Low-Complexity SQR-based Decoding Algorithm for Quasi-Orthogonal Space-Time Block Codes

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***Abstract*— In this paper, we propose a new decoding algorithm for quasi-orthogonal space-time block codes (QOSTBCs) which achieves near Maximum Likelihood (ML) performance while substantially reducing the decoding complexity. We show that for a system with rate $r = n_s/T$, where n_s is the number of transmitted symbols per T time slots; the proposed algorithm decomposes the original complex-valued system into a parallel system with n_s 2×2 real-valued components, thus allowing for a simple decoding of one real symbol at a time. For a square QAM constellation with L points (L-QAM), this algorithm achieves full diversity by properly incorporating two-dimensional rotation using the optimal rotation angle and the same rotating matrix for any number of transmit antennas ($N \geq 4$). We show that the complexity of this algorithm is linear with the number of transmitted symbols n_s, and is independent of the constellation size L.**

I. Introduction

Space-time block-codes (STBCs) from quasi-orthogonal designs achieve high symbol transmission rates compared to orthogonal STBCs. However, the Maximum Likelihood (ML) decoding of QOSTBCs is more complicated. Specifically, a joint detection of two complex symbols is required for a full-rate QOSTBC system with $N = 4$ transmit antennas, or $3/4$-rate system with $N = 8$. For the full-rate system with $N = 8$, four complex symbols are jointly detected to obtain the ML solution [1], [2]. Unlike the OSTBCs, the decoding complexity is no longer linear, but rather, increases exponentially with N, i.e., $\mathcal{O}(L^{N/2})$ where L is the size of the L-QAM constellation [1], [3].

Targeting the decoding complexity, a number of algorithms were proposed in the literature attempting to reduce that complexity [4], [5]. In [4], a decoding algorithm was proposed whose complexity is $\mathcal{O}(L^{N/4})$. Despite the fact that this complexity is lower than that of the conventional algorithm, it is still exponential in N and will increase rapidly as L becomes large. Minimum decoding complexity (MDC) for the class of QOSTBCs whose ML decoding requires only a joint detection of two real symbols was proposed in [5]. An algebraic structure for the code was constructed to guarantee that every complex transmitted symbol is orthogonal to all other complex symbols, but the in-phase and quadrature components within the same complex symbols need not be orthogonal. Sphere decoding (SD) [6]-[9], on the other hand, was proposed as a low complexity algorithm for the detection of QOSTBCs [3]. The ML metric of QOSTBCs is written into two independent Euclidean norms in [3], making it possible to apply SD to operate on each independently, thus reducing the decoding complexity. Complexity of SD has been widely studied in [7], [10], [11]. In the sequel, we will compare the complexity of our algorithm with those discussed above in detail.

In general, QOSTBCs do not achieve the full diversity provided by the channel. One way to provide full diversity using QOSTBCs is to choose half of the transmitted symbols from a signal constellation set $\mathcal{A}$ and to choose the other half from a rotated constellation set $e^{j\phi}\mathcal{A}$ where ϕ is the rotation angle [1], [2], [12]. Another way is to use multidimensional rotated constellations as discussed in [4], [13], [14].

Our new decoding algorithm is based on Sorted QR decomposition (SQR) [15] of the real-valued lattice representation for the class of QOSTBCs discussed in [5]. In [15], SQR was proposed as part of a simple decoding algorithm for that special kind of STBCs called layered space-time codes (LST) [16]. In spite of the simplicity of the algorithm presented in [15], it is not applicable to QOSTBCs and, furthermore, it will not be able to achieve full diversity. In this work, making use of the real-valued representation, we show that the detection can be performed independently considering only one real symbol at a time. Moreover, full diversity is achieved by properly incorporating two-dimensional rotation which is performed using the same rotating matrix for arbitrary N. This becomes possible by an appropriate grouping of the complex transmitted symbols. We discuss the performance and complexity results for our proposed algorithm, and compare them with conventional ML detection [17], those in [4], [5], and sphere decoding [3].

II. QOSTBCs and System Model

Consider a MIMO system with N transmit and M receive antennas, and an interval of T symbols during which the channel is constant. The received signal is given by

$$Y = \sqrt{\frac{\rho}{N}} C_N H + V \tag{1}$$

where $Y = [y_t^j]_{T\times M}$ is the received signal matrix of size $T\times M$ and whose entry y_t^j is the signal received at antenna j at time t, $t = 1, 2, \ldots, T$, $j = 1, 2, \ldots, M$; $V = [v_t^j]_{T\times M}$ is the noise matrix; and $C = [c_t^i]_{T\times N}$ is the transmitted signal

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matrix whose entry c_t^i is the signal transmitted at antenna i at time t, $t = 1, 2, \ldots, T$, $i = 1, 2, \ldots, N$. $H = [h_{i,j}]_{N\times M}$ is the channel coefficient matrix of size $N \times M$ whose entry $h_{i,j}$ is the channel coefficient from transmit antenna i to receive antenna j. The entries of the matrices H and V are independent, zero-mean, and circularly symmetric complex Gaussian random variables of unit variance; and the parameter ρ is the signal-to-noise-ratio (SNR) per receiving antenna.

In this paper, we consider the class of QOSTBCs for which decoding pairs of symbols independently is possible (see Chapter 5 of [18]). The analysis in the sequel applies to any QOSTBC that belongs to this class [5], [17]. In the following, we consider $N = 4$ and emphasize the fact that the derivation is applicable to arbitrary N.

Now, for $N = 4$, we consider the full-rate code presented in [4] with $n_s = 4$ and is defined by

$$C_4 = \begin{bmatrix} s_1 & s_3 & s_4 & s_2 \\ s_3^* & -s_1^* & s_2^* & -s_4^* \\ s_4^* & s_2^* & -s_1^* & -s_3^* \\ s_2 & -s_4 & -s_3 & s_1 \end{bmatrix}. \tag{2}$$

Note that removing one or more columns of a QOSTBC results in new STBCs for smaller numbers of transmit antennas [18].

III. Proposed Algorithm

Assuming that the channel H is known at the receiver and setting $M = 1$ for simplicity, we start by decomposing the T-dimensional complex problem defined by (1) to a $2T$-dimensional real-valued problem. Using either of the real-valued representations proposed in [4] or [9] gives the same results in terms of our proposed algorithm. Rewriting (1) in matrix form, we have

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix} = C_N \begin{bmatrix} h_{1,1} \\ h_{2,1} \\ \vdots \\ h_{N,1} \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_T \end{bmatrix}. \tag{3}$$

We specify the complex transmitted symbols of C_N by their real and imaginary parts as $s_m = x_{2m-1} + jx_{2m}$ for $m = 1, 2, \ldots, n_s$. Applying the real-valued lattice representation defined in [9] to (3), we obtain

$$\check{y} = \check{H}x + \check{v} \tag{4}$$

or equivalently

$$\begin{bmatrix} \Re(y_1) \\ \Im(y_1) \\ \vdots \\ \Re(y_T) \\ \Im(y_T) \end{bmatrix} = \check{H} \begin{bmatrix} \Re(s_1) \\ \Im(s_1) \\ \vdots \\ \Re(s_m) \\ \Im(s_m) \end{bmatrix} + \begin{bmatrix} \Re(v_1) \\ \Im(v_1) \\ \vdots \\ \Re(v_T) \\ \Im(v_T) \end{bmatrix}.$$

The real-valued fading coefficients of $\check{H}$ are defined using the complex fading coefficients $h_{i,j}$ from transmit antenna i to receive antenna j as $h_{2n-1} = \Re(h_{n,1})$, and $h_{2n} = \Im(h_{n,1})$ for $n = 1, 2, \ldots, N$ (recall that we restricted ourselves to $M = 1$, and therefore we only consider $j = 1$). Let's define the number of transmitted symbols as n_s, then the equivalent real-valued channel $\check{H}$ is a $2N \times 2n_s$ matrix. Using (2), $\check{H}$ is given by

$$\check{H} = \begin{bmatrix} h_1 & -h_2 & h_7 & -h_8 & h_3 & -h_4 & h_5 & -h_6 \\ h_2 & h_1 & h_8 & h_7 & h_4 & h_3 & h_6 & h_5 \\ -h_3 & -h_4 & h_5 & h_6 & h_1 & h_2 & -h_7 & -h_8 \\ -h_4 & h_3 & h_6 & -h_5 & h_2 & -h_1 & -h_8 & h_7 \\ -h_5 & -h_6 & h_3 & h_4 & -h_7 & -h_8 & h_1 & h_2 \\ -h_6 & h_5 & h_4 & -h_3 & -h_8 & h_7 & h_2 & -h_1 \\ h_7 & -h_8 & h_1 & -h_2 & -h_5 & h_6 & -h_3 & h_4 \\ h_8 & h_7 & h_2 & h_1 & -h_6 & -h_5 & -h_4 & -h_3 \end{bmatrix}.$$

Setting $\check{H} = \begin{bmatrix} \check{h}_1 & \check{h}_2 & \cdots & \check{h}_8 \end{bmatrix}$, where $\check{h}_k$ is the k-th column of $\check{H}$, we observe that

$$\begin{aligned} \langle \check{h}_1, \check{h}_i \rangle &= 0, i \neq 3, & \langle \check{h}_2, \check{h}_i \rangle &= 0, i \neq 4, \\ \langle \check{h}_5, \check{h}_i \rangle &= 0, i \neq 7, & \langle \check{h}_6, \check{h}_i \rangle &= 0, i \neq 8. \end{aligned} \tag{5}$$

where $\langle \check{h}_i, \check{h}_j \rangle$ is the inner product of columns $\check{h}_i$ and $\check{h}_j$.

Applying SQR decomposition to (4), we have

$$\begin{aligned} \check{y} &= QRx + \check{v} \\ Q^H\check{y} &= Rx + Q^H\check{v} \\ \bar{y} &= Rx + \bar{v} \end{aligned}$$

where $\bar{v}$ and $\check{v}$ have the same statistical properties since Q is unitary and so is Q^H. The matrix R is an 8×8 block diagonal matrix of the form

$$R = \begin{bmatrix} R_{1,2} & 0 & \cdots & 0 \\ 0 & R_{3,4} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{7,8} \end{bmatrix} \tag{6}$$

where

$$R_{i,i+1} = \begin{bmatrix} r_{i,i} & r_{i,i+1} \\ 0 & r_{i+1,i+1} \end{bmatrix} \qquad i = 1, 3, 5, 7.$$

Using (6), the ML problem is now simpler and rather than minimizing $||Y - CH||_F^2$, the solution is obtained by minimizing the metric $||\bar{y} - Rx||_2^2$ in a layered fashion over all different combinations of the vector x. Finding the optimal solution now is much simpler than in the conventional way, since R contains too many zeros which is a result of using the SQR decomposition and the observations in (5). To further reduce the complexity, we suggest that the decoding be carried out by quantization to the nearest real-valued element in the signal constellation used. To make this clearer, let the square L-QAM alphabet be given as Ω^2, where $\Omega = \{-\sqrt{L}+1, -\sqrt{L}+3, \ldots, \sqrt{L}-1\}$. Then

$$x_{i+1} = \mho\left(\frac{\bar{y}_{i+1}}{r_{i+1,i+1}}\right) \tag{7}$$

for $i = 1, 3, \ldots, 2n_s - 1$, and assuming x_{i+1} is decoded correctly, we obtain

$$x_i = \mho\left(\frac{\bar{y}_i - r_{i,i+1}x_{i+1}}{r_{i,i}}\right) \tag{8}$$

for same i, where $\mho(.)$ quantizes the value of its argument to the closest element in the set Ω. Noting that the SQR algorithm is a sorted version of the QR decomposition, we may ask this question: why use SQR rather than QR? According to (8), decoding x_{i+1} efficiently has a strong impact on the decoding of x_i. Thus, we aim that the decoding of x_{i+1} be as reliable as possible. This reliability can be improved by two means

- using SQR.
- declare a band of potential ambiguity around the boundary points of the quantizer.

The idea of SQR is to find the permutation of $\tilde{H}$ that minimizes each diagonal element $r_{i,i}$ in R with i running from 1 to $2n_s$, thus maximizing the SNR in each step of the detection process, and reducing the propagation of an error from decoding x_{i+1} to decoding x_i [15]. It is important to note that SQR finds the best upper triangular matrix R, but each time R may have a different form. All of these forms have the common property that half of the rows of R have always one nonzero element (the diagonal element), and the other half have exactly two nonzero elements, one on diagonal. This property makes the above algorithm generic by performing the decoding of the symbols that correspond to the rows that have only one nonzero element using (7), and decoding the other half using (8). For example, another form of R would be

$$R = \begin{bmatrix} r_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & r_{1,8} \\ 0 & r_{2,2} & 0 & 0 & 0 & 0 & r_{2,7} & 0 \\ 0 & 0 & r_{3,3} & 0 & r_{3,5} & 0 & 0 & 0 \\ 0 & 0 & 0 & r_{4,4} & 0 & r_{4,6} & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{5,5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_{6,6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_{7,7} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_{8,8} \end{bmatrix}. \tag{9}$$

Then, the decoded real symbols are obtained by

$$x_i = \mho(\frac{\bar{y}_i}{r_{i,i}})$$

for $i = 5, 6, 7, 8$, and

$$x_4 = \mho(\frac{\bar{y}_4 - r_{4,6}x_6}{r_{4,4}}), \quad x_3 = \mho(\frac{\bar{y}_3 - r_{3,5}x_5}{r_{3,3}}),$$
$$x_2 = \mho(\frac{\bar{y}_2 - r_{2,7}x_7}{r_{2,2}}), \quad x_1 = \mho(\frac{\bar{y}_1 - r_{1,8}x_8}{r_{1,1}}).$$

An additional way to improve the reliability aforementioned is to define a band of potential ambiguity around the boundary points of the quantizer. The band length is specified statistically. To make this clearer, consider the quantization function for a 16-QAM modulation scheme

$$\mho(z) = \begin{cases} -3, & z \le -2 \\ -1, & -2 < z \le 0 \\ 1, & 0 < z \le 2 \\ 3, & z > 2 \end{cases}.$$

If the value of z for a symbol x_k decoded by (7) is very close to one of the boundary values $\{-2, 0, 2\}$ (for e.g., $z = 2.01$), then the decoding of the real symbol x_{k+1} that is coupled with x_k and decoded by (8), is declared as ambiguous. Consequently, the quantization function returns two values for x_k for which z is close to (for $z = 2.01$, $\mho(z) = \{1, 3\}$), and the decoding of x_{k+1} is carried out twice, using one value of x_k at a time. The decoder chooses the real symbols x_k and x_{k+1} that have the minimum weight (minimum Euclidean distance). For example, for R defined in (9), if the quantizer returns a pair of estimates for x_8 namely $\{1, 3\}$, then x_1 will also have two estimates given by $\mho(\frac{\bar{y}_1 - r_{1,8}}{r_{1,1}})$ and $\mho(\frac{\bar{y}_1 - 3r_{1,8}}{r_{1,1}})$, respectively, (say $\{-1, 1\}$). The decoder, in turn, finds $|\bar{y}_8 - r_{8,8}x_8|^2 + |\bar{y}_1 - r_{1,1}x_1 - r_{1,8}x_8|^2$ for two different combinations of $\{x_8, x_1\}$ (namely $\{1, -1\}$ and $\{3, 1\}$ in this example), and returns that set which gives the minimum value. Note that if the band length is set to zero, then the quantizer always returns one value.

Now, for a given channel realization, assume that SQR creates an R matrix of the form presented in (9), then the decoding algorithm can be written in an algorithmic way as shown as **Algorithm 1** below.

Algorithm 1 Proposed decoding algorithm

step 1:
$x_8 = \mho(\frac{\bar{y}_8}{r_{8,8}})$, $x_7 = \mho(\frac{\bar{y}_7}{r_{7,7}})$, $x_6 = \mho(\frac{\bar{y}_6}{r_{6,6}})$, $x_5 = \mho(\frac{\bar{y}_5}{r_{5,5}})$.
step 2:
if (number of estimates for x_8) $= 1$ **then**
  $x_1 = \mho(\frac{\bar{y}_1 - r_{1,8}x_8}{r_{1,1}})$
else
  $x_1(1) = \mho(\frac{\bar{y}_1 - r_{1,8}x_8(1)}{r_{1,1}})$
  $x_1(2) = \mho(\frac{\bar{y}_1 - r_{1,8}x_8(2)}{r_{1,1}})$
  if $(|\bar{y}_8 - r_{8,8}x_8(1)|^2 + |\bar{y}_1 - r_{1,8}x_8(1) - r_{1,1}x_1(1)|^2) < (|\bar{y}_8 - r_{8,8}x_8(2)|^2 + |\bar{y}_1 - r_{1,8}x_8(2) - r_{1,1}x_1(2)|^2)$ **then**
    $x_8 = x_8(1)$, $x_1 = x_1(1)$
  else
    $x_8 = x_8(2)$, $x_1 = x_1(2)$
  end if
end if
{step 2 is carried out considering the remaining coupled pairs $(x_2, x_7), (x_3, x_5), (x_4, x_6)$.}

IV. QOSTBC WITH FULL DIVERSITY

In general, there are two methods to achieve full diversity and improve the BER performance at high SNR values. One way suggests choosing half of the transmitted symbols in a quasi-orthogonal design from a signal constellation set $\mathcal{A}$ and choosing the other half from a rotated constellation set $e^{j\phi}\mathcal{A}$ [1], [2]. Another approach is to apply multi-dimensional rotated constellations which exhibit full diversity and maximum coding gain [4], [13]. However, no proper expressions for the rotation matrix of sizes greater than four exist. Applying the first approach to our algorithm introduces interference among the real symbols and as a result the orthogonality properties defined in (5) do no longer hold. Instead, a two-dimensional rotation of the real symbols $(x_1, x_2, \ldots, x_{2n_s})$

that are not orthogonal to each other is applied, thus maintaining the orthogonality properties among the channel columns and maximizing the diversity on one hand, and overcoming the problem of finding proper expressions for the rotation matrix on the other hand.

The two-dimensional rotation matrix is defined by [4], [14]

$$G = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (10)$$

where the optimal angle is obtained by $\theta = \frac{1}{2}\text{atan}(\frac{1}{2})$ for square QAM constellations [5].

In the following, we discuss the application of this rotation to QOSTBCs for $N = 4$, keeping in mind that this derivation is generic and applicable to arbitrary N. Using (5), we combine every two real symbols that are not orthogonal to each other in one group, then we apply a two-dimensional rotation using the matrix G. The produced real symbols are defined as

$$\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_3 \end{bmatrix} = G \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}, \qquad \begin{bmatrix} \tilde{x}_2 \\ \tilde{x}_4 \end{bmatrix} = G \begin{bmatrix} x_2 \\ x_4 \end{bmatrix},$$

$$\begin{bmatrix} \tilde{x}_5 \\ \tilde{x}_7 \end{bmatrix} = G \begin{bmatrix} x_5 \\ x_7 \end{bmatrix}, \qquad \begin{bmatrix} \tilde{x}_6 \\ \tilde{x}_8 \end{bmatrix} = G \begin{bmatrix} x_6 \\ x_8 \end{bmatrix}.$$

Full diversity is achieved by replacing s_1, s_2, s_3, s_4 defined in (2) by $\tilde{s}_1, \tilde{s}_2, \tilde{s}_3, \tilde{s}_4$ where $\tilde{s}_m = \tilde{x}_{2m-1} + j\tilde{x}_{2m}$ for $m = 1, 2, \ldots, n_s$. Incorporating the rotation matrix G into the system changes the equivalent channel matrix $\check{H}$, but keeps the orthogonality properties defined in (5) unchanged. The equivalent channel matrix $\check{H}$ after the rotation is given by

$$\begin{bmatrix} h_1\cos\theta + h_7\sin\theta & \cdots & -h_6\cos\theta + h_4\sin\theta \\ h_2\cos\theta + h_8\sin\theta & \cdots & h_5\cos\theta - h_3\sin\theta \\ \vdots & \ddots & \vdots \\ h_8\cos\theta + h_2\sin\theta & \cdots & -h_3\cos\theta + h_5\sin\theta \end{bmatrix}.$$

Applying SQR decomposition to $\check{H}$ produces a block diagonal matrix R of the form previously discussed in (6). Thus, applying our proposed algorithm becomes possible and achieves full diversity by properly choosing the optimal rotation angle θ. To make this clearer, we define the codeword difference matrix between any pair of distinct transmitted signals, $\mathbf{C}^1$ and $\mathbf{C}^2$, as $\Delta C = \mathbf{C}^1 - \mathbf{C}^2$ where $\mathbf{C}^1 = G(\check{s}_1, \check{s}_2, \check{s}_3, \check{s}_4)$, $\mathbf{C}^2 = G(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3, \tilde{s}_4)$, and $G(.)$ is the generator matrix that produces the codeword matrices $\mathbf{C}^1$ and $\mathbf{C}^2$. In order to achieve full diversity, the minimum value of the determinant of the product $(\Delta C)^H(\Delta C)$ has to be nonzero, and should be maximized in order to obtain the optimum coding gain [1], [18], [19]. With some algebra, we obtain

$$(\Delta C)^H(\Delta C) = \begin{bmatrix} (\Delta A)I_2 & (\Delta B)I_2 \\ (\Delta B)I_2 & (\Delta A)I_2 \end{bmatrix} \quad (11)$$

where I_2 is the identity matrix of size $N/2 = 2$, and ΔA and ΔB are given by

$$\Delta A = \sum_{k=1}^{4} |\check{s}_k - \tilde{s}_k|^2$$

and

$$\Delta B = \sum_{k=1}^{2} -(\check{s}_k - \tilde{s}_k)(\check{s}_{k+2} - \tilde{s}_{k+2})^* + \sum_{k=1}^{2} (\check{s}_k - \tilde{s}_k)^*(\check{s}_{k+2} - \tilde{s}_{k+2}).$$

Now, we define $\tilde{\Delta}_i = \Delta_i \cos\theta - \Delta_j \sin\theta$ and $\tilde{\Delta}_j = \Delta_j \cos\theta + \Delta_i \sin\theta$ for $(i, j) \in \{(1, 3), (2, 4), (5, 7), (6, 8)\}$, where Δ_k and $\tilde{\Delta}_k$ are the differences between the real symbol $\check{x}_k$ in codeword $\mathbf{C}^1$ and $\tilde{x}_k$ in codeword $\mathbf{C}^2$ for $k = 1, 2, \ldots, 8$, before and after the two-dimensional rotation respectively. Thus, ΔA and ΔB can be rewritten as

$$\Delta A = \sum_{\substack{k=1 \\ k:odd}}^{8} (\tilde{\Delta}_k)^2 + (\tilde{\Delta}_{k+1})^2 \quad (12)$$

and

$$\Delta B = 2 \sum_{\substack{k=1 \\ k:odd}}^{4} -(\tilde{\Delta}_k + j\tilde{\Delta}_{k+1})(\tilde{\Delta}_{k+4} - j\tilde{\Delta}_{k+5}). \quad (13)$$

The determinant expression of $(\Delta C)^H(\Delta C)$ defined in (11) is derived in [19] as

$$\det(\Delta C)^H(\Delta C) = [(\Delta A + \Delta B)(\Delta A - \Delta B)]^4. \quad (14)$$

Substituting (12) and (13) in (14), and denoting the set of pairs (i, j) $S = \{(1, 3), (2, 4), (5, 7), (6, 8)$, and the determinant value by d, we get

$$d = \left(\left(\sum_{(i,j)\in S} (\tilde{\Delta}_i + \tilde{\Delta}_j)^2 \right) \times \left(\sum_{(i,j)\in S} (\tilde{\Delta}_i - \tilde{\Delta}_j)^2 \right) \right)^4.$$

Following [1], [2], and [19], we can assume that the worst-case (minimum) value of d is obtained when only one group of the real symbols (x_i, x_j) for $(i, j) \in S$, makes an error and the rest of the symbols are error free, hence d is simplified to

$$d = \left[\Delta_i^2 \cos(2\theta) - 2\Delta_i\Delta_j \sin(2\theta) - \Delta_j^2 \cos(2\theta)\right]^8.$$

In order to achieve full diversity, we can properly choose the rotation angle θ so that d is nonzero. It was shown in [4], [5], that $\theta = \frac{1}{2}\text{atan}(\frac{1}{2})$ is the optimal rotation angle for square QAM constellations which not only provides a nonzero value of d and therefore achieves full diversity, but also maximizes the minimum determinant value d, and consequently, provides the optimum coding gain.

Finally, we emphasize the fact that the proposed decoding algorithm is generic, works and has the same computational complexity for rotated and non-rotated QOSTBCs.

V. Computational Complexity

In this section, we compare the computational complexity of our proposed algorithm with that of conventional ML detection and the complexity of other reported algorithms in the literature. The overall complexity is measured in terms of the number of operations required to decode the transmitted signals for each block period T. Using the same notation in [4], a complex multiplication is equivalent to 4 real multiplications

C_M and 2 real additions C_A, while a complex addition is equivalent to 2 real additions. We split the complexity formula into two parts in order to represent C_M and C_A independently. We denote the complexity of our proposed algorithm by $\mathcal{C}_{PR}$, and show it as a two dimensional vector where the first dimension is the number of real multiplications and the second, the number of real additions, then

$$\mathcal{C}_{PR} = n_s(3C_M + p16C_M, 1C_A + p11C_A)$$

where p is the number of times the quantizer returns more than one value for decoding one block of transmitted symbols. Obviously, if the band of ambiguity length is zero, then $p = 0$. The maximum complexity of $\mathcal{C}_{PR}$ occurs when the quantizer always returns two values, i.e., $p = n_s$.

Conventional ML detection [17], on the other hand, performs pairwise complex symbol detection and the complexity $\mathcal{C}_{ML}$ is presented in [4] as

$$\mathcal{C}_{ML} = 2L^{\frac{N}{2}}\{(2N^2 + 4N)C_M, (2N^2 + 3N - 1)C_A\}.$$

Obviously, $\mathcal{C}_{ML}$ increases exponentially with N and polynomially with L, whereas $\mathcal{C}_{PR}$ is linear with n_s (or equivalently with N), and is independent of the constellation size L.

In [4], a decoding algorithm was proposed for the detection of QOSTBCs. Its computational complexity was given by

$$\mathcal{C}_{[4]} = 4L^{\frac{N}{4}}\{(N^2 + 4N)C_M, (N^2 + 3N - 1)C_A\}.$$

Again, this complexity is exponential in N and polynomial in L, thus $\mathcal{C}_{PR}$ is more desirable. For example, for $N = 4$ and 16-QAM modulation scheme, $\mathcal{C}_{PR}$ achieves $> 73\%$ reduction in the complexity compared to $\mathcal{C}_{[4]}$, and $> 97\%$ compared to $\mathcal{C}_{ML}$. Obviously, these gains are at the expense of a small performance loss. For instance, for $N = 8$ and 16-QAM constellation, a performance degradation of 0.2 dB is experienced compared to ML performance while a complexity reduction of 98%, and $> 91\%$, is obtained compared to conventional ML detection and [4], respectively. Note that the complexity gain obtained by $\mathcal{C}_{PR}$ becomes more substantial as N or L becomes larger. For example, for $N = 8$ and 64-QAM, $\mathcal{C}_{PR}$ achieves 99.8%, and 98% reduction in the complexity compared to conventional ML and [4], respectively.

SD was proposed in [3] to decode QOSTBCs. The computational complexity is reduced from $\mathcal{O}((n_s)^6)$ for ML to $\mathcal{O}(2(n_s/2)^6)$ using SD, where n_s is the number of transmitted symbols. This complexity is polynomial with n_s. However,, the choice of the radius is based on the Gram matrix eigenvalues, which adds a computational complexity of $\mathcal{O}((2M)^3)$. Moreover, when no point is found inside the sphere, the radius is increased and the algorithm restarts, which additionally increases the computational complexity.

Finally, the decoding algorithm presented in [5] has a complexity which is comparable to the complexity of the algorithm proposed in [4] which is a function of $\mathcal{O}(L^{N/4})$, since both algorithms perform joint detection of two real symbols. Therefore, the same complexity gain of $\mathcal{C}_{PR}$ over $\mathcal{C}_{[4]}$ is applicable also over $\mathcal{C}_{[5]}$. However, the algorithm of [5] suffers a performance loss of 0.4 dB compared to ML.

VI. Simulation Results

We provide simulation results for the proposed algorithms for $N = 8$ and $N = 4$. In all simulations, we consider one receive antenna. In order to achieve full diversity, we use the rotating matrix G defined in (10) with $\theta = \frac{1}{2}\mathrm{atan}(\frac{1}{2})$.

Figure 1 provides simulation results for the transmission of 4 bits/s/Hz with $N = 4$ and 16-QAM. We compare PR with conventional ML detection with and without rotation. PR achieves full diversity through the rotation matrix G, while ML detection achieves it by replacing the transmitted symbols s_3, s_4 by $s_3e^{j\phi}, s_4e^{j\phi}$ in (2) with $\phi = \pi/4$ (i.e., choosing half of the transmitted symbols from a rotated constellation set $e^{j\phi}\mathcal{A}$ as discussed earlier) [18]. The band of ambiguity length is set to 0.1. The performance of PR is near to conventional ML, namely there is a performance loss of $< .5$ dB with rotation and < 0.1 dB without rotation. A complexity gain of $> 97\%$ compared to conventional ML is the tradeoff in this loss.

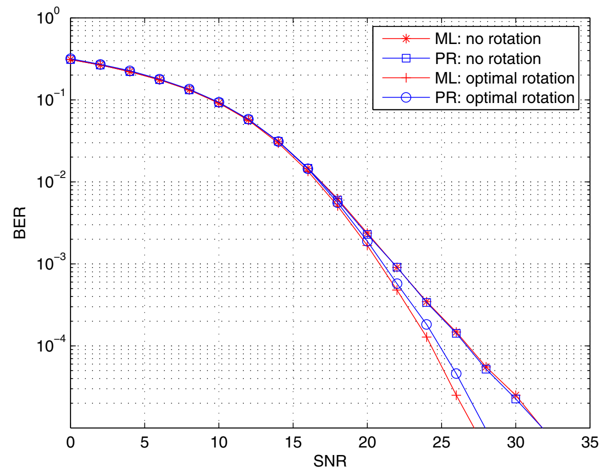

Fig. 1. BER vs SNR for the proposed algorithm PR and conventional ML for QOSTBCs at 4 bits/s/Hz; 4 transmit and 1 receive antennas, 16-QAM.

Figure 2 shows similar results for $N = 8$ and 4-QAM, at 1.5 bits/s/Hz. The band of ambiguity length is set to zero, which means that the quantizer always returns one value. It is clear that PR has a performance loss of 0.2 dB compared to conventional ML with and without rotation, while having a complexity reduction of $> 71\%$.

The complexity is measured in terms of the number of real multiplications required to decode one block of transmitted symbols as a function of the constellation size L. The complexity of [5] is in the order of $\mathcal{O}(L^{N/4})$ which is equal to $\mathcal{C}_{[4]}$ and will be denoted $\mathcal{C}_{[4],[5]}$ in the sequel. In Table I, we give a comparison between $\mathcal{C}_{ML}, \mathcal{C}_{[4],[5]}, \mathcal{C}_{PR}$ in terms of the number of real multiplication and real additions considering $N = 4$ for different constellation sizes. In Table II, we show the same comparison for $N = 8$. The number of multiplications and additions shown include the computation of SQR, and

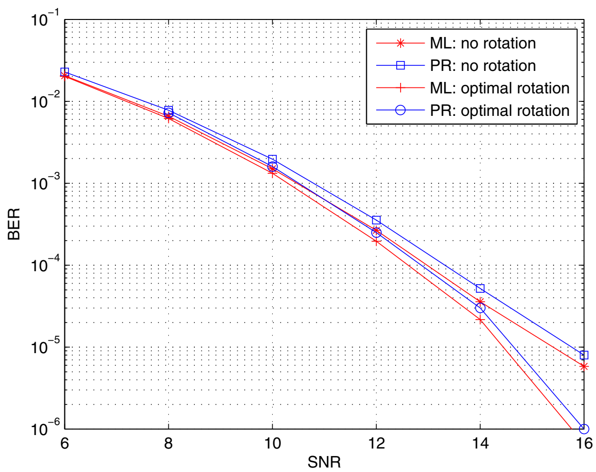

Fig. 2. BER vs SNR for the proposed algorithm PR and conventional ML for QOSTBCs at 1.5 bits/s/Hz; 8 transmit and 1 receive antennas, 4-QAM.

$\bar{y} = Q^H \tilde{y}$. Clearly, $\mathcal{C}_{PR}$ is independent of L, whereas $\mathcal{C}_{ML}$ and $\mathcal{C}_{[4],[5]}$ increases exponentially as L is larger.

TABLE I

OF REAL MULTIPLICATIONS AND REAL ADDITIONS VS L FOR $N = 4$

	L	4	16	64	256
C_M	ML	1536	24576	393216	6291456
	[4], [5]	512	2048	8192	32768
	PR	544	544	544	544
C_A	ML	1376	22016	352256	5636096
	[4], [5]	432	1728	6912	27648
	PR	388	388	388	388

TABLE II

OF REAL MULTIPLICATIONS AND REAL ADDITIONS VS L FOR $N = 8$

	L	4	16	64	256
C_M	ML	5120	81920	1.3×10^6	2.1×10^7
	[4], [5]	4608	18432	73728	294912
	PR	1488	1488	1488	1488
C_A	ML	4832	77312	1.2×10^6	1.9×10^7
	[4], [5]	4200	16800	67200	268800
	PR	1134	1134	1134	1134

Figure 3 shows a complexity comparisons between $\mathcal{C}_{ML}$, $\mathcal{C}_{[4],[5]}$, and $\mathcal{C}_{PR}$ with respect to N, considering 16-QAM. Apparently, the complexity gain obtained by PR is substantial and this gain becomes greater as N is larger.

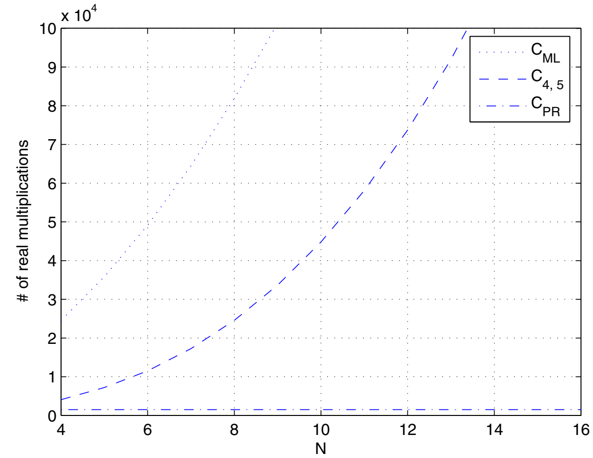

Fig. 3. # of real multiplications vs N for conventional ML, [4] and [5], and PR for QOSTBCs using 16-QAM.

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