Adaptive Modulation and Coding for Bit Interleaved Coded Multiple Beamforming

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***Abstract*— Bit interleaved coded multiple beamforming (BICMB) was previously designed to achieve full spatial multiplexing of $\min(N, M)$ and full spatial diversity of NM for N transmit and M receive antennas over flat fading channels. Furthermore, BICMB when combined with orthogonal frequency division multiplexing (OFDM) achieves full spatial multiplexing and full diversity order of NML over L-tap frequency-selective channels. BICMB requires full channel state information (CSI) both at the transmitter and receiver, however it uses uniform power and rate over the parallel channels established by multiple beamforming. In this paper, our main goal is to investigate the performance of the previously analyzed BICMB system via adaptive modulation and coding (AMC) to further utilize CSI and to improve system throughput performance. Hence, we name the new systems as Adaptive BICMB (ABICMB) over flat fading, and ABICMB-OFDM over frequency-selective channels. Simulation results show that adaptive system achieves 4-13 dB performance gain compared to non-adaptive case depending on the antenna configuration and environment. Systems analyzed require perfect CSI both at the transmitter and receiver, which may be difficult to obtain in a practical scenario. However, high performance gains achieved makes it worthwhile to study the performance of the proposed systems, leaving room for significant gain with limited feedback.**

I. Introduction

Multi-input multi-output (MIMO) systems allow significant diversity gains in fading environments. Some of the MIMO systems incorporating diversity require the channel state information (CSI) at the receiver, but not at the transmitter [1]. Another group requires perfect or partial CSI at both the transmitter and the receiver. When perfect CSI is available at both ends, two techniques that can be used are single and multiple beamforming [2], [3]. These techniques utilize singular value decomposition (SVD) which separates the MIMO channel into parallel subchannels. When only the subchannel with the largest gain is used for transmission, the technique is called *single beamforming* [2]. MIMO systems can also be used to enhance the throughput of wireless systems [4]. To that end, when more than one subchannel is used to improve the capacity, the technique is called *multiple beamforming* [2]. In other words, multiple beamforming is a special case of *spatial multiplexing* in which SVD-based linear processing is employed at the transmitter and the receiver sides [5].

We previously showed that the diversity order of uncoded multiple beamforming decreases as more symbols are transmitted simultaneously [3]. The recent results in [6] show that if a properly designed bit interleaved coded modulation (BICM) is incorporated to the SVD-based multiple beamforming, one can achieve full spatial multiplexing of $\min(N, M)$ and full spatial diversity of NM for N transmit and M receive antennas over flat fading channels. In addition, properly designed BICMB-OFDM achieves full spatial multiplexing and full diversity order of NML over L-tap frequency-selective channels [7]. However, bit interleaved coded multiple beamforming (BICMB) in [6] and [7] employ uniform power and constellation over the established subchannels. As a result, BICMB does not fully utilize the CSI at the transmitter. In this paper, we further exploit the perfect CSI available at both ends with the inclusion of adaptive modulation and coding in the system. Adaptive coding [8], adaptive modulation [9], and AMC [10], were previously analyzed for different systems, and the results showed significant gains compared to the non-adaptive case. We first analyze the pairwise error probability for adaptive BICMB for both flat fading and frequency-selective fading channels. Next, we study a rate maximization problem with total power and bit error rate (BER) constraints. Simulation results show that 4-13 dB gain is achievable via adaptivity.

Notation: N, M are the number of transmit and receive antennas, respectively. K is the number of subcarriers. Bold capital and lower case letters denote matrices and vectors, respectively. The superscript $(\cdot)^H$ denotes Hermitian transpose. $\bar{(\cdot)}$ denotes binary complement.

II. Channel Model and System Overview

For the ABICMB scheme, it is assumed that the instantaneous CSI is available both at the transmitter and the receiver as in the previously analyzed BICMB system. For the sake of clarity, we will first describe the channel model for uncoded multiple beamforming for flat fading and frequency-selective fading (with OFDM) cases individually.

A. Flat Fading

We assume a quasi-static flat fading MIMO channel model, where channel fading parameters are modeled as i.i.d. complex

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Gaussian random variables. Let $\mathbf{H}$ denote the quasi-static flat fading $N \times M$ MIMO channel. Multiple beamforming is implemented by multiplying the symbols with appropriate beamforming vectors both at the transmitter and the receiver. In this paper, we assume that CSI is available at both ends. In such a case, the beamforming vectors are obtained via the SVD of the channel, separating the MIMO channel into parallel subchannels [11], [12]. Without loss of generality, we assume $N \leq M$ throughout this paper. Then the SVD of $\mathbf{H}$ can be written as $\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H = [\mathbf{u_1}\,\mathbf{u_2}\ldots\mathbf{u_N}]\mathbf{\Lambda}[\mathbf{v_1}\,\mathbf{v_2}\ldots\mathbf{v_M}]^H$, where $\mathbf{U}$ and $\mathbf{V}$ are $N \times N$ and $M \times M$ unitary matrices, respectively, and $\mathbf{\Lambda}$ is an $N \times M$ diagonal matrix with singular values of $\mathbf{H}$, $\lambda_i \in \mathbb{R}$, $i = 1, 2, \ldots, N$, on the main diagonal with decreasing order. If N symbols are transmitted simultaneously to utilize all of the subchannels, then the system input-output relation at the t^{th} time instant and s^{th} subchannel can be written as

$$\begin{aligned}\mathbf{y}(t) &= \mathbf{x}(t)[\mathbf{u_1}(t)\ \ldots\mathbf{u_N}(t)]^H\mathbf{H}(t)[\mathbf{v_1}(t)\ldots\mathbf{v_N}(t)] \\ &\quad + \mathbf{n}(t)[\mathbf{v_1}(t)\ldots\mathbf{v_N}(t)] \\ y_{t,s} &= \lambda_{t,s}x_{t,s} + n_{t,s}\ , \quad \text{for } t = 1,\ldots,T, \text{ and } s = 1,\ldots,N\end{aligned} \tag{1}$$

where $\mathbf{n}(t)$ is $1 \times M$ additive white Gaussian noise with zero-mean and variance $N_0 = N/SNR$. Note that the total power transmitted is scaled as N. The channel elements h_{nm} are modeled as zero-mean, unit-variance complex Gaussian random variables. Consequently, the received signal-to-noise ratio is SNR.

B. Frequency-Selective Fading

When the environment is frequency-selective, then OFDM is used to combat intersymbol interference. By adding cyclic prefix (CP), OFDM converts the frequency selective channel into parallel flat fading channels for each subcarrier. Let $\mathbf{H}(k)$ denote the quasi-static, flat fading $N \times M$ MIMO channel at the k^{th} subcarrier. Then, $\mathbf{H}(k)$ can be written as

$$\mathbf{H}(k) = \mathbf{W}^H(k)\mathbf{H} \tag{2}$$

where $\mathbf{W}(k)$ denotes a block-diagonal matrix with diagonal elements $\mathbf{w}(k) = [1 \quad W_K^k \quad \ldots \quad W_K^{(L-1)k}]^H$ and $W_K = e^{-j2\pi/K}$, and $\mathbf{H}_{n,m}$ represents the L-tap frequency selective channel from the transmit antenna n to the receive antenna m. Each tap is assumed to be statistically independent and modeled as zero mean complex Gaussian random variable with variance $1/L$. With IFFT and FFT processing at the transmitter and receiver, respectively, and appropriate CP, the system input-output relation can be represented by (1), where subcarrier number k will be used instead of time subscript t, and correspondingly (k, s) represents the s^{th} spatial subchannel on k^{th} subcarrier.

For brevity, in the rest of this section, we will only explain the ABICMB system model. We consider a packet-based transmission where, for each packet transmission, the bit stream is encoded using a convolutional code C_v with rate $r(C_v)$ and the minimum Hamming distance $d_{free}(C_v)$. The output bits of the binary convolutional encoder are interleaved by a random block interleaver, spreading the bits over the parallel orthogonal subchannels created by SVD-based multiple beamforming. The bit interleaver of ABICMB can be modeled as $\pi : k' \rightarrow (t, s, i)$ where k' denotes the original ordering of the coded bits $c_{k'}$, t denotes the time ordering of the signals $x_{t,s}$ transmitted, s denotes the subchannel used to transmit $x_{t,s}$, and i indicates the position of the bit $c_{k'}$ on the symbol $x_{t,s}$.

At each channel use, the s^{th} subchannel transmits one of $M_s = 2^{m_s}$ symbols in a QAM constellation set χ_s with a transmit power P_s. The constellation set χ_s has a minimum constellation distance $d_{min,s}$. Assuming that m_s bits are loaded to the s^{th} subchannel, m_s coded and interleaved bits are mapped over a signal set $\chi_s \subseteq \mathbb{C}$ of size $|\chi_s| = 2^{m_s}$ with a binary labeling map $\mu_s : \{0,1\}^{m_s} \rightarrow \chi_s$. Gray encoding is used to map the bits onto symbols. Let $\chi_b^{(i,s)}$ denote the subset of all signals $x \in \chi_s$ whose label has the value $b \in \{0,1\}$ in position i. Then, the ML bit metrics can be given by using (1), [13], [14]

$$\gamma^i(y_{t,s}, c_{k'}) = \min_{x \in \chi_{c_{k'}}^{(i,s)}} |y_{t,s} - x\lambda_s|^2. \tag{3}$$

The ML decoder at the receiver makes the decisions according to the rule

$$\hat{\underline{\mathbf{c}}} = \arg\min_{\underline{\mathbf{c}} \in C_v} \sum_{k'} \gamma^i(y_{t,s}, c_{k'}). \tag{4}$$

Equations (3) and (4) will be valid for ABICMB-OFDM as well; one should use subscript k instead of t and channel singular values will be a function of both subchannel and subcarrier index (i.e., $\lambda_{k,s}$).

A very low complexity Viterbi decoder for BICM can be implemented as in references [15], [16]. The same decoder can be used for ABICMB as well: Instead of using the single-input single-output (SISO) channel value of BICM-OFDM for the decoder ([15] and [16]), one should use λ_s.

III. Performance Analysis

In this section, we provide the performance analyses for both ABICMB and ABICMB-OFDM systems. First, we find an upper bound for the pairwise error probability for both systems of interest. In the last subsection, we study a rate maximization problem to enhance the throughput of the systems under power and BER constraints and provide a suboptimal solution.

A. Pairwise Error Probability

1) Flat Fading: Assume the code sequence $\underline{c}$ is transmitted and the probability that $\hat{\underline{c}}$ is detected is represented by $P(\underline{c} \rightarrow \hat{\underline{c}}|\mathbf{H})$ for a given channel realization. Then, using (3), the PEP of $\underline{c}$ and $\hat{\underline{c}}$ given CSI can be written as

$$\begin{aligned}P(\underline{c} \rightarrow \hat{\underline{c}}|\mathbf{H}) = P\Bigg(&\sum_{k'} \min_{x \in \chi_{c_{k'}}^i} |y_{t,s} - x\lambda_s|^2 \geq \\ &\sum_{k'} \min_{x \in \chi_{\hat{c}_{k'}}^i} |y_{t,s} - x\lambda_s|^2 \mid \mathbf{H}\Bigg)\end{aligned} \tag{5}$$

where $s \in \{1, 2, \ldots, N\}$. Note that channel singular value has no time subscript since the channel is assumed to be constant during one packet transmission.

Without loss of generality, we assume that $\underline{c}$ and $\hat{\underline{c}}$ differ in the first d_{free} bits. Hence, the summations can be simplified to only d_{free} terms for PEP analysis

$$P(\underline{c} \to \hat{\underline{c}}|\mathbf{H}) = P\left(\sum_{k'=1}^{d_{free}} \min_{x \in \chi^i_{c_{k'}}} |y_{t,s} - x\lambda_s|^2 \geq \sum_{k'=1}^{d_{free}} \min_{x \in \chi^i_{\bar{c}_{k'}}} |y_{t,s} - x\lambda_s|^2 \mid \mathbf{H}\right). \quad (6)$$

Note that for binary codes and for the d_{free} points at hand, $\hat{c}_{k'} = \bar{c}_{k'}$. Let's denote $\tilde{x}_{t,s} = \arg\min_{x \in \chi^i_{c_{k'}}} |y_{t,s} - x\lambda_s|^2$, and $\hat{x}_{k,s} = \arg\min_{x \in \chi^i_{\bar{c}_{k'}}} |y_{t,s} - x\lambda_s|^2$. It is easy to see that $\tilde{x}_{t,s} \neq \hat{x}_{t,s}$ since $\tilde{x}_{t,s} \in \chi^i_{c_{k'}}$ and $\hat{x}_{t,s} \in \chi^i_{\bar{c}_{k'}}$, where $\chi^i_{c_{k'}}$ and $\chi^i_{\bar{c}_{k'}}$ are complementary sets of constellation points within the signal constellation set χ. Also, $|y_{t,s} - x_{t,s}\lambda_s|^2 \geq |y_{t,s} - \tilde{x}_{t,s}\lambda_s|^2$ and $x_{t,s} \in \chi^i_{c_{k'}}$.

For convolutional codes, due to their trellis structure, d_{free} distinct bits between any two codewords occur in consecutive trellis branches. Let's denote b such that d_{free} bits occur within b consecutive bits. The bit interleaver can be designed such that b consecutive bits are mapped onto distinct symbols. This guarantees that there exist d_{free} distinct pairs of $(\tilde{x}_{t,s}, \hat{x}_{t,s})$, and d_{free} distinct pairs of $(x_{t,s}, \hat{x}_{t,s})$. The PEP can be rewritten as

$$\begin{aligned} P(\underline{c} \to \hat{\underline{c}}|\mathbf{H}) \leq \\ & P\left(\sum_{k, d_{free}} |y_{t,s} - x_{t,s}\lambda_s|^2 - |y_{t,s} - \hat{x}_{t,s}\lambda_s|^2 \geq 0\right) \\ & \leq Q\left(\sqrt{\frac{\sum_{s=1}^{N} \alpha_s \lambda_s^2 d_{min,s}^2}{2N_0}}\right) \end{aligned} \quad (7)$$

where α_s denotes how many times the s^{th} subchannel is used and $\sum_{s=1}^{N} \alpha_s = d_{free}$. Note that, the upper bound in (7) can be calculated using the analysis in [10], [14], however we provide a simpler method here. Equation (7) can be further simplified using a similar analysis in [7] in order to show that using different constellations and power values over the subchannels does not effect the diversity performance of BICMB system.

2) Frequency-Selective Fading: The PEP analysis of ABICMB-OFDM system closely follows the analysis of ABICMB, since the input-output relations are equivalent. Using (1) and the steps between (5) and (7), and changing the subscripts wherever needed, it is straightforward to show that the error bound for the ABICMB-OFDM system can be expressed as

$$P(\underline{c} \to \hat{\underline{c}}|\mathbf{H}(k)) \leq Q\left(\sqrt{\frac{\sum_{k, d_{free}} \lambda_{k,s}^2 d_{min_{k,s}}^2}{2N_0}}\right) \quad (8)$$

for all k. In order to achieve high performance, i.e., to decrease the number of dominant error events, the interleaver should spread the incoming bits in such a way that consecutive bits are mapped onto distinct subcarriers and subchannels.

Note that, for a convolutional code C_v with puncturing period p, BER is bounded by [17]

$$P_b \leq \frac{1}{p} \sum_{d=d_{free}(C_v)}^{\infty} N_v(d) P(\underline{c} \to \hat{\underline{c}} \mid \mathbf{H}) \quad (9)$$

where $N_v(d)$ denotes the total input weight of the error events at the Hamming distance d of C_v and the PEP term is given by (7) and (8). The bound in (9) is loose especially for quasi-static flat fading channels. A tighter bound can be attainable using the analysis in [18] and [19]. It was not possible to use the tighter bound in our rate maximization problem in the next subsection due to its complicated formulation. The simplicity of the bound in (9) will enable us to analytically analyze the rate maximization problem in the sequel.

B. Rate Maximization

Our objective is to maximize the total rate by applying adaptive modulation and coding. For practical reasons, we assume that there are V different convolutional codes $C_1, C_2, \ldots, C_V$ available for the system and we will force the system to use only square QAM constellations with $m_{max} = 6$. The rate maximization problem for ABICMB can be expressed as

$$\begin{aligned} &\max \quad R(v) = r(C_v) \sum_{s=1}^{N} m_s \\ &\text{s. t.} \quad P = \sum_{s=1}^{N} P_s \leq P_{total},\ P_b \leq P_e \text{ , and } m_s \leq m_{max} \end{aligned} \quad (10)$$

where P_{total} is the total power constraint, P_e is the target BER, and P_b is given in (9). The rate maximization problem for ABICMB-OFDM will have a similar form, and can be expressed as

$$\begin{aligned} &\max\ R(v) = r(C_v) \sum_{k=1}^{K} \sum_{s=1}^{N} m_{k,s} \\ &\text{s. t.}\ P = \sum_{k=1}^{K} \sum_{s=1}^{N} P_{k,s} \leq P_{total},\ P_b \leq P_e, \text{ and } m_{k,s} \leq m_{max} \end{aligned} \quad (11)$$

The optimization problem in (10) and (11) is difficult to solve since the dependence between the optimization parameters is complicated. It can be observed from (7) that BER can be kept below a certain error level, if $\lambda_s^2 d_{min,s}^2$ is greater than a constant value. This means that higher constellations can be used in better channel realizations. To simplify a similar formulation that was for a single antenna BICM-OFDM system in [10], the authors chose a constant value to keep the BER below a certain level as $\lambda_s^2 d_{min,s}^2 \geq \frac{6N_0\Gamma}{d_{free}(C_v)}$, where Γ is a constant that needs to be tuned to achieve the target error rate. We will include this constraint in our formulation, since not only will it be sufficient to keep BER below a target value, but also it will greatly simplify the optimization problem. Incorporating the new constraint, the bound for BER becomes

$$P_b \leq \frac{1}{p} \sum_{d=d_{free}(C_v)}^{\infty} N_v(d) Q\left(\sqrt{\frac{3d\Gamma}{d_{free}(C_v)}}\right) \quad (12)$$

where Γ can be found using numerical methods to achieve certain target BER with a significant complexity, since it will require inversion of sum of many Q functions. Alternatively, Γ can be tuned by simulation. Using the relation $d_{min,s}^2 = 6P_s/(2^{m_s} - 1)$ [20], $\lambda_s^2 d_{min,s}^2 \geq \frac{6N_0\Gamma}{d_{free}(C_v)}$ will be equivalent to $m_s \leq \log_2(1 + \frac{d_{free}(C_v)\lambda_s^2 P_s}{\Gamma N_0})$ for ABICMB. With the addition of this new constraint, the modification to the optimization problem in (10) will be suboptimal. Considering only the first term in P_b, the modified problem for both flat and frequency-selective cases can be expressed as

$$\begin{aligned} &\max \quad R(v) \\ &\text{s. t.} \quad P \leq P_{total} \ , \ \ m_s \leq \log_2(1 + \frac{d_{free}(C_v)\lambda_s^2 P_s}{\Gamma N_0}). \quad (13) \end{aligned}$$

For a given convolutional encoder C_v, the problem (13) has the same form of a discrete-rate capacity maximization problem with an SNR gap of $\Gamma/d_{free}(C_v)$, which was originally studied for uncoded systems [20], [21]. This problem is optimally solved with the greedy-based-loading algorithm [21] which will determine the corresponding rate and power values. Therefore the original problem in (10) can be suboptimally solved with running the greedy-based-loading algorithm for each element of the convolutional code set and choosing the convolutional encoder which maximizes the data rate. Although the solution is suboptimal, simulation results in the next section show that significant gains are attainable via adaptive modulation and coding.

IV. Simulation Results

In this section we provide simulation results that will quantify the analytical analysis for the ABICMB and ABICMB-OFDM systems. In the simulations below, the industry standard 64-state 1/2-rate (133,171) $d_{free} = 10$ convolutional code is used as the mother code. The rates $2/3$ and $3/4$ are constructed from the 1/2 rate mother code via puncturing. Therefore the cardinality of the encoder set used for adaptive loading is 3. The minimum Hamming distances are 10, 6, and 5 for the encoders respectively. For the high SNR region, we also allow uncoded adaptive loading to achieve rate 1. The channel is assumed to be constant during one packet transmission. The target BER is set as 10^{-5} for all of the simulation scenarios.

We ran simulations for ABICMB with 2×2 antenna configuration. The parameter Γ is tuned as 4.5 dB for the 2×2 case to achieve the corresponding BER level. This tuning is done by simulation. In a practical communication system, Γ can be tuned by observing the error rates using a frame error detector (e.g., cyclic redundancy check). In Fig. 1, the

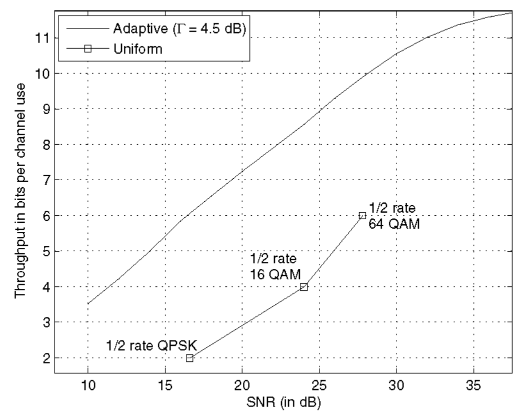

Fig. 1. Throughput performance of ABICMB and uniform BICMB for 2×2 antenna configuration at 10^{-5} BER in flat fading channel.

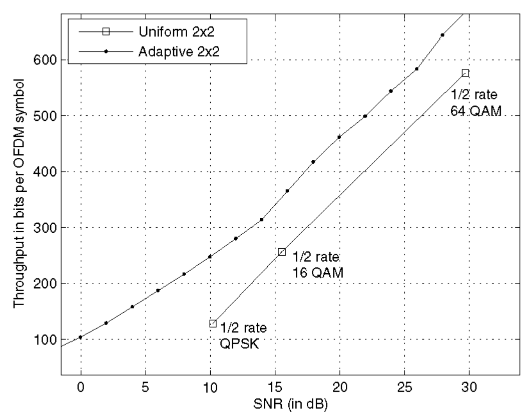

Fig. 2. Throughput performance of ABICMB-OFDM and uniform BICMB-OFDM for 2×2 antenna configuration at 10^{-5} BER in 50ns rms delay spread exponential channel.

throughput performance of ABICMB is compared with that of BICMB which uses uniform power and constellation over the established subchannels by multiple beamforming for the 2×2 case. The throughput definition used in the figure corresponds to the average number of information bits transmitted per each channel use. For BICMB, we used 3 different modes to plot its throughput curve which are 1/2-rate 4 QAM, 16 QAM and 64 QAM. For these 3 modes, BICMB achieves the target of 10^{-5} BER with transmitting 2 streams (equivalently 2 symbols simultaneously). The performance gain of ABICMB is 11-13 dB compared to uniform BICMB, which is significant.

Fig. 2 shows the performance results for ABICMB-OFDM and uniform BICMB-OFDM over a 50ns rms delay spread channel with 2×2 antenna configuration. In the simulations, for the 2×2 case, we observed that using different Γs for the first and second singular values of subcarriers gives better performance compared to using single Γ for the whole system. To achieve the target BER, Γ_1 is set as 6.3 dB and Γ_2 is set as 9 dB. As shown in Fig. 2, the adaptive system achieves 4-7

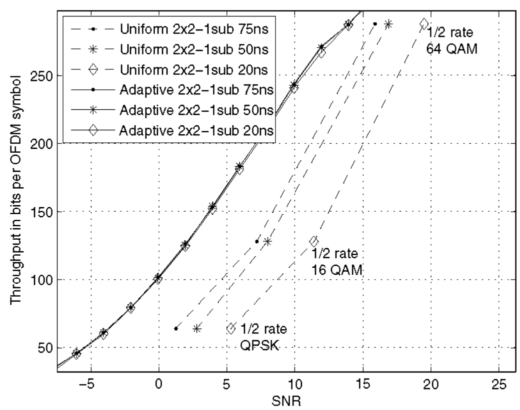

Fig. 3. Throughput performance of ABICMB-OFDM and uniform BICMB-OFDM for 2×2 antenna configuration at 10^{-5} BER for different rms delay spreads, where $\Gamma = 6.3$ dB.

dB performance gain compared to uniform BICMB-OFDM.

Finally, Fig. 3 demonstrates the robustness of the ABICMB-OFDM system against a change in the delay spread of the channel. In Fig. 3, only the largest singular value of the sub-carriers are used for transmission, therefore adaptive loading is employed only over frequency. Actually, this is a performance comparison of adaptive bit interleaved single beamforming and uniform bit interleaved single beamforming, which was previously analyzed in [22]. As seen from the figure, the adaptive system is robust against delay spread changes and the same Γ parameter is used independently from the delay spread to achieve 10^{-5} BER. As the delay spread decreases, the non-adaptive system has a performance loss, accordingly, the performance gain of the adaptive system increases.

V. Conclusion

Bit interleaved coded multiple beamforming (BICMB) was previously analyzed and was shown to attain significant performance gains compared to other spatial multiplexing systems. In particular, properly designed BICMB achieves full spatial multiplexing of $\min(N, M)$ and full spatial diversity of NM for N transmit and M receive antennas over flat fading channels. Furthermore, BICMB when combined with orthogonal frequency division multiplexing (OFDM) achieves full spatial multiplexing and full diversity order of NML over L-tap frequency-selective channels.

BICMB exploits the channel state information both at the transmitter and the receiver using transmit and receive beamforming vectors with a properly designed interleaver. However, it uses uniform power and constellation over the subchannels. In this paper, we analyzed the performance when adaptive modulation and coding is introduced to the previously designed BICMB system. First, we analyzed the pairwise error probability of adaptive BICMB (ABICMB) for both flat and frequency-selective channels. Later, we described a rate maximization problem and showed that a greedy-based adaptive loading algorithm can be suboptimally used to maximize the throughput under BER and total power constraints. Simulation results showed that adaptive system outperforms uniform BICMB 4-13 dB depending on the antenna configuration and environment.

The ABICMB and ABICMB-OFDM systems analyzed in this paper use perfect channel information at the transmitter and the receiver. This may not be the case for a practical scenario. However, the performance gains are encouraging to analyze the BICMB-based systems with limited feedback.

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