A Low Complexity Progressive Bitstream Transmission System for Hybrid Channels with Correlated Loss

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Outline

- Motivation and introduction
- Hybrid channels
- Product codes
- Proposed solution
- Numerical results
- Summary and conclusion
Motivation

- Multi-media content delivery over fading channels and packet networks
- Lossy compression + channel coding
- Design metric: End-to-end expected distortion

**Goal**: Distortion minimization for a given transmission rate

**Solution**: Joint source-channel coding with progressive transmission
Progressive Transmission

- Embedded source coding
  - Rate-distortion trade-off through bitstream truncation
  - Image: SPIHT, JPEG2000
  - Video: 3D SPIHT, MPEG4-FGS
- Joint source-channel coding
  - Unequal protection
  - Maximize the expected rate (rate-based)
  - Minimize the expected distortion (distortion-based)
Hybrid Channel

- Simultaneous bits errors and packet erasures
  - Wireless device connected to the internet
- Correlated loss
  - Burst of errors (fading)
  - Bursts of packet loss (congestion)
- Finite-state channel model
Finite-State Channel Model

- Bit errors
  - Gilbert-Elliot
    - Error rates: $\varepsilon_G \ll \varepsilon_B$, SNR-dependent
- Packet erasures
  - Gilbert
    - Error rates: $\varepsilon_G = 0$, $\varepsilon_B = 1$
Product Code RS2D

- Well-known solution [Sherwood & Zeger 1997]

\[ \begin{align*}
N & \quad \text{Number of packets} \\
L & \quad \text{Number of symbols/packet} \\
B_T = NL & \quad \text{Total budget} \\
X & \quad \text{Error protection parity} \\
O & \quad \text{Erasure protection parity} \\
1, 2, \ldots & \quad \text{Source symbols}
\end{align*} \]

- Optimize \( L_s \) and the number of erasure parity symbols
Product Code RS2D Optimization

- Optimization solutions [Stankovic et. al. 2004]
  - Optimal rate-based solution $O(NL)$
  - Near-optimal distortion-based solution $O(NL^2)$
- Existing work: memoryless channels
  - We extend it to correlated channels
- Compare RS2D vs. proposed solution
Extension of RS2D to Correlated Channels

- Correlated channel statistics
  
  \[ P_{err}(n, m) = \text{Prob}\{m \text{ bit errors out of } n \text{ bits}\} \]
  \[ P_{ers}(n, m) = \text{Prob}\{m \text{ packet erasures out of } n \text{ packets}\} \]

- \( \Phi_C = \text{Prob}\{\text{row code decoding failure with } C \text{ parity symbols}\} \)
  
  - Symbol error rates (RS codes)
  - Long bursts: \( \Phi_C \) from simulations
  - Short bursts: \( \Phi_C = 1 - \sum_{i=0}^{\left\lfloor \frac{C}{2} \right\rfloor} \binom{L}{i} P_{se}^i (1 - P_{se})^{L-i}, P_{se} = 1 - P_{err}(s, 0), s: \text{symbol size (bits)} \)

\[
P_N(n, C) = \sum_{m=0}^{n} p(N_f = n - m | N_{ers} = m) P_{ers}(N, m)
\]

\[
p(N_f = n - m | N_{ers} = m) = \binom{N - m}{n - m} \Phi_C^{n-m} (1 - \Phi_C)^{N-n}
\]
Product Code Issues

- Complexity
  - Encoding/decoding (2D)
  - Quadratic optimization complexity $O(NL^2)$
- Poor burst error performance
- Unequal protection for erasures only
- Large rate/distortion-based performance gap

| N | Number of packets |
| L | Number of symbols/packet |
| $B_T = NL$ | Total budget |

$L_S$ and $L$ denote the number of symbols and packets, respectively. $N$ and $L$ are used to allocate resources for protection. $X$ indicates error protection parity, $O$ indicates erasure protection parity, and $1, 2, \ldots$ are source symbols.
What’s Wrong with the Product Code?

• **Row code**
  - 2D code, high encoding/decoding complexity
  - Poor performance for long bursts
  - Optimal rate: $O(NL^2)$ optimization

• **Solution:**
  - Get rid of the row code!
Proposed Solution RS1D

- Simultaneous error/erasure correction using RS codes

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* : Error/erasure parity
1, 2, ...: Source symbols
Proposed Solution RS1D

- Complexity
  - Encoding/decoding (1D)
  - Optimization $O(NL)$
- Burst error interleaving
- Unequal protection for errors AND erasures
- Small rate/distortion-based performance gap
RS1D Optimization

- Error+erasure equivalent channel with failure rate $\Psi_C$
- Distortion-based cost function:

$$\mathcal{E}_D = \sum_{i=1}^{L+1} D_{i-1} \Psi_C \prod_{j=1}^{i-1} (1 - \Psi_{C_j})$$

$D_i$: Distortion with the first $i$ codewords

- Known linear complexity optimization techniques
- No explicit error vs. erasure budget allocation (linear complexity)
RS1D Failure Probability

\[ \Psi_C = 1 - \sum_{i=0}^{N} p(N_{err} \leq \left\lfloor \frac{C-i}{2} \right\rfloor | N_{ers} = i) P_{ers}(N, i) \]

\[ p(N_{err} \leq \left\lfloor \frac{C-i}{2} \right\rfloor | N_{ers} = i) = \sum_{j=0}^{\left\lfloor \frac{C-i}{2} \right\rfloor} p(N_{err} = j | N_{ers} = i) \]

\[ p(N_{err} = j | N_{ers} = i) = \binom{N-i}{j} P_{se}^j (1 - P_{se})^{N-i-j} \]

where

\[ P_{se} = 1 - P_{err}(s, 0) : \text{Symbol error probability} \]

\[ P_{err}(n, m) = \text{Prob}\{m \text{ bit errors out of } n \text{ bits}\} \]

\[ P_{ers}(n, m) = \text{Prob}\{m \text{ packet erasures out of } n \text{ packets}\} \]

\[ s : \text{Symbol size in bits} \]
Numerical Results

- $512 \times 512$ gray scale Lena image
- SPIHT encoder
- $PSNR = 10 \log_{10} \frac{255^2}{D}$, $D$:expected distortion
- Total budget $B_T = LN = 10^4$ bytes
  - Max source coding rate: 0.3 bits-per-pixel
  - Various $\frac{L}{N}$ ratios
N=100, L=100

[Graph showing error rate vs. SNR for different bitstream transmission systems.]

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N=200, L=50
N=50, L=200
Summary and Conclusion

- Extended product code analysis to correlated channels
- Simple 1D coding scheme for hybrid channel
  - Performance gain (up to 8dB)
  - Low coding/optimization complexity
- More results*
  - Low rate/distortion-based gap (0.5dB vs. 7dB for 2D)
  - Memoryless channels: No distortion gain, but reduced complexity

Thank You!