A}_1(t)\mathbf{H}_{A_11}(t-1) + \mathbf{H}_{B_1A_2}(t)\mathbf{A}_2(t)\mathbf{H}_{A_21}(t-1))}_{\mathbf{E}_1(t)}\mathbf{X}_1(t-1)$$
$$+\underbrace{(\mathbf{H}_{B_1A_1}(t)\mathbf{A}_1(t)\mathbf{H}_{A_12}(t-1) + \mathbf{H}_{B_1A_2}(t)\mathbf{A}_2(t)\mathbf{H}_{A_22}(t-1))\mathbf{X}_2(t-1)}_{\mathbf{I}_1(t)=\mathbf{0}} + \mathbf{N}'_{B_1}(t) \quad (13)$$

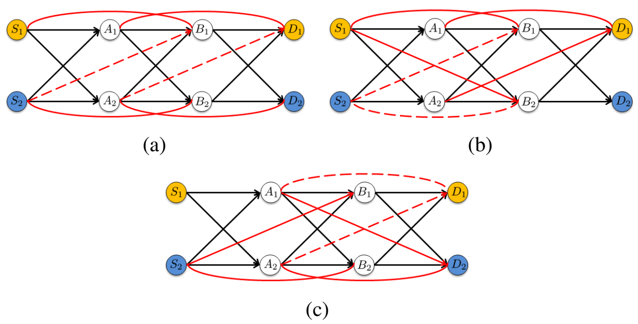


Fig. 11. Networks with $(L_1, L_2) = (2, 3)$

only one additional term $\mathbf{H}_{B_12}\mathbf{X}_2(t)$ compared to (6). These two nodes will use the same procedure to cancel $\mathbf{X}_1(t-1)$ and $\mathbf{X}_2(t-1)$ at B_1 and B_2, respectively. In time slot $t = 1$, there is no inter-symbol interference and the received signals are given by $\mathbf{Y}_{B_1}(1) = \mathbf{H}_{B_11}(1)\mathbf{X}_1(1) + \mathbf{H}_{B_12}(1)\mathbf{X}_2(1) + \mathbf{N}'_{B_1}(1)$ and $\mathbf{Y}_{B_2}(1) = \mathbf{H}_{B_22}(1)\mathbf{X}_2(1) + \mathbf{N}'_{B_2}(1)$. In time slot $t = 2$, B_1 cancels $\mathbf{X}_1(1)$ as follows:

$$\begin{aligned}\tilde{\mathbf{Y}}_{B_1}(2) &= \mathbf{Y}_{B_1}(2) - \mathbf{E}_1(2)\mathbf{H}^{-1}_{B_11}(1)\mathbf{Y}_{B_1}(1)\\ &= \mathbf{H}_{B_11}(2)\mathbf{X}_1(2) + \mathbf{H}_{B_12}(2)\mathbf{X}_2(2)\\ &\quad -\mathbf{E}_1(2)\mathbf{H}^{-1}_{B_11}(1)\mathbf{H}_{B_12}(1)\mathbf{X}_2(1) + \tilde{\mathbf{N}}_{B_1}(2)\end{aligned}$$

where $\tilde{\mathbf{N}}_{B_1}(2)$ is the effective noise. Similarly, in time $t > 2$, B_1 cancels $\mathbf{X}_1(t-1)$ as shown in (14). For B_2, it cancels $\mathbf{X}_2(1)$ in time $t = 2$ as follows: $\mathbf{Y}'_{B_2}(2) = \mathbf{Y}_{B_2}(2) - \mathbf{E}_2(2)\mathbf{H}^{-1}_{B_22}(1)\mathbf{Y}_{B_2}(1) = \mathbf{H}_{B_22}(2)\mathbf{X}_2(2) + \mathbf{N}''_{B_2}(2)$. In time $t > 2$, B_2 cancels $\mathbf{X}_2(t-1)$ as follows:

$$\begin{aligned}\mathbf{Y}'_{B_2}(t) &= \mathbf{Y}_{B_2}(t) - \mathbf{E}_2(t)\mathbf{H}^{-1}_{B_22}(t-1)\mathbf{Y}'_{B_2}(t-1)\\ &= \mathbf{H}_{B_22}(t)\mathbf{X}_2(t) + \mathbf{N}''_{B_2}(t). \quad (15)\end{aligned}$$

From $\mathbf{Y}'_{B_2}(t)$, we can resolve $\mathbf{X}_2(t)$ by inverting the effective channel, i.e., $\bar{\mathbf{Y}}_{B_2}(t) = \mathbf{H}^{-1}_{B_22}(t)\mathbf{Y}'_{B_2}(t) = \mathbf{X}_2(t) + \mathbf{H}^{-1}_{B_22}(t)\mathbf{N}''_{B_2}(t)$.

So far, interference with delay two has been canceled. We still need to cancel interference with delay one. First consider interfering signals from S_1 to D_2, which are along paths $S_1 \to B_1 \to D_2$ and $S_1 \to A_2 \to D_2$. B_1 will send $\tilde{\mathbf{Y}}_{B_1}(t)$ in the next block by multiplying a matrix $\mathbf{B}_1(t)$, i.e., $\mathbf{X}_{B_1}(t) = \mathbf{B}_1(t)\tilde{\mathbf{Y}}_{B_1}(t-1)$. With this transmission scheme, the interfering signal $\mathbf{X}_1(t)$ is received at D_2 as $\mathbf{H}_{2B_1}(t)\mathbf{B}_1(t)\mathbf{H}_{B_11}(t-1) + \mathbf{H}_{2A_2}(t)\mathbf{A}_2(t)\mathbf{H}_{A_21}(t-1))\mathbf{X}_1(t-1)$, which can be canceled by setting $\mathbf{B}_1(t) = -\mathbf{H}^{-1}_{2B_1}(t)\mathbf{H}_{2A_2}(t)\mathbf{A}_2(t)\mathbf{H}_{A_21}(t-1)\mathbf{H}^{-1}_{B_11}(t-1)$. After canceling the interference from S_1, D_2 can cancel the ISI of its desired signal using memory and then decode its message.

Next consider canceling interference from S_2 to D_1. The interference arriving at D_1 along paths $S_2 \to A_1 \to D_1$ and $S_2 \to B_1 \to D_1$ is given by

$$\begin{aligned}\mathbf{I}_2(t) &= (\mathbf{H}_{1A_1}(t)\mathbf{A}_1(t)\mathbf{H}_{A_12}(t-1)\\ &\quad +\mathbf{H}_{1B_1}(t)\mathbf{B}_1(t)\mathbf{H}_{B_12}(t-1))\mathbf{X}_2(t-1)\\ &\quad +\mathbf{H}_{1B_1}(t)\mathbf{B}_1(t)\tilde{\mathbf{X}}_2(t-1).\end{aligned}$$

To cancel it, B_2 will generate its transmitted signal in such a manner that it is arrived at D_1 as $-\mathbf{I}_2(t)$. It can be seen that from $\bar{\mathbf{Y}}_{B_2}(1), \bar{\mathbf{Y}}_{B_2}(2), \cdots, \bar{\mathbf{Y}}_{B_2}(t-1)$, B_2 can construct the signal given in (16). Then the transmitted signal from B_2 is $\mathbf{X}_{B_2}(t) = \mathbf{H}^{-1}_{1B_2}(t)\tilde{\mathbf{Y}}_{B_2}(t)$. With this transmission scheme, all interference is canceled at D_1 and the received signal at D_1 becomes $\mathbf{Y}_1(t) = (\mathbf{H}_{1B_1}(t)\mathbf{B}_1(t)\mathbf{H}_{B_11}(t-1) + \mathbf{H}_{1A_1}(t)\mathbf{A}_1(t)\mathbf{H}_{A_11}(t-1))\mathbf{X}_1(t-1) + \mathbf{N}'_1(t)$. After canceling all interference, D_1 can decode its message.

Next consider the case in which the dashed link from A_2 to D_1 in Fig. 11(a) is added. The achievable scheme is very similar to the case without this link. Specifically, all nodes except B_2 use the same scheme as the case in which that link is absent. The only difference is that the amplifying matrix $\mathbf{B}_2(t)$ should be changed to account for the additional interference due to the additional link. In this network, the interference from S_2 to D_1 is the sum of signals arriving along three paths $S_2 \to A_1 \to D_1$, $S_2 \to A_2 \to D_1$ and $S_2 \to B_2 \to D_1$: $(\mathbf{H}_{1A_1}(t)\mathbf{A}_1(t)\mathbf{H}_{A_12}(t-1) + \mathbf{H}_{1A_2}(t)\mathbf{A}_2(t)\mathbf{H}_{A_22}(t-1) + \mathbf{H}_{1B_2}(t)\mathbf{B}_2(t)\mathbf{H}_{B_22}(t-1))\mathbf{X}_2(t-1)$, which can be canceled by choosing $\mathbf{B}_2(t) = -\mathbf{H}^{-1}_{1B_2}(t)(\mathbf{H}_{1A_1}(t)\mathbf{A}_1(t)\mathbf{H}_{A_12}(t-1) + \mathbf{H}_{1A_2}(t)\mathbf{A}_2(t)\mathbf{H}_{A_22}(t-1))\mathbf{H}^{-1}_{B_22}(t-1)$.

Next consider the networks shown in Fig. 11(b). Let us first use A_1 and A_2 to cancel interfering signals along paths $S_2 \to A_1 \to D_1$ and $S_2 \to A_2 \to D_1$. Once this interfering signal is canceled, links $A_1 \to D_1$ and $A_2 \to D_1$ are effectively removed from the network. The remaining interference can be canceled using the transform approach by B_1 and B_2.

Finally, for the networks shown in Fig. 11(c), B_1 and B_2 will be used to cancel interference along paths $S_2 \to B_1 \to D_1$ and $S_2 \to B_2 \to D_1$. The remaining interference can be canceled using A_1 and A_2 by exploiting the transform approach.

When $(L_1, L_2) = (2, 4)$, there are a total of 6 possible cases. Three basic connectivities are shown in Fig. 12. The remaining three possible connectivities can be obtained by switching labels A_1 and A_2, or B_1 and B_2, or both in the network shown in Fig. 12(c). For the case shown in Fig. 12(a), by shutting down nodes A_1 and A_2, the network becomes the $2 \times 2 \times 2$ IC for which 2 DoF can be achieved [5]. Similarly, by shutting down nodes B_1 and B_2 in the network shown in Fig. 12(b), it becomes the $2 \times 2 \times 2$ IC as well. For the network shown in Fig. 12(c), by shutting down nodes A_2 and B_2, the network becomes the $2 \times 2 \times 2$ IC with interfering relays, for

$$\begin{aligned}\tilde{\mathbf{Y}}_{B_1}(t) &= \mathbf{Y}_{B_1}(t) - \mathbf{E}_1(t)\mathbf{H}_{B_11}^{-1}(t-1)\tilde{\mathbf{Y}}_{B_1}(t-1) \\ &= \mathbf{H}_{B_11}(t)\mathbf{X}_1(t) + \mathbf{H}_{B_12}(t)\mathbf{X}_2(t) + \underbrace{\sum_{n=1}^{t-1}(-1)^{t-n}\left(\prod_{m=n}^{t-1}\mathbf{E}_1(m+1)\mathbf{H}_{B_11}^{-1}(m)\right)\mathbf{H}_{B_12}(n)\mathbf{X}_2(n)}_{\tilde{\mathbf{X}}_{\mathbf{2}}(t)} + \tilde{\mathbf{N}}_{B_1}(t) \quad (14)\end{aligned}$$

$$\begin{aligned}-\tilde{\mathbf{Y}}_{B_2}(t) &= (\mathbf{H}_{1A_1}(t)\mathbf{A}_1(t)\mathbf{H}_{A_12}(t-1) + \mathbf{H}_{1B_1}(t)\mathbf{B}_1(t)\mathbf{H}_{B_12}(t-1))\bar{\mathbf{Y}}_{B_2}(t-1) \\ &\quad + \mathbf{H}_{1B_1}(t)\mathbf{B}_1(t)\left(\sum_{n=1}^{t-2}(-1)^{t-n-1}\left(\prod_{m=n}^{t-2}\mathbf{E}_1(m+1)\mathbf{H}_{B_11}^{-1}(m)\right)\mathbf{H}_{B_12}(n)\bar{\mathbf{Y}}_{B_2}(n)\right) = \mathbf{I}_{\mathbf{2}}(t) + \tilde{\mathbf{N}}_{B_2}(t) \quad (16)\end{aligned}$$

which 2 DoF can be achieved as shown before.

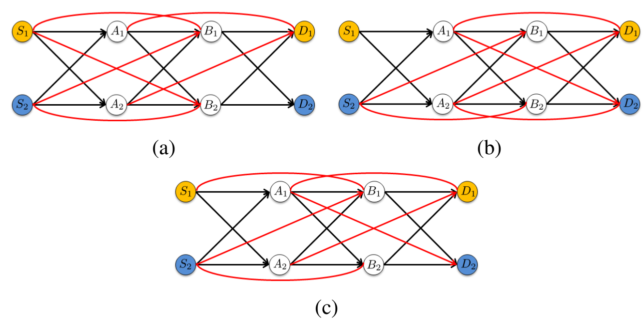


Fig. 12. Networks with $(L_1, L_2) = (2, 4)$

When $(L_1, L_2) = (3, 3)$, the total number of possible connectivities is 12. Four basic networks are shown in Fig. 13. By switching the labels of B_1 and B_2 in Fig. 13(a) and labels of A_1 and A_2 in Fig. 13(c), two more connectivities are obtained. Switching A_1 and A_2, or B_1 and B_2 or both in Fig. 13(b) and 13(d) will give us three more connectivities for each figure for a total of six possible connectivities. By shutting down B_1 and B_2 in Fig. 13(a) and shutting down A_1 and A_2 in Fig. 13(c), we obtain the $2 \times 2 \times 2$ IC for which 2 DoF can be achieved [5]. By shutting down A_1 and B_2 in Fig. 13(c) and shutting down A_1 and B_1 in Fig. 13(d), we obtain the $2\times2\times2$ IC with interfering relays for which 2 DoF can be achieved. Achievable schemes for other cases follow directly.

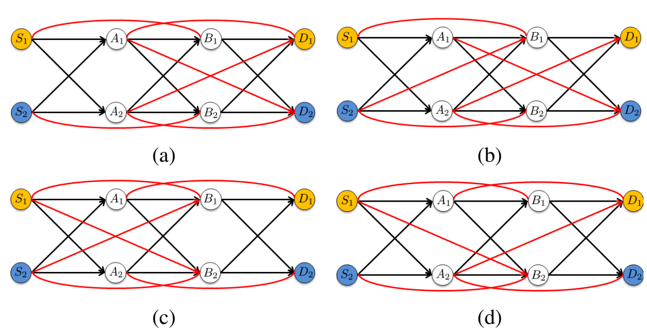


Fig. 13. Networks with $(L_1, L_2) = (3, 3)$

When $(L_1, L_2) = (3, 4)$, there are a total of four possible connectivities as shown in Fig. 14. By shutting down A_1 and A_2 in Fig. 14(a) and 14(d), and shutting down B_1 and B_2 in Fig. 14(b) and 14(c), we obtain the $2 \times 2 \times 2$ IC for which 2 DoF can be achieved.

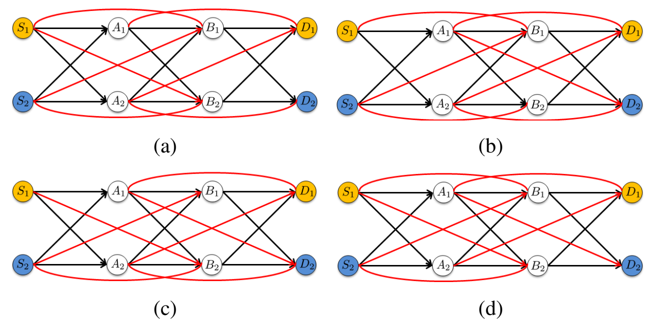


Fig. 14. Networks with $(L_1, L_2) = (3, 4)$

When $(L_1, L_2) = (4, 4)$, simply shutting down node A_1 and A_2 gives us the $2 \times 2 \times 2$ IC for which 2 DoF can be achieved.

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