Link Failure Recovery in Large Arbitrary Networks via Network Coding

Serhat Nazim Avci and Ender Ayanoglu

Center for Pervasive Communications and Computing
Department of Electrical Engineering and Computer Science
University of California, Irvine
Irvine, CA 92697-2625

***Abstract*—Network coding-based link failure recovery techniques provide near-hitless recovery and offer high capacity efficiency. In this paper, we propose a simple column generation-based design algorithm and a novel advanced diversity (network) coding technique to achieve near-hitless recovery in large networks. The main problem is solved with Linear Programming (LP) and Integer Linear Programming (ILP), whereas the subproblem can be solved with different methods. Simulation results suggest that both the novel coding structure and the novel design algorithm lead to higher capacity efficiency for near-hitless recovery. The novel design algorithm simplifies the capacity placement problem which enables implementing diversity coding-based techniques in large networks with arbitrary topology.**

I. Introduction

The protection of the data in wide area networks is very important since the network failures, which happen regularly, pose social, economical, and security threats. A breakdown of the network failure statistics can be found in [1]. In this paper, we focus on single link failure recovery, which makes up to 70% of all network failures [2]. Various protection and recovery techniques are developed to minimize the costs of such failures, each offering a tradeoff in terms of different recovery metrics. The two main recovery metrics are restoration speed and capacity efficiency. Capacity efficiency is calculated by the total required capacity, in fiber miles, to route and protect the data streams. Restoration speed is measured by the total outage duration between the instant of failure and the restoration of failed traffic. Capacity efficiency has a pre-failure cost, whereas restoration speed has a post-failure cost. The goal is to minimize both of these costs.

Recovery techniques differ from each other depending on if they dedicate the spare resources to single demands or share them among different failure scenarios and traffic demands. The biggest advantage of dedicated recovery techniques is the near-hitless recovery without signaling and rerouting operations. 1+1 Automatic Protection Switching (APS) has two link-disjoint dedicated paths for each connection demand and those paths are employed to transmit the same data to the destination node [3]. The destination node switches to the protection path and restores the traffic nearly instantaneously in the case of a link failure over the primary path. However, 1+1 APS is capacity-inefficient since it requires more than 100% capacity. In [4], it is cited that 1+1 APS is currently employed in today's networks despite its low capacity efficiency, which is an indication of the need for nearly instantaneous link failure recovery.

This work was partially supported by NSF under Grant No. 0917176.

Coding-based recovery techniques emerged to improve the capacity efficiency of dedicated protection techniques. In a coding-based recovery technique, the dedicated protection paths share the spare resources by coding operations, in particular, erasure coding [5], [6]. The first coding-based recovery technique is called diversity coding. This technique has two advantages. First, like 1+1 APS, it offers nearly instantaneous recovery. Second, like rerouting-based restoration schemes, it is capacity-efficient. The first works of diversity coding [5], [6] predate network coding, usually considered to be introduced in [7].

In [8], a heuristic algorithm is developed to implement diversity coding over arbitrary networks. In [1], optimal algorithms for the diversity coding technique are developed. It is shown that diversity coding can offer competitive capacity efficiency while providing near-hitless recovery. In [9], a coding-based solution named Coded Path Protection (CPP) is developed by converting a solution of Shared Path Protection [10]. In CPP, sharing of the spare resources is replaced with the employment of these resources to code different paths, which results in higher restoration speed, higher transmission integrity, and lower error signaling complexity. The bidirectional nature of CPP allows encoding and decoding inside the network for unicast demands.

In [11] and [12], network coding-based protection schemes, similar to diversity coding, are proposed in which coding operations are carried out over trees and trails, respectively. These schemes are called 1+N protection and differ from diversity coding due to their bidirectional nature. In [13], the cost efficiencies of a network coding-based recovery technique and a simpler version of diversity coding technique are evaluated.

The coding-based techniques mentioned above have certain assumptions to make them easier to implement. First, in systematic coding, primary paths are exempt from coding operations. Second, in these techniques, coding operations are bound to specific topologies. Third, these protection schemes require strict link-disjointness between each primary path and the protection paths. However, those assumptions restrict their capacity efficiencies. In [4], an argument that 1+N coding requires high nodal degree, which reduces its efficiency on sparse topologies, was made.

Nonsystematic coding, where both primary and protection paths are incorporated into coding operations, is implemented in wireless mesh networks for single link failure recovery in [14]. In [15], nonsystematic diversity coding is implemented using a heuristic algorithm for static provisioning. The connection demands are added to the existing coding groups one by one ensuring the decodability of the coding structures. A coding group is a set of connection demands that are coded and protected together. In [15], it is shown that nonsystematic diversity coding has more coding flexibility than conventional diversity coding resulting in higher capacity efficiency. In [16], a general network-coding based approach is presented which employs nonsystematic coding and does not explicitly require link-disjointness between primary paths and protection paths. However, this approach can protect at most two connection demands in restricted specific topologies. In [17], the restriction over the number of protected connection demands is removed for bidirectional networks.

Due to high design complexity limitations, the coding-based recovery techniques in the literature, such as [11], [16], [17], fail to offer solutions in large realistic networks even though they have potential in terms of capacity efficiency and restoration time. These techniques are tested on relatively small networks and with relatively few traffic demands compared to the long-distance networks of the U.S. and France, to be discussed in the sequel. In [18], a novel two step approach is presented to cope with high design complexity in realistic networks. The first step of this algorithm is the pre-processing phase in which all candidate coding groups are calculated and enumerated. In the second step, some of those candidate coding groups are selected and placed on the networks to meet the traffic demand. This approach overcomes the complexity incurred by the size of the traffic matrix. However, the number of coding groups is exponentially dependent on the network size and the nodal degree of the destination node.

This paper contributes to the field of diversity coding-based (or network coding-based) link failure recovery in two novel ways. First, we introduce an optimal, simple, and modular design algorithm that provisions the static traffic in large arbitrary networks. The design algorithm uses the column generation technique which does not require explicit enumeration of the coding groups. It starts the problem with a small set of coding groups and creates new coding groups when they are needed. The underlying coding structure of this algorithm is arbitrary as long as the destination nodes of the connections are the same, which offers a solution for different techniques under the same framework. Second, we improve the coding structure of simple diversity coding by offering a technique we call coherent diversity coding. This coding structure is implemented using an Integer Linear Programming (ILP) formulation. In a coherent diversity coding structure, we implement a more relaxed link-disjointness criterion between the paths in a coding group. This enables one to form coding groups with higher flexibility and bigger size. The decodability is preserved while the high nodal degree requirement is mitigated. Moreover, coherent diversity coding incorporates nonsystematic coding.

In this paper, the performance of the new proposed coding technique and the column generation-based design algorithm are investigated compared to conventional (systematic or non-systematic) diversity coding and *p*-cycle protection [3]. The simplicity of the new design algorithm is also tested based on a set of simulations over the relatively large long-distance networks of the U.S. and France.

II. Column Generation Method

The column generation method is an effective technique to solve relatively large linear programming (LP) formulations without explicitly enumerating all possible variables [19]. In some problems, only a small subset of the variables are nonzero in the final solution. In those problems, column generation starts with a small set of variables and creates new and useful variables (columns) which will be likely employed in the final solution. In general, column generation dramatically decreases the time and space complexity depending on the nature of the problem. In the network-coding based link failure recovery problem, we have observed that column generation technique results in significant time and memory savings, and therefore it enables the optimal implementation of efficient network coding-based techniques over large realistic networks.

Column generation has been used for different LP problems, including the well-known cutting stock problem [19]. The cutting stock problem is to satisfy paper demand of different widths by cutting fixed width rolls in different patterns. The goal is to use a minimum number of rolls. The problem starts with a small set of basic cutting patterns. The useful cutting patterns are generated one-by-one. We observed that the diversity coding-based link failure recovery problem is very similar to the cutting stock problem. Diversity coding over networks can be implemented like the cutting stock problem as long as the cutting patterns are replaced by coding groups and the demands for different widths of paper are replaced with the traffic demands of a single destination node. The only difference is the fact that coding groups can have different costs, whereas in the cutting stock problem, each cutting pattern is cut from rolls with the same total width. Other advanced methods developed for the cutting stock problem, such as extended Dantzig-Wolfe decomposition [19], can also be applied to the implementation of diversity coding.

The column generation technique is also applied to the *p*-cycle protection [20] and SPP [21] problems resulting in significant time and memory savings. It is a better fit to diversity coding technique than *p*-cycle protection and SPP since, in diversity coding, there is a single subproblem that generates coding groups. However, in *p*-cycle protection, there is a subproblem for both generating *p*-cycles and generating candidate paths for each connection demand. Likewise, in SPP, there is a different subproblem for generating candidate path pairs for each connection demand.

The column generation method for diversity coding is visualized in Fig. 1. There are two main components of this method. The main problem, which is also called the *Coding Groups Placement Problem*, inputs the traffic demands and a subset of the basic coding groups. This set includes coding groups consisting of a single connection demand originating from each source node to the destination node. The main problem in this step is an LP formulation that finds the optimal coding group combinations to meet the traffic demands. After

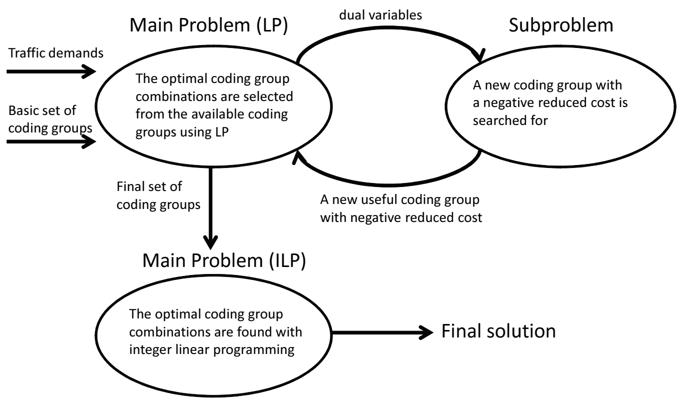


Fig. 1. Steps in the column generation method.

the first run, it passes the dual variables of the solution to the subproblem. The subproblem, which is also called the *Coding Group Generation Problem*, attempts to find a new useful coding group. The subproblem generates a reduced cost. A new useful coding group has a negative reduced cost given the dual variables of the main problem. The new useful coding group is input to the main problem iteratively. In the next round, optimal coding group combinations are found given the expanded coding group set. The dual variables of this run are input to the subproblem as before. This iterative operation is carried out until the subproblem cannot find any new coding group with a negative reduced cost. The main problem is then solved one last time as an ILP. The gap between ILP and LP solutions of the main problem is generally very small, as will be discussed in Section IV.

A. Example 1

As an example, assume there are 2 connection demands from S_1 and S_2 to D. Each has a unit traffic demand. The cost of coding groups that protect only $S_1 - D$ and only $S_2 - D$ are 10 and 7, respectively. The coding groups combination problem employs one of each coding group to satisfy the traffic demands at a total cost of 17. The values of the dual variables of this solution are input to the subproblem. The subproblem finds a coding group solution with a negative reduced cost -3, which means the new coding group will be useful in the main problem. It returns the new coding group consisting of both $S_1 - D$ and $S_2 - D$ at a total cost of 15. The main problem is run one more time and decreases the total cost from 17 to 15 since it employs only the new coding group created by the subproblem. Note that the negative reduced cost of the subproblem is not linearly related to the decrease of the total cost in the main problem. The dual variables of the new solution of the main problem are input to the subproblem again. This time, the subproblem cannot find any coding group solution with a negative reduced cost, which indicates that the optimal result has been achieved.

III. ILP Formulations

In this section, we present the algorithms that realize the main problem and the subproblem. The main problem finds the optimal combination of coding groups out of a given set and places them on the network to meet the traffic demands. Throughout the iterative process, the main problem is realized with an LP formulation, whereas in the last step, the formulation is converted to an ILP since in the final solution coding groups must be replaced in integer numbers. On the other hand, the realization of the subproblem is not unique. The coding group generation algorithm depends on the adopted coding structure. In addition, the way new coding groups are generated can be realized by heuristic techniques, which does not violate the optimality of the whole algorithm. In this section, we present three different coding group generation algorithms using mixed integer programming (MIP) or ILP formulations.

A. Main Problem (Coding Groups Placement Problem)

An LP formulation is developed to implement the coding groups placement algorithm, which serves the main problem of the column generation method. The goal is to place the coding groups with minimum total cost while meeting the traffic demands. The input parameters of the LP are

- CG : The set of coding groups, this set is expanded at each iteration,
- V : The set of nodes,
- t_f : The traffic demand from source node f to destination node d,
- $cost_i$: The cost of coding group i,
- $CG_{i,f}$: The number of connections originating from node f in coding group i.

The variables related to the coding groups placement problem are

- $n(i)$: Keeps the number instances of coding group i placed on the network, normally a continuous variable.

The variables $n(i)$ are continuous when the main problem is LP. They are converted to integer variables at the final ILP step of column generation.

The objective function is

$$\min \sum_{i \in CG} cost_i \times n(i). \quad (1)$$

The following inequalities ensure a sufficient number of coding groups are placed to protect all of the traffic demands

$$\sum_{i \in CG} CG_{i,f} \times n(i) \geq t_f \qquad \forall f \in V, f \neq d. \quad (2)$$

A flow diagram of the column generation method in terms of the parameters and variables of the LP formulations is shown in Fig. 2, where π_f are the dual variables of the constraints in 2. The traffic demand parameters t_f and an initial basic coding group set $CG^{initial}$ are input to the main problem. After the first run, the main problem inputs the resulting dual values of the constraints to the subproblem. The subproblem returns a new coding group with negative reduced cost, if available. The iterative process terminates when the subproblem cannot produce any more new coding groups with reduced cost. Then the variables $n(i)$ are converted to integer variables and ILP is run at the last step to get the final solution.

B. Subproblem (Coding Group Generation Problem)

The objective of the subproblem is to find a new coding group in each iteration that will be useful in the main problem.

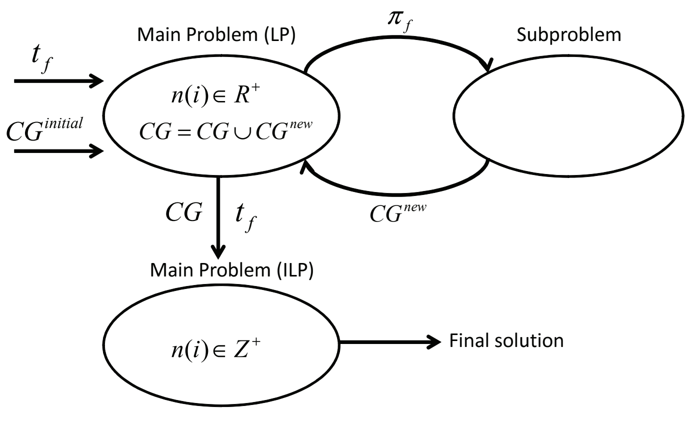
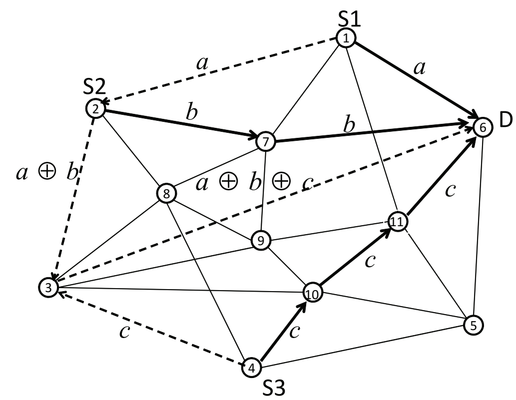
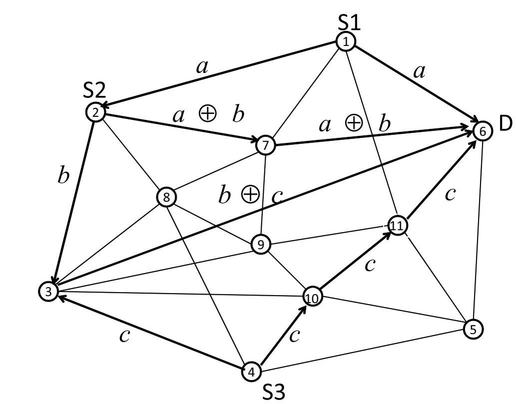


Fig. 2. Steps in the column generation method in terms of LP and ILP variables.



(a)



(b)

Fig. 3. (a) An example of the systematic diversity coding tree structure. There are three link-disjoint primary paths spanned by the primary tree and there is a link-disjoint protection tree, (b) An example of nonsystematic diversity coding structure for the same set of connections.

The subproblem inputs the dual variables of the main problem and returns a new coding group. A new coding group can be selected among many which have negative reduced costs. In this paper, we opt to search for a new coding group with the minimum negative reduced cost until there is none. We present three different coding group generation algorithms, each implementing a different version of diversity coding. These versions have the tradeoff of simplicity versus capacity efficiency. In the following subsections, they are presented in increasing order of capacity efficiency and design complexity.

1) Systematic Diversity Coding: In this algorithm, we adopt systematic diversity coding where only protection paths are encoded. The core algorithm is adopted from the diversity coding tree algorithm in [22]. In a coding group, there is a primary tree serving as the union of the primary paths of the protected connections. There is also a link-disjoint protection tree whose branches originate from the source nodes of the protected connections. Those branches merge when they come together until they reach at the destination node. An example is taken from [22] and is shown in Fig. 3(a). There are three connection demands originating from S_1, S_2, and S_3 going to node D. The solid black lines represent the primary tree whereas dashed lines represent the protection tree.

The input parameters required in the MIP formulation of the coding group generation algorithm based on systematic diversity coding are

- $G(V, E)$: Network graph,
- S : The set of spans in the network, a span consists of two links in the opposite directions,
- a_e : Cost associated with link e,
- $\Gamma_i(f)$: The set of incoming links of each node f,
- $\Gamma_o(f)$: The set of outgoing links of each node f,
- d : The common destination node,
- ND : The nodal degree of the destination node d,
- α : A constant employed in the algorithm where $\frac{1}{|V|} \geq \alpha \geq 0$,
- β : A constant employed in the algorithm, $\beta \geq 2 \times \max(|V|, \max_i(ND_i))$,
- π_f : The values of the dual variables of the main problem.

The set of variables of this MIP formulation are

- CG_f^{new} : Integer variable, equals to the number of connections originating from node f in the new coding group,
- $d_e \in \{0, 1\}$: Integer variable, equals 1 iff the primary tree of the new coding group passes through link e,
- $c_e \in \{0, 1\}$: Integer variable, equals 1 iff the protection tree of the new coding group passes through link e,
- p_f : A continuous variable between 0 and 1. It keeps the "voltage" value of node f [22] in the protection tree of the new coding group,
- g_f : Same description as p_f except it is used for the primary tree of the new coding group.

The objective function minimizes the reduced cost of a new coding group

$$\min \sum_{e \in E} (d_e + c_e) \times a_e - \sum_{f \in V} CG_f^{new} \times \pi_f. \qquad (3)$$

If the value of the objective function comes out to be negative, then a new coding group is found and input to the main problem.

$$\sum_{f \in V} CG_f^{new} \leq ND - 1, \qquad (4)$$

$$\sum_{e\in\Gamma_o(f)} d_e = CG_f^{new} + \sum_{e\in\Gamma_i(f)} d_e \quad \forall f \in V, f \neq d, \tag{5}$$

$$\sum_{e\in\Gamma_i(d)} d_e = \sum_{f\in V} CG_f^{new}, \tag{6}$$

$$\sum_{e\in\Gamma_o(d)} d_e + c_e = 0, \tag{7}$$

$$\sum_{e\in\Gamma_o(f)} c_e \geq \frac{CG_f^{new}}{\beta} + \frac{\sum_{e\in\Gamma_i(f)} c_e}{\beta} \qquad \forall f \in V, f \neq d. \tag{8}$$

$$\sum_{e\in\Gamma_i(d)} c_e \geq \frac{\sum_{f\in V} CG_f^{new}}{\beta}, \tag{9}$$

$$d_{e1} + d_{e2} + c_{e1} + c_{e2} \leq 1 \quad \forall e1, e2 \in g, \forall g \in S, \tag{10}$$

$$g_f - g_f \geq \alpha \cdot d_e - (1 - d_e) \quad \forall e \in E, \tag{11}$$

$$p_f - p_f \geq \alpha \cdot c_e - (1 - c_e) \quad \forall e \in E. \tag{12}$$

Inequality (4) ensures that the size of the new coding group does not exceed $ND - 1$. Equation (5) carries out the origination and continuation of the primary tree, whereas equation (6) and equation (7) carry out the termination of the primary tree. Inequality (8) is responsible for the origination and continuation of the protection tree, whereas inequality (9) and equation (7) are responsible for the termination of the protection tree. Inequality (10) makes sure that primary and protection trees are link-disjoint. Inequalities (11) and (12) assign voltage values to nodes to prevent getting cyclic structures in primary and protection trees, respectively.

2) Nonsystematic Diversity Coding: In this section, the coding groups are generated based on a more generic coding structure where both primary and protection paths can be encoded. We refer to *Lemma 1* from [14] while building valid nonsystematic diversity coding. This coding structure increases the capacity efficiency of systematic diversity coding with extra design complexity. An example is shown in Fig. 3(b). Different from systematic diversity coding, the primary paths of $S_1 - D$ and $S_2 - D$ are encoded. The core algorithm to generate new coding groups in the column generation method is an ILP formulation taken from [22] with small changes. Reference [22] presents how to optimally build nonsystematic diversity coding structures. The algorithm in [22] looks for every possible coding scenario by eliminating the invalid cases that can be identified as *coding cycles.* The ILP formulation of the nonsystematic diversity coding group generation algorithm has a set of binary integer variables taking values from the set $\{0, 1\}$

- $x_e(i)$: Equals 1 iff the path i passes through link e,
- $n(i, s)$: Equals 1 iff path i is in subgroup s,
- $m(i, j)$: Equals 1 iff path i and path j are in the same subgroup so are coded together,
- $r(i, f)$: Equals 1 iff path i and connection demand f are indirectly related,
- $t_e(s)$: Equals 1 iff one of the paths in subgroup s traverses over link e,
- $\sigma_{f,i}$: Equals 1 iff node f is the source node of demand i.

The objective function is

$$\min \sum_{e\in E} \sum_{s=1}^{2N} t_e(s) \times a_e - \sum_{f\in V} CG_f^{new} \times \pi_f. \tag{13}$$

The constraints are

$$\sum_{f\in V} \sigma_{f,i} \leq 1, \quad 1 \leq i \leq ND - 1, \tag{14}$$

$$CG_f^{new} = \sum_{i=1}^{ND-1} \sigma_{f,i} \quad \forall f \in V, f \neq d, \tag{15}$$

$$\sum_{f\in V} \sum_{i=1}^{ND-1} \sigma_{f,i} \leq ND - 1 \tag{16}$$

$$\sum_{e\in\Gamma_i(f)} x_e(j) - \sum_{e\in\Gamma_o(f)} x_e(j) = \begin{cases} -\sigma_{f,i} & \text{if } v \neq d, \\ \sum \sigma_{f,i} & \text{if } v = d, \end{cases} \quad j = 2i,\ j = 2i - 1. \tag{17}$$

$$\sum_{s=1}^{2(ND-1)} n(i, s) = 1, \quad 1 \leq i \leq 2(ND - 1), \tag{18}$$

$$n(i, s) + n(i - 1, s) \leq 1, \quad 1 \leq i, s \leq 2(ND - 1) : \mathrm{mod}(i, 2) = 0, \tag{19}$$

$$t_e(s) \geq x_e(i) + n(i, s) - 1 \quad \forall e, i, s \tag{20}$$

$$t_e(s_1) + t_e(s_2) + t_k(s_1) + t_k(s_2) \leq 1 \quad \forall e, k \in g, \forall g \in S, \forall s_1, s_2 \tag{21}$$

$$m(i, j) \geq n(i, s) + n(j, s) - 1 \quad \forall i \neq j, s. \tag{22}$$

$$r(i, f) \geq m(i, j) + m(j^*, 2f) + m(j^*, 2f - 1) - m(i, 2f) - m(i, 2f - 1) - 1 \quad \forall i, j, f, : i \neq j \tag{23}$$

such that $j^* = j - 1$ if $\mathrm{mod}(j, 2) = 0$ and $j^* = j + 1$ otherwise.

$$r(i, f) \geq r(i, g) + m(2g, 2f) + m(2g, 2f - 1) + m(2g - 1, 2f) + m(2g - 1, 2f - 1) - 1 \ \forall i, f \neq g : i \neq 2f, i \neq 2f - 1, i \neq 2g, i \neq 2g - 1. \tag{24}$$

$$r(2f, g) + r(2f - 1, g) + m(2f, 2g) + m(2f - 1, 2g) + m(2f, 2g - 1) + m(2f - 1, 2g - 1) \leq 1 \ \forall g, f : g \neq f, \tag{25}$$

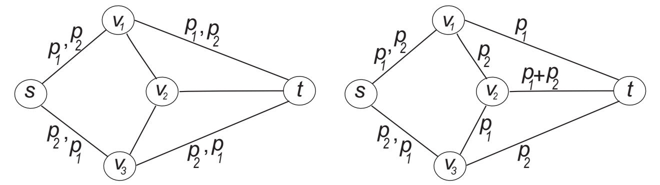
Inequality (14) ensures that each demand has at most one source node. Some connection demands may be empty. Equation (15) calculates the number of connection demands originating from each node at the new coding group. Inequality (16) bounds the total number of connection demands in the new coding group by the nodal degree of the destination node minus 1. Equation (17) carries out the origination, continuation, and termination of the paths of each connection demand. Each connection demand has two paths in a coding group. Equation (18) ensures that each connection demand is a part of a coding subgroup. Inequality (19) ensures that paths belonging to the same connection cannot be a part of the same subgroup. Inequality (20) compiles the topologies of the subgroups by combining the paths of the demands in that subgroup. Inequality (21) satisfies the link-disjointness criterion between the topologies of different subgroups. Inequality (22) says that if two paths are in the same subgroup then they are assumed to be coded together. In inequality (23), path i becomes indirectly related to demand f if there exists a path j that is coded with both path i and one of the paths carrying demand f. Moreover, path i must not be coded with either paths of demand f. In inequality (24), path i becomes indirectly related to demand f if there exists a demand g that is indirectly related to path i, and one of the paths of demand g must be coded together with one of the paths of demand f. Inequality (25) ensures that two different connection demands either can be indirectly related or one of their paths are encoded together. Otherwise, a coding cycle occurs which is a violation of the validity of the coding structure.

3) Coherent Diversity Coding: In this section, we introduce a novel coding structure that can mitigate the limiting link-disjointness criterion to the optimal extent. It is called *Coherent Diversity Coding*. This coding structure is optimal under the conditions

- There is a single destination node,
- There are two link-disjoint paths for each connection demand,
- The coding operations are within $GF(2)$.

It enables one to achieve more capacity-efficient results than conventional diversity coding. Conventional diversity coding, systematic or nonsystematic, requires two paths to be either coded or to be link-disjoint. This prevents applying conventional diversity coding at the destination nodes with a nodal degree of 2, even though the rest of the network is highly connected. There is room for improvement in the capacity efficiency of coding groups by relaxing the link-disjointness criterion between different paths. Fig. 4 is taken from [16] and shows how the strict link-disjointness criterion for two connections can be relaxed in order to save capacity. The connection demands are from node s to node t, carrying signals p_1 and p_2, respectively. There is no available nontrivial solution for conventional diversity coding on this topology since there are only N, which is 2 in this case, number of link-disjoint paths, less than the required $N+1$ ($N+1=3$), from source to destination. Therefore, in Fig. 4(a), the solution of conventional diversity coding is identical to that of 1+1 APS. The low nodal degree of the source node is a bottleneck for conventional diversity coding. On the other hand, the network-coding based technique proposed by [16] shows that these two data signals can be coded to save capacity in Fig. 4(b). However, the technique in [16] is nontractable for more than two connection demands and lacks an efficient capacity placement algorithm.

Therefore, we developed the optimal link-disjointness criteria between paths in the same coding group that can mitigate



(a) (b)

Fig. 4. Effect of low nodal degree on coding (a) Diversity coding solution (identical to 1+1 APS), (b) Network coding-based solution [16].

the effects of low nodal degree in the network. The coherent diversity coding enables paths sharing the same link, even if they are not coded together, up to the extent that decodability is preserved. Therefore, it is both optimal and feasible. Under the optimality conditions stated above, the necessary and sufficient conditions of decodability are to ensure that at least one copy of each signal is alive and any subset of k signals resides in at least k subgroups after any single link failure. The resulting coding structure will be decodable according to *Lemma 1* in [14]. Therefore, we build the coding structure of coherent diversity coding such that after any single link failure, there will be at least one copy of each signal and any subset of k signals reside in at least k subgroups. The terms of coherent and noncoherent paths are coined to keep the track of link-disjointness relationship between paths. If two paths are coherent to each other, then they can fail simultaneously, therefore they can share the same links. Otherwise, their simultaneous failure will impair the decodability as will be shown with an example. The proposed technique is nearly as simple to implement as diversity coding.

The received vector of systematic diversity coding for two connection demands

$$\begin{bmatrix} p_1 \\ p_2 \\ p_1+p_2 \end{bmatrix} \tag{26}$$

where p_1 and p_2 are the data signals of two different connection demands. Each symbol on the received vector represents a single path and each data signal is carried with two different paths. The paths carrying the same signal are complementary of each other. If two paths have to be link-disjoint, then they are defined as noncoherent to each other. Assume the path carrying p_1 in the first subgroup and the path carrying p_2 in the third subgroup fail simultaneously, then the received vector is

$$\begin{bmatrix} 0 \\ p_2 \\ p_1+0 \end{bmatrix}. \tag{27}$$

The destination node will still be able to decode symbols p_1 and p_2. It is clearly seen that conventional diversity coding can tolerate failure of symbols in more than one subgroup. Therefore, the path carrying p_1 in the first subgroup and path carrying p_2 in the third subgroup can share some of the links. Therefore, they are coherent to each other. Similarly, the path carrying p_2 in the second subgroup and the path carrying p_1 in the third subgroup can share links. After those relaxations, the solution in Fig. 4(b) is achieved with a modified diversity coding approach. This approach is simpler to keep track

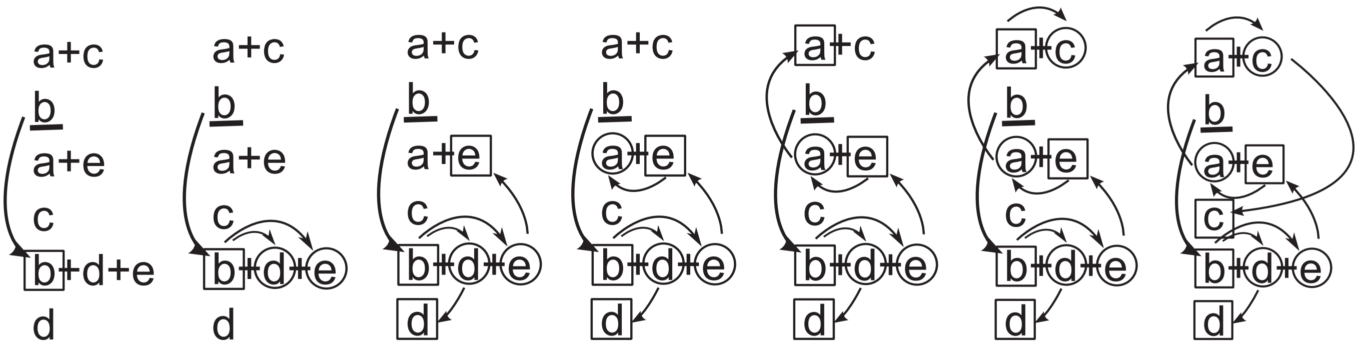


Fig. 5. The process of finding the coherent and noncoherent paths to the underlined path in the second subgroup. Coherent paths are depicted in a circle and noncoherent paths in a square.

of since there are at most $2 \times N$ paths for N connection demands. Intuitively, in systematic diversity coding, a path can be link-joint with the paths that are combined with its complementary path. However, to implement those relaxations over nonsystematic codes with an arbitrary number of data signals, a general strategy is needed. The set of rules that define the general strategy are

1. A path is link-disjoint (noncoherent) with its complementary path,
2. A path is coherent with the path that is coded with its complementary path,
3. A path is noncoherent with the complementary paths of its coherent paths,
4. A path is coherent with the paths that are coded with its noncoherent paths.

The logic behind these rules is to make sure that at least one path carrying each data signal survives and any subset of k signals are found within at least k subgroups under any single link failure scenario. It is also important to keep the number of nonzero subgroups greater than or equal to N under any failure scenario. The following example visualizes how coherent and noncoherent relationships between paths are found. A valid nonsystematic code is

$$\begin{bmatrix} a+c \\ b \\ a+e \\ c \\ b+d+e \\ d \end{bmatrix} \tag{28}$$

with five connection demands. The procedure to find the set of coherent and noncoherent paths of the path carrying b in the second subgroup is shown in Fig. 5. In the first step, the complementary path of underlined b is set as a noncoherent path in Fig. 5(a) following *Rule 1*. The coherent paths are placed in a circle, whereas noncoherent paths are placed in a square. In Fig. 5(b), paths that are combined with a noncoherent path are set as coherent paths following *Rule 2*. In Fig. 5(c) and in Fig. 5(d), the third and fourth rules of the general strategy are applied, respectively. The process is carried out by following *Rule 3* and *Rule 4* interchangeably until those rules are no longer applicable. At the end, if there is any nonvisited path in the coding group, it is assumed to be coherent. In that case, the rest of the paths are set as coherent paths to the path of interest.

For example, in Fig. 5, assume that the underlined path carrying signal b fails simultaneously with the path carrying signal a in the first subgroup which is noncoherent to itself. If so, the received vector at the destination node becomes

$$\begin{bmatrix} c \\ a+e \\ c \\ b+d+e \\ d \end{bmatrix}. \tag{29}$$

This vector clearly violates one of the conditions of decodability because the set of four signals $\{a, e, b, d\}$ is bounded within only three subgroups $\{\{a+e\}, \{b+d+e\}, \{d\}\}$. Therefore, the resulting decoding vector is not decodable. The other scenarios can also be checked to confirm that simultaneous failures of noncoherent paths impair the survivability, unlike the simultaneous failures of the coherent paths. If more than two paths are supposed to share the same link then each pair of paths must be coherent to each other. To find the coherent and noncoherent set of paths of each path, this process is repeated starting with the path of interest.

We developed an ILP formulation to generate new coding groups based on the principles of coherent diversity coding. The ILP formulation of this coding structure inherits all of the variables, parameters, objective function, and constraints of Section III-B2. The extra variables needed for this ILP formulation are

- $\theta(i,j) \in \{0,1\}$: Binary variable, equals 1 iff the path i and path j are noncoherent, in other words, they cannot fail simultaneously.

The objective function to find a new coding group with the most negative reduced cost is

$$\min \sum_{e \in E} \sum_{s=1}^{2N} t_e(s) \times a_e - \sum_{f \in V} CG_f^{new} \times \pi_f. \tag{30}$$

The additional constraints are

$$\theta(i,i-1) = \theta(i-1,i) = 1 \quad \forall i : mod(i,2) = 0, \tag{31}$$

$$\theta(i,j) \geq m(i^*, j^*) \quad \forall i, j, \tag{32}$$

$$\theta(i,k) \geq \theta(i,j) + m(j,k^*) - 1 \quad \forall i \neq j \neq k. \tag{33}$$

$$x_e(i)+x_e(j)+x_{e^*}(i)+x_{e^*}(j) \leq 2-\theta(i,j) \qquad \forall i,j,e \quad (34)$$

such that link e and link e^* are links of the same span in the opposite directions. Equation (31) makes sure that complementary paths have to be link-disjoint with each other according to *Rule 1*. Inequality (32) ensures both *Rule 2* and *Rule 3* are satisfied. In addition, inequality (33) ensures that both *Rule 3* and *Rule 4* are satisfied. Two paths cannot share a link if they are noncoherent, which is guaranteed by inequality (34).

IV. Simulation Results

In this section, we present simulation results to investigate the performance of the novel design algorithm and the new coding structure differentially. The first test network is the NSFNET network, which is depicted in Fig. 6. The numbers next to the nodes are the index of those nodes and the numbers next to the edges are the length of those edges. The traffic matrix of the NSFNET network consists of 3000 random unit-sized demands, which are chosen using a realistic gravity-based model [23]. Each node in the NSFNET network represents a U.S. metropolitan area and their population is proportional to the weight of each node in the connection demand selection process. In this network, we simulated TSA from [18], *p*-cycle protection [20], diversity coding tree from [22], and the proposed CGM. CPLEX 12.2 is used for the simulations. We also adopted different coding structures for TSA and CGM. There are three different tables that present the simulation results of this network. In Table I, the performance metrics are the total cost (capacity) (TC) and the runtime. The first technique in this table is the diversity coding tree algorithm [22]. TSA-SDC refers to the two-step approach implementing systematic diversity coding, whereas TSA-NSDC means TSA for nonsystematic diversity coding. CGM-SDC, CGM-NSDC, and CGM-CDC correspond to the CGM implementing systematic diversity coding, nonsystematic diversity coding, and coherent diversity coding. In our implementation, in order to reuse previous results to save time, these three algorithms are implemented sequentially. The coding groups (columns) generated by CGM-SDC are inherited by CGM-NSDC. Likewise, CGM-CDC inherits the coding groups generated by CGM-NSDC. The *p*-cycle algorithm is taken from [20], which is also based on column generation.

Table I presents various trade-offs between protection techniques. First of all, coding-based techniques offer near-hitless recovery. Their restoration speed is at least two orders of magnitude higher than that of *p*-cycle protection [22]. On the other hand, *p*-cycle protection has higher capacity efficiency than the tested coding-based methods. As it is seen, the diversity coding tree algorithm has the highest complexity which keeps it from achieving optimal results even though it implements the same systematic diversity coding like TSA-SDC and CGM-SDC do. The proposed CGM is more scalable than the diversity coding tree algorithm and TSA, as seen from the runtime column. In both TSA and CGM, nonsystematic diversity coding is more capacity-efficient than systematic diversity coding. In addition, proposed coherent diversity coding is the most capacity-efficient among coding-based methods. However, the increase in capacity efficiency is negligible compared to the savings in runtime. Network designers can opt to carry out the implementations of CGM-NSDC and CGM-CDC after the implementation of CGM-SDC. We believe that CGM-SDC is the most efficient coding-based technique in terms of restoration speed, capacity efficiency, and design complexity.

The second test network is the U.S. long-distance network, taken from [24], which is depicted in Fig. 7. The traffic matrix is created using a gravity-based model [23]. In total, there are 23,204 static unit connection demands. This setup is chosen in order to observe the performance of the new design algorithm in a large realistic network with a dense traffic scenario. We compare the performance of CGM with TSA and the *p*-cycle algorithm from [20] in terms of spare capacity percentage (SCaP) defined in [8]. The other coding-based recovery design algorithms are too complex to implement in this setup. The results are presented in Table II.

As seen from the results, the proposed design algorithm can achieve optimal results with different versions of diversity coding even in a large realistic network with a dense traffic scenario. Proposed coherent diversity coding technique performs best compared to other coding-based recovery techniques at the expense of higher complexity. The increase in capacity efficiency due to the advanced coding technique is more significant than it is in the NSFNET network. The implementation of systematic diversity coding with the proposed CGM is highly scalable since its runtime does not increase as much as others when the network size gets bigger. The TSA approach is not as scalable as CGM since the number of candidate paths in TSA increases exponentially with the nodal degree and the number of nodes, whereas the number of candidate paths in CGM increases linearly with the number of nodes. The SCaP result of the new technique is better than that of the column generation based *p*-cycle algorithm. It should be noted that, *p*-cycle algorithm carries out Spare Capacity Placement (SCP) [3] due to its high complexity, whereas the proposed algorithm carries out Joint Capacity Placement (JCP) [3]. Even with that adjustment, the proposed CGM is simpler than the *p*-cycle algorithm.

The third network is the long-distance network of France with 43 nodes and 142 unidirectional links taken from [25]. It is depicted in Fig. 8. There are a total number of 4,518,318 unit connection demands. The traffic scenario is created following the same gravity-based model. The reason to select this network is to test the performance of CGM in very large

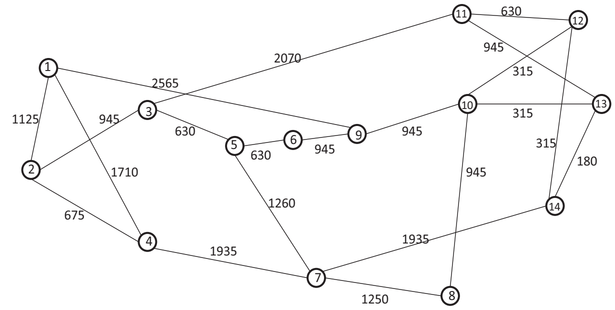


Fig. 6. NSFNET network.

TABLE I
COST AND RUNTIME COMPARISON BETWEEN DIFFERENT TECHNIQUES

Protection Technique	TC	Runtime
Diversity Coding Tree	16410400	≈ 6 hours
TSA-SDC	15788730	≈ 6 minutes
TSA-NSDC	15742000	≈ 9 minutes
CGM-SDC	15793170	≈ 10 seconds
CGM-NSDC	15742000	≈ 5 minutes
CGM-CDC	15674520	≈ 1 hour
P-cycle algorithm	14814350	≈ 3 minutes

TABLE II
COMPARATIVE PERFORMANCE OF THE NEW ALGORITHMS IN U.S. LONG-DISTANCE NETWORK

Protection Technique	SCaP	Runtime	No. of Coding Groups
TSA-SDC	105.6%	≈ 3 hours	31464
CGM-SDC	105.6%	≈ 2 minutes	61
CGM-NSDC	105.5%	≈ 2 hours	72
CGM-CDC	102.4%	≈ 9 hours	79
P-cycle algorithm	107.0%	≈ 2.5 hours	32 (*p*-cycles)

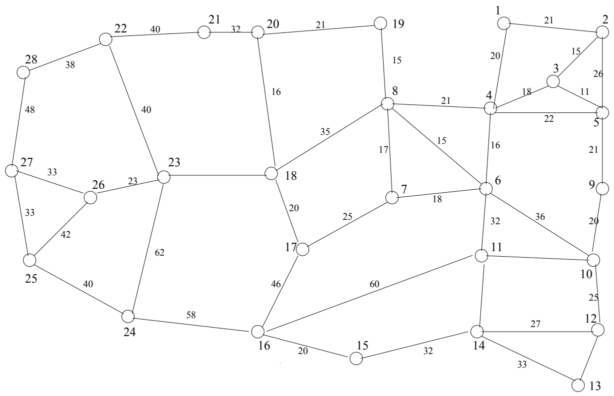


Fig. 7. U.S. long-distance network.

realistic networks. Therefore, we only simulate CGM-SDC to investigate the runtime performance of the column generation method without extra complexity due to the advanced coding structure. It is compared to 1+1 APS. We also break down the results in terms of the nodal degree of the nodes to see the effect of the nodal degree on both capacity efficiency and runtime. The results are presented in Table III. The runtime of 1+1 APS is equal to 1 minute.

CGM can achieve the optimal result in such a large network with over four million unit demands. The capacity efficiency of CGM-SDC improves as the nodal degree increases with the exception of nodal degree being equal to 5. It may be seen as an exception due to the small sample size. According to the Table III, there is a trade-off between the runtime and the capacity improvement over 1+1 APS. When the nodal degree increases, the SCaP improvement of CGM-SDC over 1+1 APS increases at the expense of increased runtime of CGM-SDC with some exceptions due to the small sample size. When the nodal degree is equal to 2, systematic diversity coding acts the same as 1+1 APS as we mentioned before.

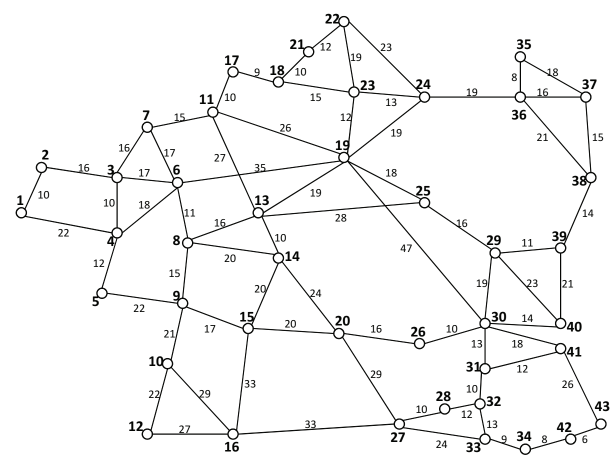


Fig. 8. Long-distance network of France.

V. CONCLUSION

In this paper, we introduced an advanced version of diversity coding and an optimal and simple design algorithm to achieve near instantaneous recovery with higher capacity efficiency. The proposed coherent diversity coding method employs nonsystematic coding, which enables all paths to be encoded, and relaxes the link-disjointness criterion between paths to cope with the low nodal degree in the network. The code is developed with the objective of minimum capacity. The design algorithm consists of two parts, namely a main problem and a subproblem. These two advanced techniques combined achieve results with higher capacity efficiency in a much shorter amount of time in relatively large networks. The advantages of both techniques are shown with examples and simulation results.

The new design framework is based on the column-generation method and consists of two parts, a main problem where the traffic demands are met with the available coding groups and the subproblem where new useful coding groups

TABLE III
SCaP PERFORMANCE OF THE NEW ALGORITHM WITH RESPECT TO THE NODAL DEGREE

Nodal Degree	CGM-SDC (SCaP)	1+1 APS (SCaP)	Runtime of CGM-SDC	Sample Size
2 links	155.3%	155.3%	$\approx$ 2 minutes	12 nodes
3 links	125.5%	149.4%	$\approx$ 16 minutes	13 nodes
4 links	106.7%	140.6%	$\approx$ 39 minutes	14 nodes
5 links	146.4%	184.5%	$\approx$ 26 minutes	2 nodes
6 links	89.5%	126.5%	$\approx$ 85 minutes	1 node
7 links	86.6%	136.6	$\approx$ 53 minutes	1 node
Total	105.7%	141.0%	$\approx$ 85 minutes	43 nodes

are generated at each iteration. The main problem starts with a set of dummy coding groups and inputs new coding groups at each iteration. The subproblem creates a new coding group depending on the information coming from the main problem. The iterations are terminated when a new useful coding group cannot be found. The main problem is formulated as LP throughout the iteration process. At the end, the main problem is solved via ILP which creates a very small optimality gap. We have formulated the subproblem different for different coding techniques based on either ILP or MIP. There is a complexity versus capacity efficiency tradeoff in formulating the subproblem. The main problem consists of only $|V|-1$ constraints. It finds and places the optimal coding group combinations to match the traffic demands, which takes sub-ms to run. The new algorithm can be implemented over networks with arbitrary topology and it can achieve optimal results in very large arbitrary networks for arbitrary traffic scenarios.

We ran various sets of simulations to investigate the performance of the new coding structure and the new design algorithm differentially. The coherent diversity coding has a higher capacity efficiency then both the nonsystematic and systematic diversity coding. The improvement is very small in some networks but is more significant in other networks. The most important observation of the paper is how the new column generation-based design method simplifies implementation of coding-based recovery techniques in very large arbitrary networks. The new technique can find optimal solutions in a much shorter time than the competitive techniques. The complexity of the new technique is more scalable than the competitive techniques depending on the network size, the size of the traffic demands, and the nodal degree of the nodes in the network.

REFERENCES

[1] S. N. Avci and E. Ayanoglu, "Optimal algorithms for near-hitless network restoration via diversity coding," in *Proc. IEEE GLOBECOM*, December 2012, pp. 1–7.

[2] M. Menth, M. Duelli, and J. Milbrandt, "Resilience analysis of packet-switched communication networks," *IEEE/ACM Trans. Netw.*, vol. 17, no. 6, p. 1, December 2009.

[3] W. D. Grover, *Mesh-Based Survivable Networks: Options and Strategies for Optical, MPLS, SONET, and ATM Networking*. Prentice-Hall PTR, 2004.

[4] I. B. Barla, F. Rambach, D. A. Schupke, and G. Carle, "Efficient protection in single-domain networks using network coding," in *Proc. IEEE GLOBECOM*, December 2010, pp. 1–5.

[5] E. Ayanoglu, C.-L. I, R. D. Gitlin, and J. E. Mazo, "Diversity coding: Using error control for self-healing in communication networks," in *Proc. IEEE INFOCOM '90*, vol. 1, June 1990, pp. 95–104.

[6] ——, "Diversity coding for transparent self-healing and fault-tolerant communication networks," *IEEE Trans. Commun.*, vol. 41, pp. 1677–1686, November 1993.

[7] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network information flow," *IEEE Trans. Inf. Theory*, vol. 46, pp. 1204–1216, July 2000.

[8] S. N. Avci, X. Hu, and E. Ayanoglu, "Recovery from link failures in networks with arbitrary topology via diversity coding," in *Proc. IEEE GLOBECOM*, December 2011, pp. 1–6.

[9] S. N. Avci and E. Ayanoglu, "Coded path protection: Efficient conversion of sharing to coding," *IEEE Trans. Comm.(to be published)*, 2013.

[10] S. Ramamurthy, L. Sahasrabuddhe, and B. Mukherjee, "Survivable WDM mesh networks," *J. Lightwave Technol.*, vol. 21, no. 4, pp. 870–883, April 2003.

[11] A. E. Kamal and O. M. Al-Kofahi, "Efficient and agile 1+N protection," *IEEE Trans. Comm.*, vol. 59, no. 1, pp. 169–180, January 2011.

[12] A. E. Kamal, A. Ramamoorthy, L. Long, and S. Li, "Overlay protection against link failures using network coding," *IEEE/ACM Trans. Netw.*, vol. 19, no. 4, pp. 1071–1084, Aug. 2011.

[13] H. Øverby, G. Bóizck, P. Barbarczi, and J. Tapolcai, "Cost comparison of 1+1 path protection schemes: A case for coding," in *Proc. IEEE ICC*, June 2012.

[14] O. M. Al-Kofahi and A. E. Kamal, "Network coding-based protection of many-to-one wireless flows," *IEEE J. Sel. Areas in Commun.*, vol. 27, no. 5, pp. 787–813, June 2011.

[15] S. N. Avci and E. Ayanoglu, "Extended diversity coding: Coding protection and primary paths for network restoration," in *Proc. of the International Symposium on Network Coding*, June 2012, pp. 1–6.

[16] S. E. Rouayheb, A. Sprinston, and C. Georghiades, "Robust network codes for unicast connections: A case study," *IEEE/ACM Trans. Netw.*, vol. 19, no. 3, pp. 644–656, June 2011.

[17] A. Sprinston, S. E. Rouayheb, and C. Georghiades, "Robust network coding for bidirected networks," in *Proc. of the Information Theory and Applications Workshop*, Jan.-Feb. 2007, pp. 378–383.

[18] S. N. Avci and E. Ayanoglu, "Network coding-based link failure recovery over large arbitrary networks," in *Proc. IEEE GLOBECOM*, December 2013, pp. 1–7.

[19] G. Desaulniers, J. Desrosiers, and M. Solomon, *Column Generation*. Springer, 2005.

[20] T. Stidsen and T. Thomadsen, "Joint routing and protection using p-cycles," Informatics and Mathematical Modelling, Technical University of Denmark, DTU, Kgs. Lyngby, Denmark, Tech. Rep., May 2005.

[21] C. Rocha and B. Jaumard, "Revisiting p-cycles / FIPP p-cycles vs. shared link / path protection," in *Proc. 17th International Conference of Computer, Communications and Networks (ICCCN 2008)*, Virgin Island, USA, August 2008.

[22] S. N. Avci and E. Ayanoglu, "Optimal algorithms for near-hitless network restoration via diversity coding," *IEEE Trans. Comm.*, vol. 61(9), pp. 3878 – 3893, September 2013.

[23] Y. Zhang, M. Roughan, N. Duffield, and A. Greenberg, "Fast accurate computation of large-scale IP traffic matrices from link loads," in *Proc. ACM SIGMETRICS*, June 2003.

[24] Y. Xiong and L. G. Mason, "Restoration strategies and spare capacity requirements in self-healing ATM networks," *IEEE/ACM Trans. Netw.*, vol. 7, pp. 98–110, February 1999.

[25] J. Doucette, D. He, W. D. Grover, and O. Yang, "Algorithmic approaches for efficient enumeration of candidate p-cycles and capacitated p-cycle network design," in *Proc. DRCN 2003*, Banff, Canada, October 2003, pp. 212–220.