MIMO BICM-OFDM Beamforming with Full and Partial CSIT

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***Abstract*— In this paper we analyze single beamforming in combination with bit interleaved coded modulation (BICM) and OFDM. We show that BICM-Beamforming-OFDM (BBO) achieves full diversity in space and frequency independent of the power delay profile of the channel. Since only one stream of data is transmitted over all transmit antennas, a simple interleaver is shown to be sufficient to achieve full space and frequency diversity. Simulation results show that beamforming-based systems introduce substantial coding gain, even with partial channel state information at the transmitter (CSIT), when compared to other systems based on space time block codes (STBC) with the same full diversity order.**

I. Introduction

Multi-input multi-output (MIMO) wireless systems allow significant diversity gains for wireless communications. In general, MIMO systems that achieve full spatial diversity with channel state information only at the receiver (CSIR) are known as space-time coding. Some important results of space-time coding systems can be found in [1].

Another technique, which can be used to achieve full spatial diversity over Rayleigh fading channels by utilizing channel state information (CSI) both at the transmitter and the receiver, is known as beamforming. Beamforming separates the MIMO channel into parallel subchannels. When the best subchannel is used, the technique is called single beamforming [2]. If more than one subchannel is used, the technique is called multiple beamforming [2]. In this letter we focus on single beamforming, and from now on we will refer to it as beamforming for simplicity.

It is known that a widely used technique, bit interleaved coded modulation (BICM) with orthogonal frequency division multiplexing (OFDM), can achieve the maximum frequency diversity order that is inherited in the channel [3], [4]. In Section III we present a system as a combination of BICM [5], beamforming, and OFDM, named BBO. In [6], [7], [8] a more general system transmitting multiple streams of data while maintaining full diversity order was presented. However, BBO presented in this letter transmits only one stream of data, and a much simpler interleaver than that of [6], [7], [8] is shown to be sufficient to achieve the full frequency and space diversity. In fact, in the simulations of Section IV, we used the simple interleaver of the IEEE 802.11a standard. As a result, BBO achieves a diversity order of NML over L-tap frequency selective channels with N transmit and M receive antennas regardless of the power delay profile (PDP) of the channel. Simulation results using the IEEE MIMO wireless channel models [9], [10], [11] are also presented in Section IV.

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II. Beamforming

A. Overview

The beamforming vector is obtained via the singular value decomposition (SVD) of the channel. The $M \times N$ matrix $\mathbf{H}$ denotes the quasi-static Rayleigh flat fading MIMO channel. The SVD of $\mathbf{H}$ can be written as [12]

$$\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H = [\underline{u}_1\ \underline{u}_2 \ldots \underline{u}_M]\mathbf{\Lambda}[\underline{v}_1\ \underline{v}_2 \ldots \underline{v}_N]^H \quad (1)$$

where $\mathbf{\Lambda}$ is a $M \times N$ matrix with singular values, $\{\lambda_i\}_{i=1}^{\min(N,M)}$, in decreasing order on the main diagonal. $\mathbf{U}$ and $\mathbf{V}$ are two unitary matrices of size $M \times M$ and $N \times N$, respectively. The overall system can be represented by

$$y = \underline{u}_1^H \mathbf{H} \underline{v}_1 x + \underline{u}_1^H \underline{n} = \lambda_1 x + n \quad (2)$$

where λ_1 is the largest singular value of $\mathbf{H}$, y is the received symbol, x is the transmitted symbol, and $\underline{n}$ is complex additive white Gaussian noise (AWGN) vector of size $M \times 1$ with zero mean and variance $N_0 = 1/SNR$. The elements of $\mathbf{H}$ are modeled as complex Gaussian random variables with zero mean and 0.5 variance per complex dimension.

B. Performance

By analyzing the pairwise error probability (PEP), it can be shown that beamforming achieves the diversity order of NM for arbitrary N and M. An analysis is given in [13], [14].

III. BICM Beamforming OFDM (BBO)

A. System Model

The system consists of N transmit and M receive antennas; and one OFDM symbol has K subcarriers. The output bits of a convolutional encoder are interleaved within one OFDM symbol to avoid extra delay requirement to start decoding at the receiver. As will be shown in the next section, a simple interleaver which spreads the consecutive coded bits over subcarriers is sufficient to achieve full frequency and spatial diversity. Also, for practical systems, we recommend that the consecutive coded bits are transmitted over subcarriers that are further away from one another instead of adjacent subcarriers in order to lower the correlation between the channels that is seen at each subcarrier. After interleaving, the output bit $c_{k'}$ is mapped onto the symbol $x(k)$ at the ith bit location using the signal map χ with Gray labeling and transmitted on the kth subcarrier. It is assumed that an appropriate length of CP is used for each OFDM symbol. By doing so, OFDM converts the frequency selective channel into parallel flat fading channels, denoted as $M \times N$ matrix $\mathbf{H}(k)$ for $k = 1, 2, \ldots, K$ and given by

$$\mathbf{H}(k) = \mathbf{W}_K^H(k)\mathbf{P}_F\mathbf{h}$$

$$\mathbf{W}_K(k) = \mathbf{I}_M \otimes \underline{\mathrm{W}}_K(k) = \begin{bmatrix} \underline{\mathrm{W}}_K(k) & \underline{0}_{L\times1} & \cdots & \underline{0}_{L\times1} \\ \underline{0}_{L\times1} & \underline{\mathrm{W}}_K(k) & \cdots & \underline{0}_{L\times1} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{0}_{L\times1} & \underline{0}_{L\times1} & \cdots & \underline{\mathrm{W}}_K(k) \end{bmatrix}_{ML\times M}, \quad (3)$$

$$\mathbf{P}_F = \mathbf{I}_M \otimes \mathbf{P},$$

$$\mathbf{P} = \begin{bmatrix} p_0 & 0 & \ldots & 0 \\ 0 & p_1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & p_{(L-1)} \end{bmatrix}_{L\times L},$$

$$\mathbf{h} = \begin{bmatrix} \underline{\mathrm{h}}_{11} & \underline{\mathrm{h}}_{12} & \cdots & \underline{\mathrm{h}}_{1N} \\ \underline{\mathrm{h}}_{21} & \underline{\mathrm{h}}_{22} & \cdots & \underline{\mathrm{h}}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{\mathrm{h}}_{M1} & \underline{\mathrm{h}}_{M2} & \cdots & \underline{\mathrm{h}}_{MN} \end{bmatrix}_{ML\times N}$$

where $\otimes$ denotes the Kronecker product of two matrices, $\mathbf{I}_N$ is the $N \times N$ identity matrix, $\underline{\mathrm{W}}_K(k) = [1 \;\; W_K^k \;\; W_K^{2k} \;\; \ldots \;\; W_K^{(L-1)k}]^H$ is an $L \times 1$ vector with $W_K \triangleq e^{-i2\pi/K}$, the $L \times L$ PDP matrix is denoted by $\mathbf{P}$, and $\underline{\mathrm{h}}_{mn}$ is an $L \times 1$ vector representing the L-tap frequency selective channel from the transmit antenna n to the receive antenna m and given by

$$\underline{\mathrm{h}}_{mn} = [h_{mn}(0) \; h_{mn}(1) \; \ldots \; h_{mn}(L-1)]^T. \quad (4)$$

Each tap is modeled as independent, identically distributed complex Gaussian random variable with zero-mean and unit variance. Note that the PDP matrix for each channel from each transmit antenna and to each receive antenna is assumed to be the same. The entries of the PDP matrix, p_ls, are real and strictly positive and

$$\sum_{l=0}^{L-1} p_l^2 = 1. \quad (5)$$

The fading model is assumed to be quasi-static, i.e., the fading coefficients are constant over the transmission of one packet, but independent from one packet transmission to the next.

When the beamforming of Section II is applied on each subcarrier, the received signal is given by

$$y(k) = \lambda_1(k)x(k) + n(k) \quad (6)$$

where $n(k) = \underline{\mathrm{u}}_1(k)^H\underline{\mathrm{n}}(k)$, and $\underline{\mathrm{n}}(k)$ is complex additive white Gaussian noise of size $M \times 1$ with zero mean and variance $N_0 = 1/SNR$.

More generalized systems known as bit interleaved coded multiple beamforming (BICMB), and as BICMB-OFDM were presented in [6], [7], [8]. It was shown that BICMB, and BICMB-OFDM achieve full diversity orders while transmitting multiple streams of data. In order to maintain full diversity with increasing number of streams, specialized design criteria for the interleavers were presented in [6], [7], [8]. However, since there is only one stream of data transmitted in BBO, a much simpler interleaver is shown to be sufficient to achieve full diversity order in Section III-B. Also, as will be shown in Section III-B, BBO achieves full diversity order independent of the PDP of the channel.

B. Performance Analysis

It was shown in [15] and [16] that the maximum achievable diversity order in MIMO frequency selective fading channels is NML. Let's denote the minimum Hamming distance of the convolutional code used for BICM as d_{free}. In this section we are going to provide two approaches to illustrate the diversity order of BBO. Without loss of generality it is assumed that $N \leq M$.

Without loss of generality, we assume $N \leq M$. Since λ_1 is the maximum singular value, then

$$\lambda_1^2 \geq \frac{\left(\lambda_1^2 + \lambda_2^2 + \cdots + \lambda_N^2\right)}{N} = \frac{\left(\|\mathbf{H}\|_F^2\right)}{N} = \frac{\left(\sum_{i,j}|h_{ij}|^2\right)}{N} = \frac{\Theta}{N}. \quad (7)$$

PEP can be upper bounded as [17]

$$P(\underline{\mathrm{c}} \to \underline{\hat{\mathrm{c}}}) = E\left[P(\underline{\mathrm{c}} \to \underline{\hat{\mathrm{c}}}|\mathbf{H}(k), \forall k)\right] \leq E\left[\frac{1}{2}\exp\left(-\frac{\sum_{k,d_{free}} d_{min}^2\lambda_1(k)^2}{4N_0}\right)\right]. \quad (8)$$

Following (7) and (8), the PEP of BBO can be rewritten as

$$P(\underline{\mathrm{c}} \to \underline{\hat{\mathrm{c}}}) \leq E\left[\frac{1}{2}\exp\left(-\frac{d_{min}^2\sum_{k,d_{free}}\|\mathbf{H}(k)\|_F^2}{4NN_0}\right)\right]. \quad (9)$$

In order to calculate the PEP, one has to calculate

$$\sum_{k,d_{free}} \|\mathbf{H}(k)\|_F^2 = Tr\{\mathbf{h}^H\mathbf{P}_F(\mathbf{I}_M \otimes \mathbf{A})\mathbf{P}_F\mathbf{h}\} = Tr\{\mathbf{h}^H\mathbf{Z}\mathbf{h}\}, \quad (10)$$

where

$$\mathbf{Z} = \mathbf{I}_M \otimes \mathbf{B}, \quad \mathbf{B} = \mathbf{PAP},$$
$$\mathbf{A} = \sum_{k,d_{free}} \mathbf{A}_k, \quad \mathbf{A}_k = \underline{\mathbf{W}}_K(k)\underline{\mathbf{W}}_K^H(k). \tag{11}$$

The matrix $\mathbf{A}$ was shown to have rank $r = \min(d_{free}, L)$ [3], [4]. Note that, the matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{Z}$ are all positive semidefinite.

According to Ostrowski's theorem, for each $i = 0, 1, \ldots, L-1$, there exists a positive real number θ_i such that $\lambda_{L-1}(\mathbf{P}^2) \leq \theta_i \leq \lambda_0(\mathbf{P}^2)$ and $\lambda_i(\mathbf{B}) = \theta_i \lambda_i(\mathbf{A})$ [12]. Since $\mathbf{P}^2$ is a diagonal matrix, the minimum eigenvalue of $\mathbf{P}^2$, $\lambda_{L-1}(\mathbf{P}^2) = \lambda_{min}(\mathbf{P}^2) = \min_i p_i^2 \triangleq p_{min}^2$. Consequently, $\lambda_i(\mathbf{B}) \geq p_{min}^2 \lambda_i(\mathbf{A})$ for $i = 0, 1, \ldots, L-1$.

The SVD and the eigenvalues of $\mathbf{Z}$ can be given as (recalling Ostrowski's theorem):

$$\mathbf{Z} = \mathbf{V}_Z \mathbf{\Lambda}_Z \mathbf{V}_Z^H$$
$$\lambda_i(\mathbf{Z}) = \lambda_{\lfloor i/M \rfloor}(\mathbf{B}) \geq p_{min}^2 \lambda_{\lfloor i/M \rfloor}(\mathbf{A}) \tag{12}$$

for $i = 0, \ldots, ML-1$ where $\lfloor (\cdot) \rfloor$ is the floor function, and the eigenvalues are ordered in decreasing ordered with index i.

Let's denote the elements of $\mathbf{V}_Z^H \mathbf{h}$ with v_{ij}, for $i = 0, 1, \ldots, ML-1$, and $j = 0, 1, \ldots, N-1$. Since $\mathbf{V}_Z^H$ is unitary and the elements of $\mathbf{h}$ are independent and identically distributed complex Gaussian random variable, $|v_{ij}|$s are Rayleigh distributed with $2|v_{ij}|e^{-|v_{ij}|^2}$.

$$\begin{aligned}
&P(\underline{\mathbf{c}} \rightarrow \hat{\underline{\mathbf{c}}}) \\
&\leq E\left[\frac{1}{2}\exp\left(-\frac{\kappa d_{min}^2 p_{min}^2 \sum_{j=0}^{N-1}\sum_{i=0}^{ML-1} \lambda_{\lfloor i/M \rfloor}(\mathbf{A})|v_{ij}|^2}{4N_0}\right)\right] \\
&= \frac{1}{2}\prod_{i=0}^{r-1}\left(1 + \frac{d_{min}^2 p_{min}^2 \lambda_i(\mathbf{A}) SNR}{4N}\right)^{-NM} \\
&\simeq \frac{1}{2}\left(\prod_{i=0}^{r-1}\lambda_i(\mathbf{A})\right)^{-NM}\left(\frac{d_{min}^2 p_{min}^2 SNR}{4N}\right)^{-NMr}
\end{aligned} \tag{13}$$

for high SNR, where p_{min}^2 is the minimum tap power of the frequency selective channel. The equation (13) illustrates that BBO achieves a diversity order of $NM\min(d_{free}, L)$ independent of the PDP of the channel.

It is shown in [18] and [19] that a very low complexity decoder for BICM-OFDM can be implemented. The same decoder can be used for BBO as well: Instead of using the single-input single-output (SISO) channel value of BICM-OFDM for the decoder ([18] and [19]), one should use $\lambda_1(k)$.

IV. Simulation Results

Perfect CSIR and CSIT is assumed for all the simulations below. For the 2 transmit antenna cases 16 QAM is used for all the systems, and the Alamouti code [20] is deployed for STBC. If the number of transmit antennas is 4, then beamforming systems use QPSK whereas the systems deploying 1/2 rate STBC [1] for 4 antennas use 16 QAM to achieve the same data rate.

Beamforming Results: In order to illustrate the validity of the theoretical results in Section II, simulations were carried out for 2×1, 2×2 and 4×4 systems. Performance of both beamforming and STBC systems are compared. As shown in Figure 1, both beamforming and STBC achieve diversity order of 2, 4, and 16 for 2×1, 2×2, and 4×4 scenarios, respectively. In addition, beamforming has an 2.5 dB coding gain when compared to the Alamouti code. For the 4×4 case, beamforming provides about 7 dB performance gain when compared to STBC.

BBO Results: The BBO system used in the simulations has 64 subcarriers for one OFDM symbol. Each OFDM symbol has a duration of $4\mu s$ of which $0.8\mu s$ is CP. With this setup, channels with rms delay spread of 10 ns and 50 ns correspond to 3 and 11 taps, respectively. The output bits of the encoder are interleaved using the interleaver of IEEE 802.11a [21] within one OFDM symbol. This interleaver satisfies the simple design criterion for BBO of Section III-B.

1) Results for frequency selective channels with equal power taps: Figure 2 illustrates the results of BBO as compared to a system as a combination of BICM, STBC, and OFDM [4], [22]. Both systems use the same convolutional code with 4 states and $d_{free} = 5$, which is picked from the tables of [23]. As can be seen from the figures, as the delay spread of the channel increases, the diversity order of BBO increases as well. The figures also illustrate that BBO introduces a coding gain when compared to BICM-STBC-OFDM. Although both systems succeed in achieving the same diversity order under the same conditions, BBO shows about 2-3 dB improvement for the 2 transmit antenna case.

Figure 3 presents the simulation results for BBO as compared to BICM-STBC-OFDM when both systems use the industry standard 1/2 rate (133,171) 64 state $d_{free} = 10$ convolutional code. The diversity order of BBO increases with the increasing delay spread of the channel. It can be seen that with increasing transmit and receive antennas, the diversity order of BBO increases as well. As mentioned earlier, BBO introduces a coding gain of 2-3 dB when compared to BICM-STBC-OFDM for two transmit antennas case. The coding gain for the 4×4 case is 6 dB.

2) Results using the IEEE MIMO Channel Models: Figure 4 illustrates the simulations results using the IEEE channel models [9], [10], [11]. PDPs of IEEE channel models are presented in [9]. BBO shows 2-3 dB coding gain for the 2×2 case when compared to BICM-STBC-OFDM over IEEE Channel Models B and D. Note that, in [4], BICM-STBC-OFDM system was shown to achieve full spatial and frequency diversity independent of the PDP of the channel. As can be seen from figure 4, results for BBO and BICM-STBC-OFDM go parallel to one another confirming that BBO achieves full diversity in space and frequency independent of the PDP of the channel.

In Figures 1–4, we also present results based on limited CSIT with codebooks generated by the technique in [24]. In Figures 1–4, B bits means there are 2^B entries in the codebook for the quantized version of the $\mathbf{V}$ matrix. In all cases, this approach provides satisfactory performance with limited CSIT.

V. Conclusion

In this paper we first formally showed that single beamforming can achieve the maximum spatial diversity order over Rayleigh fading channels. Simulation results showed significant coding gains for beamforming as compared to STBC where the channel state information is not required at the transmitter.

We investigated beamforming in combination with a widely deployed technique BICM-OFDM, named BBO. We provided analysis showing that BBO can achieve the maximum diversity order in space and frequency by using an appropriate convolutional code and a simple interleaver. In other words, for N transmit and M receive antennas, BBO can achieve a diversity order of NML over L-tap frequency selective channels independent of the power delay profile of the channel.

In addition to achieving full diversity order, simulation results showed that BBO introduces significant coding gains when compared to other systems based on STBC with the same diversity order. As the number of transmit antennas increases, the coding gains increase as well.

Simulations showed that, for BBO, limited CSIT with codebooks generated using [24] provides satisfactory results even with very few bits.

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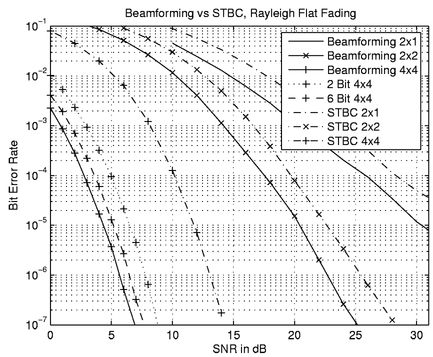

Fig. 1. Beamforming vs STBC results for flat fading channels

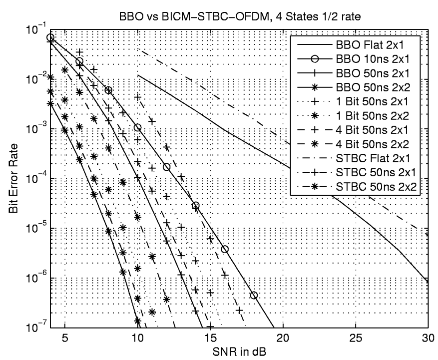

Fig. 2. BBO vs BICM STBC OFDM over different frequency selective channels. 2 transmit 1 receive antennas, 1/2 rate, 4 states $d_{free} = 5$.

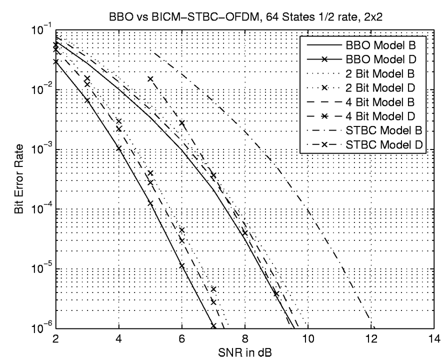

Fig. 4. BBO vs BICM STBC OFDM over IEEE MIMO channel models. 1/2 rate, 64 states, $d_{free} = 10$.

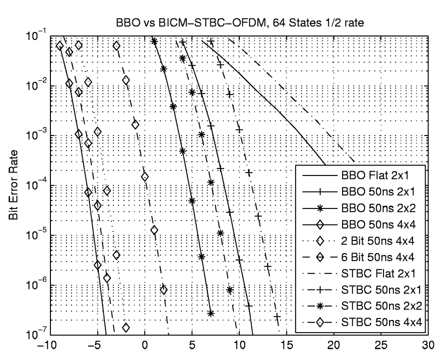

Fig. 3. BBO vs BICM STBC OFDM over different frequency selective channels. 1/2 rate, 64 states, $d_{free} = 10$.