Bit-Interleaved Coded Modulation: Low Complexity Decoding

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***Abstract*—It has been shown by Zehavi that the performance of coded modulation can be improved over a Rayleigh fading channel by bit-wise interleaving at the encoder output, and by using an appropriate soft-decision metric for a Viterbi decoder at the receiver. Caire *et al* presented the details of the theory behind bit-interleaved coded modulation (BICM).**

In this paper we show that for Gray encoded M-ary quadrature amplitude modulation (QAM) systems, the bit metrics of BICM can be further simplified. In QAM systems, the maximum likelihood (ML) detector for BICM uses the minimum distance between the received symbol and $M/2$ constellation points on the complex plane as soft-decision metrics. We show that soft-decision bit metrics for the ML decoder can be further simplified to the minimum distance between the received symbol and $\sqrt{M}/2$ constellation points on the real line $\mathbb{R}^1$. This reduces the number of calculations needed for each bit metric substantially, and therefore reduces the complexity of the decoder without compromising the performance. Simulation results for single carrier modulation (SCM), and multi-carrier modulation (MCM) systems over additive white Gaussian noise (AWGN) and Rayleigh fading channels agree with our findings. In addition, we tie this result to the decoding methods for bit interleaved convolutional code standards used in industry.

I. Introduction

The increasing interest and importance of wireless communications over the past couple of decades have led the consideration of coded modulation [1] for fading channels. It is known that, even for fading channels, the probability of error can be decreased exponentially with average signal to noise ratio using optimal diversity. Naturally, at first, several approaches using Ungerboeck's method of keeping coding combined with modulation are applied over fading channels, as summarized in [2]. These approaches considered the performance of a trellis coded system that is based on a symbol-by-symbol interleaver with a trellis code. The order of diversity for any coded system with a symbol interleaver is the minimum number of distinct symbols between codewords. Thus, diversity can only be increased by preventing parallel transitions and increasing the constraint length of the code.

In 1989 Viterbi *et al* [3] introduced a different approach. They designed schemes to keep their basic engine an off-the-shelf Viterbi decoder. This resulted in leaving the joint decoder/demodulator for two joint entities.

Zehavi [4] later realized that the code diversity, and therefore the reliability of coded modulation over a Rayleigh channel, could be improved. Using bit-wise interleaving and an appropriate soft-decision bit metric[1] at a Viterbi decoder, Zehavi achieved to make the code diversity equal to the smallest number of distinct bits, rather than channel symbols, along any error event. This leads to a better coding gain over a fading channel when compared to TCM, [4].

Following Zehavi's paper, Caire *et al* [5] presented the theory behind BICM. Their work illustrated tools to evaluate the performance of BICM with tight error probability bounds, and design guidelines. In Section II we present a brief overview of BICM, and refer the reader to [5] for details.

In QAM systems, the ML detector for BICM uses the minimum distance between the received symbol and $M/2$ constellation points on the complex plane as soft-decision metrics. In Section III, we show that soft-decision bit metrics for the ML decoder can be further simplified to the minimum distance between the received symbol and $\sqrt{M}/2$ constellation points on the real line $\mathbb{R}^1$. This reduces the number of calculations needed for each bit metric substantially, and therefore reduces the complexity of the decoder without compromising the performance.

Simulation results supporting our findings for SCM and MCM over AWGN and Rayleigh channels are presented in Section IV. We finish our paper with a brief conclusion in Section V, where we summarize our findings.

II. Bit-Interleaved Coded Modulation (BICM)

BICM can be obtained by using a bit interleaver, π, between an encoder for a binary code $\mathcal{C}$ and an N-dimensional memoryless modulator over a signal set $\chi \subseteq \mathbb{C}^N$ of size $|\chi| = M = 2^m$ with a binary labeling map $\mu : \{0,1\}^m \rightarrow \chi$. During transmission, the code sequence $\underline{c}$ is interleaved by π, and then mapped onto signal sequence $\underline{x} \in \chi$. The signal sequence $\underline{x}$ is then transmitted over the channel.

The bit interleaver can be modeled as $\pi : k \rightarrow (k', i)$ where k denotes the original ordering of the coded bits c_k, k' denotes

[1]Note the use of the metric in this paper follows convolutional coding nomenclature and is not in the strict mathematical sense.

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Fig. 1. Block diagram of transmission with BICM

the time ordering of the signals $x_{k'}$ transmitted, and i indicates the position of the bit c_k in the label of $x_{k'}$.

Let χ_b^i denote the subset of all signals $x \in \chi$ whose label has the value $b \in \{0,1\}$ in position i. Then, the ML bit metrics can be given by [5]

$$\lambda^i(\boldsymbol{y}_{k'}, b) = \begin{cases} \max\limits_{x \in \chi_b^i} \log p_{\boldsymbol{\theta}_{k'}}(\boldsymbol{y}_{k'}|\boldsymbol{x}), & \text{perfect CSI} \\ \max\limits_{x \in \chi_b^i} \log p(\boldsymbol{y}_{k'}|\boldsymbol{x}), & \text{no CSI} \end{cases} \tag{1}$$

where $\boldsymbol{\theta}_{k'}$ denotes the channel state information (CSI) for the time order k'.

The ML decoder at the receiver can make decisions according to the rule

$$\hat{\underline{c}} = \arg\max_{\underline{c} \in \mathcal{C}} \sum_k \lambda^i(\boldsymbol{y}_{k'}, c_k). \tag{2}$$

III. One-Dimensional BICM Metric for M-ary QAM

For M-ary QAM constellations $\chi \subseteq \mathbb{C}$. From this point forward we denote bold symbols $\boldsymbol{y}_{k'}$ and $\boldsymbol{x}$ as $y_{k'}$ and x which are complex numbers.

One can show using the ML criterion [6] that maximizing the probabilities in equation (1) is equal to minimizing the distance between the received symbol and the signal constellation points,

$$\lambda^i(y_{k'}, b) = \min_{x \in \chi_b^i} \|y_{k'} - x\|^2 \tag{3}$$

where $\|(\cdot)\|^2$ denotes the Euclidean distance square of $(\cdot)$ and $y_{k'}$ is the output of an equalizer or the received signal if channel is unknown. Then, the ML decision rule given in (2) can be rewritten as

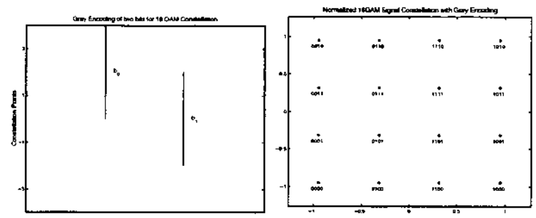
$$\begin{aligned} \hat{\underline{c}} &= \arg\min_{\underline{c} \in \mathcal{C}} \sum_k \lambda^i(y_{k'}, c_k) \\ &= \arg\min_{\underline{c} \in \mathcal{C}} \sum_k \min_{x \in \chi_{c_k}^i} \|y_{k'} - x\|^2. \end{aligned} \tag{4}$$

This metric solves the difficult problem of the different ordering of the bits before and after the interleaver at the transmitter in decoding by associating a contribution to the metric for each bit, associated with the channel symbol received while that bit is transmitted. In other words, consecutive sections of the trellis employ different channel symbols depending on the interleaver, and the metric is different than that used in conventional Viterbi decoding.

As mentioned in [5] Gray encoding is used for BICM, and plays a key role in its performance. In terms of BICM notation of this paper, we rephrase the definition of Gray encoding for the reader's convenience.

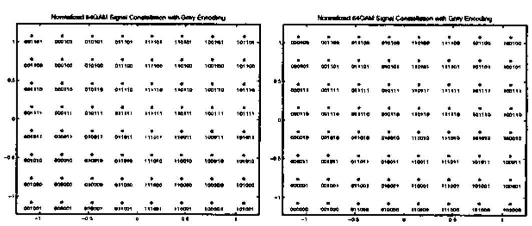
Definition: Gray Encoding: Let χ denote a signal set of size $M = 2^m$, with minimum Euclidean distance d_{min}. A binary map $\mu : \{0,1\}^m \rightarrow \chi$ is a Gray encoding for χ if, for all $i = 1, \ldots, m$ and $b \in \{0,1\}$, each $x \in \chi_b^i$ has at most one $z \in \chi_{\bar{b}}^i$ at distance d_{min}.

There are many different ways of Gray encoding an M-ary QAM constellation. One way is to separate the m bits into two, $m/2$ bits for the in-phase and $m/2$ bits for the quadrature components of a symbol. Then encode the $m/2$ bits onto $2^{m/2}$ levels on the real line $\mathbb{R}^1$ according to Gray encoding rule for each in-phase and quadrature component. Combining in-phase and quadrature components results in an M-ary QAM constellation on the complex plane. Such an encoding is shown in Figure 2 for a 16 QAM constellation[2]. The bars in Figure 2 (a) represent where the bit (b_0 or b_1) is one. Similarly, such two different encodings for 64 QAM are given in Figure 3.



(a) (b)

Fig. 2. 16 QAM constellation with Gray encoding, (a) Encoding of two bits into four levels (b) Two-Dimensional Constellation



(a) (b)

Fig. 3. Two examples of 64 QAM constellation with Gray encoding.

In order to find the bit metrics given in equation (1) or equivalently in equation (3), one has to have the subsets $\chi_b^i; i = 0, 1, \ldots, m-1, b \in \{0,1\}$ of the signal map χ. Figures 4 (a)-(h) show the subsets of the signal map of Figure 2 (b). Decision regions for the constellation points in the subsets are also shown. It is easy to find the subsets of 64 QAM constellations of Figures 3 (b) and (d) in the same manner. As given in (3) and (4), for M-ary QAM systems, each soft-decision bit metric of BICM is the minimum distance square between $y_{k'}$ and $M/2$ constellation points of $\chi_{c_k}^i$. The square distance on the complex plane in (3) can be calculated

[2]All the QAM constellations presented here are normalized so that the average energy of the signal constellation is one.

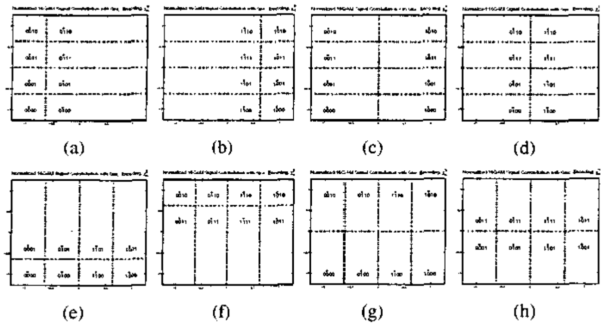


Fig. 4. Subsets of normalized 16 QAM constellation with Gray encoding. Decision regions for the constellation points are shown with dotted lines. (a) χ_0^0, (b) χ_1^0, (c) χ_0^1, (d) χ_1^1, (e) χ_0^2, (f) χ_1^2, (g) χ_0^3, (h) χ_1^3

by adding the square distances of in-phase and quadrature components.

$$\begin{aligned} \|y_{k'} - x\|^2 &= |re(y_{k'}) - re(x)|^2 + |im(y_{k'}) - im(x)|^2 \\ &= d_{in}^2(y_{k'}, x) + d_q^2(y_{k'}, x) \end{aligned} \quad (5)$$

where $re(\cdot)$ and $im(\cdot)$ are the real and imaginary parts of a complex number $(\cdot)$, and $d_{in}(\cdot)$ and $d_q(\cdot)$ represents the distance in in-phase and quadrature axes. So, the equation (4) becomes,

$$\underline{\hat{c}} = \arg\min_{\underline{c} \in C} \sum_k \min_{x \in \chi^i_{c_k}} \left[d_{in}^2(y_{k'}, x) + d_q^2(y_{k'}, x) \right]. \quad (6)$$

A Viterbi decoder at the receiver decodes the original bit sequence $c_k \in \{0,1\}$ through the trellis by calculating the bit metrics using $y_{k'}$ and i^{th} bit location. One way to do this is to generate the trellis with the original ordering of c_k's. It is known at the receiver that the bit c_k is in the symbol $y_{k'}$ at the i^{th} bit location. Through the trellis, for each branch from one stage to another, the bit metric for each c_k can be calculated with this knowledge and whether c_k is zero or one on that particular branch. Adding the bit metrics gives the branch metric, and the Viterbi algorithm can be applied through the trellis.

Let's define $x^i_{c_k}$ as the constellation point where the metric (3) is minimum $\forall x \in \chi^i_{c_k}$, and assume that $0 \le i \le m/2 - 1$. Then, it is easy to see from the decision regions in Figures 4 (a)-(d) that for a fixed i, the quadrature values of $x^i_{c_k=0}$ and $x^i_{c_k=1}$ are the same[3]. This is due to the fact that for $0 \le i \le m/2 - 1$ subsets χ_0^i and χ_1^i covers all the constellation points of χ in the quadrature axis over the given $\sqrt{M}/2$ points of the in-phase axis. Hence, $d_q(y_{k'}, x^i_{c_k})$ is the same for $c_k = 0$ and $c_k = 1$. Therefore for $i = 0, 1, \ldots, m/2 - 1$, $d_q(y_{k'}, x^i_{c_k})$ has no effect on making a decision about c_k in (6). Similarly, for $i = m/2, \ldots, m-1$ $d_{in}(y_{k'}, x^i_{c_k})$ is the same for $c_k = 0$ and $c_k = 1$ (see Figures 4 (e)-(h) for 16 QAM case), and therefore has no effect on making a decision about c_k in (6). Consequently, the two-dimensional metric given in equation (3) reduces to one-dimensional distance square.

[3]Note that this result can be easily generalized to any M-ary QAM constellation

$$\lambda^i(y_{k'}, b) = \begin{cases} \min\limits_{\tilde{x} \in \tilde{\chi}_b^{\tilde{i}}} |re(y_{k'}) - \tilde{x}|^2, & i = 0, 1, \ldots, m/2 - 1 \\ \min\limits_{\tilde{x} \in \tilde{\chi}_b^{\tilde{i}}} |im(y_{k'}) - \tilde{x}|^2, & i = m/2, \ldots, m - 1 \end{cases} \quad (7)$$

where

$\tilde{\chi}$: set of constellation points on the real line $\mathbb{R}^1$

$\tilde{\chi}_b^{\tilde{i}}$: subset of $\tilde{\chi}$ where the $\tilde{i}^{th}$ bit is equal to $b \in \{0,1\}$

$\tilde{i}$= $0, 1, \ldots, m/2 - 1$

$\tilde{i}$= $\begin{cases} i, & i = 0, 1, \ldots, m/2 - 1 \\ i - m/2, & i = m/2, \ldots, m - 1 \end{cases}$

$\tilde{x}$: elements of $\tilde{\chi}$

$|(\cdot)|$: absolute value of real number $(\cdot)$

Since $\forall a, b \in \mathbb{R}$ if $|a|^2 \le |b|^2$, then $|a| \le |b|$ holds and (6) is in summation form; one can, in addition, simplify the bit metrics to one-dimensional distance,

$$\lambda^i(y_{k'}, b) = \begin{cases} \min\limits_{\tilde{x} \in \tilde{\chi}_b^{\tilde{i}}} |re(y_{k'}) - \tilde{x}|, & i = 0, 1, \ldots, m/2 - 1 \\ \min\limits_{\tilde{x} \in \tilde{\chi}_b^{\tilde{i}}} |im(y_{k'}) - \tilde{x}|, & i = m/2, \ldots, m - 1 \end{cases}. \quad (8)$$

Figures 5 (a)-(d) shows the soft-decision bit metrics of equation (8). A minimum path Viterbi decoder can be used with the soft-decision bit metrics of equation (8) to decode the original bit sequence.

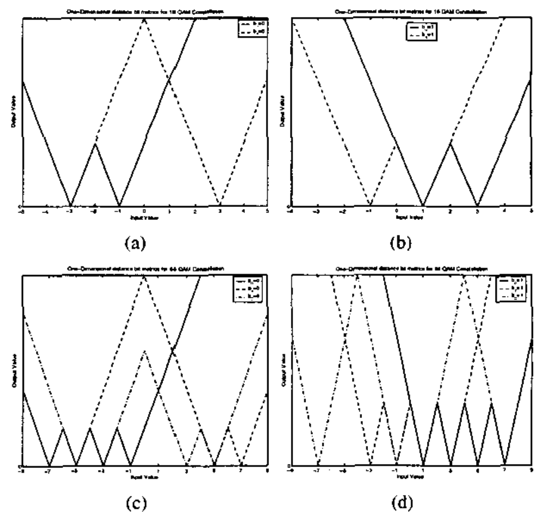
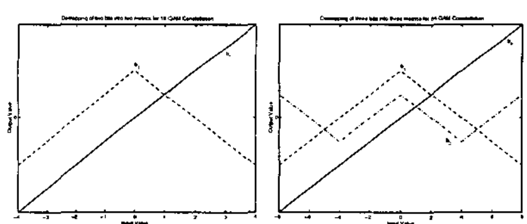


Fig. 5. Bit metrics given in (8) (a) 16 QAM, $c_k = 0$ (b) 16 QAM, $c_k = 1$ (c) 64 QAM, $c_k = 0$ (d) 64 QAM, $c_k = 1$

As a result, soft-decision bit metrics of BICM are simplified to the minimum distance between in-phase or quadrature component of $y_{k'}$ and $\sqrt{M}/2$ points of $\tilde{\chi}^{\tilde{i}}_{c_k}$ on the real line $\mathbb{R}^1$, instead of the minimum distance between $y_{k'}$ and $M/2$ points of $\chi^i_{c_k}$ on the complex plane. This reduces the number of calculations needed for each soft-decision bit metric substantially as tabulated in Table I.

Several industry standards, for example IEEE 802.11a, employ an encoder structure that is essentially equivalent to the encoder of BICM: an industry standard convolutional encoder followed by an interleaver. Typically standards leave the decoding operation to vendors. One possibility in this case is to employ hard decision decoding with its well-known performance degradation from the optimum, typically more than 2 dB. There are other techniques used in industry based on individual bit metrics. Bit metrics for one such technique are plotted in [7], as shown in Figure 6. Although one can find intuitive explanations, these bit metric plots formally correspond to the following definition.

- Define $I_b^{\tilde{i}}$ as the union of intervals on the real line $\mathbb{R}^1$ where the $\tilde{i}^{th}$ bit has the value $b \in \{0,1\}$ (the bars of Figure 2 (a) represent $I_1^{\tilde{i}}$).
- Define y as $re(y_{k'})$ for $0 \leq i \leq m/2-1$ and as $im(y_{k'})$ for $m/2 \leq i \leq m-1$.
- For given $\tilde{i}$, define $I(y,\tilde{i})$ as the interval from the set of intervals $\{I_b^{\tilde{i}}\}_b, b \in \{0,1\}$ on the real line $\mathbb{R}^1$ that the real number y belongs to.
- Define $I^c(y,\tilde{i})$ as the complement of $I(y,\tilde{i})$ on the real line. $I^c(y,\tilde{i}) = \mathbb{R}^1 - I(y,\tilde{i})$.
- Then the bit metrics can be defined as the distance from y to $I^c(y,\tilde{i})$.



(a) (b)

Fig. 6. Bit metrics given in [7] (a) 16 QAM (b) 64 QAM

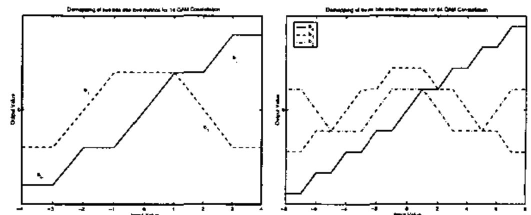
Note that the bit metrics given in [7] (Figure 6) can be interpreted as approximations to the optimum BICM bit metrics presented in this paper (equation (8) and Figure 5).

In a similar manner, we define another set of metrics that can be used with BICM for M-ary QAM systems.

- Define $\tilde{x}_b(y,\tilde{i}) \in \tilde{\chi}_b^{\tilde{i}}, b \in \{0,1\}$ as the closest constellation point to the real number y.
- Define the distance $d_b^{\tilde{i}}(y)$ as $d_b^{\tilde{i}}(y) = |y - \tilde{x}_b(y,\tilde{i})|$.
- Define the bit metrics as $d_0^{\tilde{i}}(y) - d_1^{\tilde{i}}(y)$, where y is either $re(y_{k'})$ or $im(y_{k'})$ depending on i.

Figure 7 illustrates this set of soft-decision bit metrics. These are again approximations to the metrics of (8) with the same high performance as will be shown in the next section.

The results presented here are valid for any scheme (SCM or MCM) with QAM that deploys bit interleaving at the transmitter over any type of communication channel. We showed both mathematically and via simulations (see Section IV) that the simplified soft-decision bit metrics of this paper are equal to the original ones given in [5] in terms of decoding the information bits. Therefore, with the new simplified bit metrics, there is no performance degradation in the decoder over AWGN or Rayleigh fading channels.



(a) (b)

Fig. 7. Bit metrics, difference of distances (a) 16 QAM (b) 64 QAM

IV. Simulation Results

We ran simulations for SCM and MCM systems. In both cases, the channel is modeled either as AWGN or as Rayleigh fading. Rayleigh channel is modeled as complex Gaussian random variables with zero mean and variance one.

For both systems, we ran simulations using the bit metrics given in equations (3), (7), (8), and in Figures 6 and 7. We also ran simulations using a hard decision Viterbi decoder. In hard decision Viterbi decoder case, the symbols $\{y_{k'}\}$ are first passed through a demodulator. The demapped bits are then deinterleaved and used as inputs to a hard decision Viterbi decoder.

A. SCM Results

In SCM simulations, we used the industry standard $1/2$ rate (133,171) convolutional encoder with constraint length $k=7$. The bit interleaver given in IEEE 802.11a documentation, [8], is used before modulating the bits onto 64 QAM constellation. Puncturing is used to achieve $3/4$ rate for the simulations. We assumed perfect knowledge of the channel and the received signal is equalized with this knowledge to obtain $\{y_{k'}\}$.

Simulation results for SCM are given in Figures 8 (a) and (b) for AWGN and Rayleigh channel, respectively. It is easy to see that simulation results agree with our findings as given in equations (3), (7) and (8). Hence, by using the proposed metrics in (7) or in (8), the complexity of the decoder is lowered significantly without compromising the performance. Our simulation results also showed that the bit metrics given in Figures 6 and 7 gives the same performance as the metrics of (3), (7) and (8).

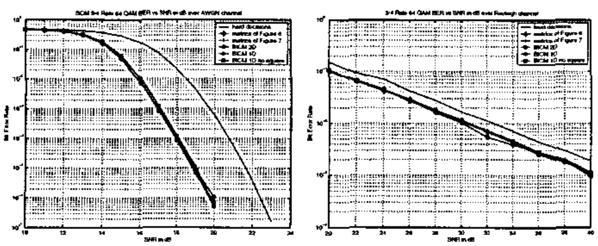
B. MCM Results

For MCM simulations, we used the wireless local area network (WLAN) standard IEEE 802.11a, [8]. IEEE 802.11a deploys orthogonal frequency division multiplexing (OFDM) with 48 data carriers. Bit-interleaving is deployed at the transmitter for IEEE 802.11a systems. At the receiver, the channel is estimated using the special training sequences of

M-ary	Multiplications		Additions		Subtractions		Comparisons	
	original	low complexity	original	low complexity	original	low complexity	original	low complexity
4	4	0	2	0	4	1	2	1
16	16	0	8	0	16	2	8	2
64	64	0	32	0	64	4	32	4
256	256	0	128	0	256	8	128	8
1024	1024	0	512	0	1024	16	512	16

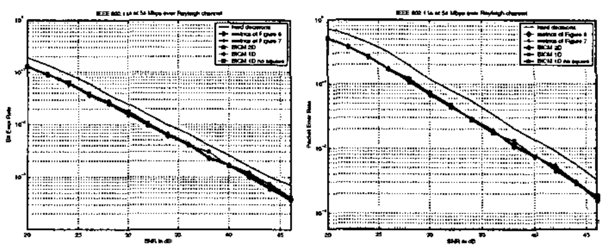
TABLE I

THE NUMBER OF REAL SUBTRACTIONS, MULTIPLICATIONS, ADDITIONS AND COMPARISONS NEEDED FOR EACH BIT METRIC USING THE ORIGINAL BICM METRIC (3) AND THE LOW COMPLEXITY METRIC (8)



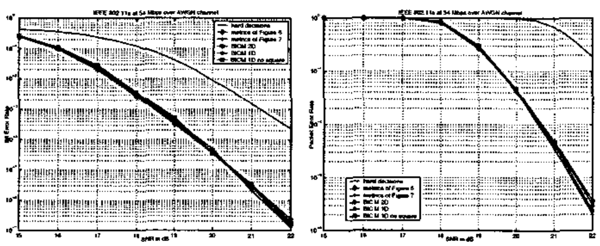
(a) AWGN Channel (b) Rayleigh Channel

Fig. 8. SCM 3/4 Rate 64 QAM, BER vs SNR in dB curves, (a) over AWGN (b) over Rayleigh



(a) BER vs SNR (b) PER vs SNR

Fig. 10. IEEE 802.11a at 54 Mbps mode over Rayleigh channel, (a) BER vs SNR (b) PER vs SNR



(a) BER vs SNR (b) PER vs SNR

Fig. 9. IEEE 802.11a at 54 Mbps mode over AWGN channel, (a) BER vs SNR (b) PER vs SNR

an IEEE 802.11a package. Received signal is equalized using this channel estimation to obtain $\{y_{k'}\}$. We ran the simulations on IEEE 802.11a at 54 Mbps mode (3/4 rate, 64 QAM).

Bit error rate (BER) vs SNR, and packet error rate (PER) vs SNR curves for AWGN channel are given in Figure 9. BER vs SNR, and PER vs SNR curves for Rayleigh channel are given in Figure 10. As expected, the bit metrics given in the equations (3), (7) and (8), and in the Figures 6 and 7 have the same performance.

V. CONCLUSION

BICM plays an important role in wireless communications. In this paper we showed that for M-ary QAM systems the complexity of a Viterbi decoder used for BICM can be significantly lowered without compromising the performance. This is achieved by Gray encoding the in-phase and quadrature components of a QAM signal separately, and then combining them to have an M-ary QAM constellation. As a result, soft-decision bit metrics are simplified to the minimum distance between the received symbol and $\sqrt{M}/2$ points on the real line $\mathbb{R}^1$, instead of the minimum distance between the received symbol and $M/2$ points on the complex plane. This reduces the complexity of the decoder substantially without compromising the performance.

Simulation results for SCM and MCM systems agreed that the proposed new metrics have the same performance as the original ones while the complexity of a decoder is reduced significantly.

In addition, we showed that the optimum BICM metrics can be simplified for implementation without degrading performance.

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