An Energy-Efficient Resource Allocation Algorithm with QoS Constraints for Heterogeneous Networks

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***Abstract*—In this paper, we propose an energy-efficient resource allocation scheme for multi-channel multi-cell networks. Our objective is to maximize the network energy efficiency subject to the minimum rate constraints for each user. We take into account both the static and transmit power consumption at base stations. We employ interference pricing and dual decomposition methods. The interference pricing scheme penalizes the base stations based on their marginal costs of interference to other users. This is leveraged to mitigate inter-sector interference and determine the optimal power levels, realizing the objective. Using dual decomposition techniques, we enforce the minimum rate constraints to be satisfied. To achieve faster convergence, we employ the Levenberg-Marquardt method in the power control phase. The proposed framework employs the fractional frequency reuse (FFR) scheme. We sequentially determine the cell-center boundaries, schedule the users in the frequency domain, and solve the power control problem. Simulation results demonstrate that the network energy efficiency can be significantly improved while reducing the user outages compared to the non-cooperative power allocation scheme.**

***Index Terms*—Lagrangian method, constrained optimization, energy efficiency, heterogeneous cellular networks, Levenberg-Marquardt method, power control, resource allocation.**

I. Introduction

In recent years, the explosive growth of demand for ubiquitous and reliable wireless data has led to an increasing power consumption in the wireless networks. Increased power consumption has both economical and environmental effects. In wireless networks, more than 80% of energy is consumed at base stations. Energy consumption increases the operational expenses. In addition, due to the elevated power consumption, generation of greenhouse gases may reach undesired levels. This problem is studied in the literature under the general topic of green communications research area, see, e.g., [1].

In a densely deployed network, increasing the transmission power of the base station only marginally increases the throughput. However, increased inter-cell interference affects the transmissions in the neighboring cells. In addition, it significantly degrades the energy efficiency of the network. To overcome this problem, several inter-cell interference cancellation and mitigation techniques are investigated in the literature. In this paper, we concentrate on the fractional frequency reuse (FFR) scheme to reduce inter-cell interference [2]. For multi-tier networks, the spectrum allocation problem has been investigated in [3]. We inherit same frequency allocation scheme in this paper. In this scheme, we first set a virtual cell-center radius for each MeNB to differentiate macrocell associated users (MUEs) into cell-center and cell-edge users based on the reference signal received power (RSRP). Then, by using this boundary, picocell base stations (pico-eNBs) are categorized into two groups: cell-center and cell-edge. Our prior work in [4] has employed the same FFR scheme and studied the energy efficiency of HetNets using different cell-center radius selection algorithms. In this paper, we employ the algorithm that maximizes the fairness among users to satisfy the minimum rate constraints of users.

The optimal power allocation strategy for a multi-channel multi-cell network is multilevel water-filling [5]. However, due to the LTE standards and the requirement of large control overhead, this solution is impractical to implement. In [5], [6], the authors show that constant power allocation across the subbands simplifies the problem significantly while the performance loss is negligible. In this paper, we allocate constant power over all resource blocks (RBs) throughout the subband. This approach significantly reduces the complexity of the problem which can be solved in a limited time. However, the power level on each subband still has to be determined by each base station. The multi-channel multi-cell power allocation problem is non-convex. Therefore, finding the global solution is difficult. On the other hand, the sector energy efficiency is a quasiconcave function over transmit power levels of the MeNBs keeping the inter-cell interference constant [4], [7]. Then, using this property and the Karush-Kuhn-Tucker (KKT) conditions, the main problem can be decomposed into successively solved quasiconcave subproblems. As we will show in Section III, the KKT conditions of the main problem and the subproblems are equivalent. In addition, we introduce pricing mechanisms which reflect the marginal cost of interference, mitigating the inter-sector interference.

Utility maximization and pricing in power control has been studied in [8]–[11]. In [8], the users are penalized based on their transmit power times a predefined constant. By this way, users incur penalties when they transmit at high power levels. In [9]–[11], the authors suggest to penalize transmissions

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according to the interference they create. Even if a base station transmits at a low power level but it significantly interferes with the other transmissions, the penalty will be high. In our paper, we use a similar approach in which MeNBs are penalized proportionally to the interference they create.

Energy-efficient resource allocation problem with rate constraints has been investigated in the literature, see, e.g., [7], [12], [13]. However, most of the works in the literature, including these three studies, have neglected the inter-cell interference conditions, which are detrimental in HetNets. In this paper, we take the inter-cell interference into account while maximizing the energy efficiency and satisfying the Quality of Service (QoS) constraints of users. We incorporate the QoS constraints per user into the problem using the dual decomposition method. In order to maximize the energy efficiency of the network, satisfy the minimum rate constraints of the users, and limit the inter-cell interference in the network, the transmissions of the users are penalized with the interference they create to other users with a certain price. The power control is only employed at the MeNBs. Pico-eNBs transmit at full power. The proposed algorithm uses a Levenberg-Marquardt method-based power control algorithm to determine optimal power levels for each sector. By this way, we develop an efficient and realistic framework for multi-cell OFDMA networks.

The remainder of this paper is organized as follows. Section II introduces the system model, power consumption model, and energy-efficiency definition. Section III presents the energy-efficient resource allocation problem with rate constraints and the proposed solution. Numerical results are discussed in Section IV and Section V concludes the paper.

II. System Model

In this section, we first present the system model along with a base station power consumption model, and then we provide our particular definition of energy efficiency.

Consider a wireless network with 19 hexagon cells as shown in Fig. 1. In this paper, we employ the FFR scheme which is described in [3] for a multi-tier network. The cell-center MUEs are allocated on Subband A in all sectors, whereas the cell-edge MUEs are transmitting on either Subband B, C, or D depending on their sector. The cell-center PUEs are scheduled on subbands which the MeNB in the same sector does not transmit on. In addition to these subbands, the cell-edge PUEs are also scheduled on Subband A. The transmission power levels of pico-eNBs are 16 dB less than MeNBs. The intra-sector interference of the cell-edge pico-eNBs to the cell-center MUEs will be limited. In this paper, we denote the sets of macro and pico-eNs by $\mathcal{B}_m$ and $\mathcal{B}_p$, respectively. In addition, the transmission power of an MeNB m and pico-eNB p on subcarrier n is denoted by $P_m^{(n)}$ and $P_p^{(n)}$, respectively. Then, the signal-to-interference-plus-noise ratio (SINR) of an

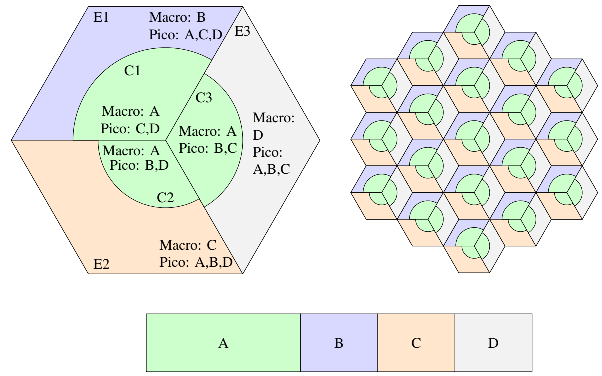

Figure 1. A multi-tier FFR scheme with dynamic cell-center region boundaries is depicted in a uniform hexagonal grid of 19 cells. The MeNBs employ three-sector antennas, whereas pico-eNBs have omnidirectional antennas.

MUE k on subcarrier n can be written as

$$\gamma_k^{(n)} = \frac{P_m^{(n)}\, g_{k,m}^{(n)}}{\sum\limits_{\substack{m'\neq m,\\ m'\in\mathcal{B}_M}} P_{m'}^{(n)}\, g_{k,m'}^{(n)} + \sum\limits_{p\in\mathcal{B}_P} P_P^{(n)}\, g_{k,p}^{(n)} + N_0\Delta_f}. \quad (1)$$

The channel gain between user k and MeNB m on subcarrier n is given by $g_{k,m}^{(n)}$. The same gain for pico-eNB p is denoted by $g_{k,p}^{(n)}$. The thermal noise power per Hz is denoted by N_0 and the bandwidth of a subcarrier is shown by Δ_f. Due to space limitations, the SINR expressions for PUEs are not presented. However, they can be easily generated by using the same approach.

In this paper, constant power allocation across the subbands is studied. Two power control parameters β and ε are introduced to characterize the power assignment of MeNBs. The first variable β scales the transmission power of MeNBs. This parameter not only optimizes the transmission power of the MeNBs and brings the energy savings to the system, but also adjusts the interference created by a particular MeNB. The second variable ε is the ratio of the transmission power of cell-edge and cell-center subcarriers. This parameter is introduced to adjust the power levels between cell-edge and cell-center MUEs. The following equation characterizes the transmissions of an MeNB in Sector 1 as

$$\beta P_{\max,M} = P_t N_A + \varepsilon P_t N_B \Leftrightarrow P_t = \frac{\beta P_{\max,M}}{N_A + \varepsilon N_B}, \quad (2)$$

where $P_{\max,M}$ is the maximum transmit power of an MeNB. The numbers N_A and N_B are the number of subcarriers on Subbands A and B, respectively. The P_t and εP_t is the transmission power of the MeNBs in the cell-center region and cell-edge, respectively. We can write a similar expression for the other sectors by replacing N_B with the number of subcarriers of Subband C, denoted by N_C, and Subband D, denoted by N_D. Similarly, the transmission power of the cell-center and cell-edge pico-eNBs in Sector 1, denoted by $P_{t,p}^C$

Table I
BASE STATION POWER CONSUMPTION MODEL PARAMETER VALUES [14]

Base Station Type	P_0 (W)	P_{sleep} (W)	$P_{\max}$ (W)	Δ
MeNB	130	75.0	20	4.7
Pico-eNB	56	39.0	6.3	2.6

and $P_{t,p}^{E}$, can be written as

$$P_{t,p}^{C} = \frac{P_{\max,P}}{N_C + N_D}, \quad P_{t,p}^{E} = \frac{P_{\max,P}}{N_A + N_C + N_D}. \tag{3}$$

For the pico-eNBs in other sectors, the transmission power can be calculated similarly, by replacing N_C and N_D with corresponding symbols.

A. Base Station Power Consumption Model

The power consumption of a base station includes the contribution of the several components such as power amplifier, radio frequency (RF) transceiver parts, baseband unit, power supply, and cooling devices [14]. In the literature, several models have been proposed to model the power consumption of base stations, see, e.g., [14]–[16]. In order to evaluate the power control gains, the model must quantify the contributions of the static power consumption and the load-dependent transmission separately. Therefore, we employ the model in [14] in this paper. The total power consumption of the MeNB M and the pico-eNB P can be written as

$$\begin{aligned} P_{Macro,M} =& N_{TRX,M}\left(P_{0,M} + \Delta_M P_{TX,M}\right) \\ P_{Pico,P} =& N_{TRX,P}\left(P_{0,P} + \Delta_P P_{TX,P}\right), \end{aligned} \tag{4}$$

where $P_{0,M}$ and $P_{0,P}$ are the static power consumption of MeNB M and pico-eNB P, respectively. The transmission power of MeNB M and pico-eNB P are $P_{TX,M}$ and $P_{TX,P}$, respectively. The numbers $N_{TX,M}$ and $N_{TX,P}$ are the number of transmit antennas at the MeNB M and pico-eNB P, respectively. The slopes of the power consumption depending on the transmission power of MeNB M and pico-eNB P are denoted by Δ_M and Δ_P, respectively. Note that, due to the fact that pico-eNBs always transmit at the full power, $P_{TX,P}$ will be equal to $P_{\max,P}$ for all pico-eNBs, whereas $P_{TX,M}$ will be set between 0 and $P_{\max,M}$. The total energy consumption in Sector s can be calculated as

$$\psi_s(\beta_s) = P_{Macro,M} + \sum_{P \in \mathcal{N}_{\text{pico},s}} P_{Pico,P}, \tag{5}$$

where $\mathcal{N}_{\text{pico},s}$ is the set of pico-eNBs in Sector s. The corresponding values for the power consumption model are presented in Table I.

B. Energy Efficiency Definition

Let $R_k^{(n)}$ denote the throughput of user k on subcarrier n, which can be expressed as

$$R_k^{(n)} = \Delta_f \log_2\left(1 + \gamma_k^{(n)}\right). \tag{6}$$

Then, the energy efficiency of Sector s can be calculated as

$$\eta_s(\boldsymbol{\varepsilon}, \boldsymbol{\beta}) = \frac{\sum\limits_{k \in \mathcal{K}_{m,s}} \sum\limits_{n \in \mathcal{N}_{M,k}} R_k^{(n)} + \sum\limits_{k \in \mathcal{K}_{p,s}} \sum\limits_{n \in \mathcal{N}_{P,k}} R_k^{(n)}}{\psi_s(\beta_s)}, \tag{7}$$

where the vectors $\boldsymbol{\varepsilon}$ and $\boldsymbol{\beta}$ are the optimization variables for all sectors in the network. The sets $\mathcal{K}_{m,s}$ and $\mathcal{K}_{p,s}$ are the sets of MUEs and PUEs in Sector s, respectively. The sets $\mathcal{N}_{M,k}$ and $\mathcal{N}_{P,k}$ are the sets of the subcarriers assigned to the MUE k and PUE k, respectively.

III. PROBLEM FORMULATION AND PROPOSED ALGORITHM

Two important aspects of resource allocation are the interference management and the QoS constraints of the users. In this paper, both of these aspects are addressed. For interference management, we employ both the FFR scheme and propose an efficient power control algorithm maximizing the energy efficiency term in (7). The QoS requirements impose the minimum rate constraints for guaranteed bit-rate (GBR) applications. These are included in the problem formulation in (8). In this paper, our objective is to maximize the aggregate sector energy efficiency while satisfying the minimum rate constraints of each MUE. In addition to the minimum rate constraints, we add two more constraints to this problem. The first constraint forces ε to be larger than 0. The second constraint ensures that the total transmission power of MeNBs is in certain limits. We formulate this problem as follows:

$$\begin{aligned} \mathbf{P}: \quad \max_{\mathbf{x}} \quad & \sum_{s \in \mathcal{S}} \eta_s(\mathbf{x}) \\ & R_k(\mathbf{x}) \geq R_{\min,k}, \quad \text{for all } k \in \mathcal{K}_m \\ & \boldsymbol{\varepsilon} \succeq \mathbf{0} \text{ and } \mathbf{0} \preceq \boldsymbol{\beta} \preceq \mathbf{1}, \end{aligned} \tag{8}$$

where $\mathcal{S}$ is the set of all sectors in the network and

$$R_k(\mathbf{x}) = \sum_{n \in \mathcal{N}_{M,k}} R_k^{(n)}. \tag{9}$$

For simplicity, we will use the vector $\mathbf{x}_s$ for the (ε_s, β_s) couple and the vector $\mathbf{x}$ for the vector couple $(\boldsymbol{\varepsilon}, \boldsymbol{\beta})$ throughout the rest of the paper.

The Lagrangian of multi-cell multi-channel energy efficiency maximization problem can be written as

$$\mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}) = \sum_{s \in \mathcal{S}} \eta(\mathbf{x}) - \sum_{k \in \mathcal{K}_m} \lambda_k c_k + \sum_{s \in \mathcal{S}} \mathcal{F}_s(\mathbf{x}_s, \nu_s, \tau_s, \rho_s), \tag{10}$$

where λ_k denotes the Lagrange multipliers associated with the QoS constraint and c_k is equal to $(R_{\min,k} - R_k(\mathbf{x}))$. The function $\mathcal{F}_s(\mathbf{x}_s, \nu_s, \tau_s, \rho_s)$ denotes the Lagrange multipliers for the boundary conditions of $\mathbf{x}_s$ which is given by

$$\mathcal{F}_s(\mathbf{x}_s, \nu_s, \tau_s, \rho_s) = \nu_s \beta_s + \tau_s(1 - \beta_s) + \rho_s \varepsilon_s, \tag{11}$$

where ν_s and τ_s are the Lagrange multipliers associated with the lower and upper bounds of β_s, respectively, and ρ_s denotes the one corresponding to the lower bound of ε_s. Then, the

KKT conditions of Problem $\mathbf{P}$ can be written as

$$\nabla_{\mathbf{x}_s}\mathcal{L}(\mathbf{x}^*,\boldsymbol{\lambda}^*) =\nabla_{\mathbf{x}_s}\eta_s(\mathbf{x}^*) + \sum_{k\in\mathcal{K}_{m,s}} \lambda_k^*\nabla_{\mathbf{x}_s}R_k(\mathbf{x}^*) + \sum_{\substack{s'\in\mathcal{S}\\ s'\neq s}}\left(\nabla_{\mathbf{x}_s}\eta_{s'}(\mathbf{x}^*) + \sum_{k\in\mathcal{K}_{m,s'}}\lambda_k^*\nabla_{\mathbf{x}_s}R_k(\mathbf{x}^*)\right) + \nabla_{\mathbf{x}_s}\mathcal{F}_s(\mathbf{x}_s^*,\nu_s^*,\tau_s^*,\rho_s^*) = \mathbf{0}, \forall s\in\mathcal{S} \tag{12a}$$

$$\lambda_k^*\left(R_{\min,k} - R_k(\mathbf{x}^*)\right) = 0 \text{ and } \lambda_k \geq 0,\ \forall k\in\mathcal{K}_m \tag{12b}$$

$$\rho_s^*\varepsilon_s^* = 0,\ \nu_s^*\beta_s^* = 0,\ \tau_s^*(1-\beta_s^*) = 0,\ \forall s\in\mathcal{S} \tag{12c}$$

$$\rho_s^* \geq 0,\ \nu_s^*\geq 0,\ \tau_s^*\geq 0,\ \forall s\in\mathcal{S} \tag{12d}$$

Note that $\mathbf{x}^*$ is the optimal solution of the primal problem and ($\boldsymbol{\lambda}^*$, $\boldsymbol{\rho}^*$, $\boldsymbol{\nu}^*$, $\boldsymbol{\tau}^*$) denotes the one for the dual problem. Equation (10) is called as the dual feasibility condition. It identifies the condition where the gradient of the Lagrangian in (10) vanishes at $\mathbf{x}=\mathbf{x}^*$.

The objective function of problem $\mathbf{P}$ is non-convex over the transmission power levels of MeNBs. The solution can be obtained by exhaustive search over all power levels of MeNBs. Instead of solving this problem, we divide this problem into $|S|$ subproblems and solve them simultaneously in each time instant and continue this process until convergence. The number $|S|$ corresponds to number of sectors in the network. Each sector maximizes its own energy efficiency while satisfying the minimum rate constraints of its users. As stated earlier, in order to prevent MeNBs from increasing their transmission powers imprudently, an interference pricing mechanism is introduced. In this mechanism, each MeNB is penalized proportionally to interference it creates. In cases where the minimum rate constraint of user k is not satisfied, its Lagrange multiplier λ_k is increased. In [4], it is shown that the energy efficiency of Sector s is quasiconcave over ε_s and β_s, keeping the other power levels constant.

Under the assumption that interference conditions remain constant, we solve the following problem in each sector:

$$\begin{aligned} \max_{\mathbf{x}_s}\quad & \eta_s(\mathbf{x}_s) - \theta_s(\mathbf{x}_s) \\ \text{s.t.}\quad & R_k(\mathbf{x}_s) \geq R_{\min,k}, \quad \text{for all } k\in\mathcal{K}_{m,s} \\ & \varepsilon_s \geq 0 \text{ and } 0\leq\beta_s\leq 1, \end{aligned} \tag{13}$$

where $\theta_s(\mathbf{x}_s)$ denotes the interference pricing function which accounts for the interference that MeNB M creates of Sector s. This interference pricing function can be expressed as

$$\theta_s(\mathbf{x}_s) = \mathbf{x}_s^T\sum_{\substack{s'\in\mathcal{S}\\ s'\neq s}}\left(\nabla_{\mathbf{x}_s}\eta_{s'}(\mathbf{x}) + \sum_{k\in\mathcal{K}_{m,s'}}\lambda_k\nabla_{\mathbf{x}_s}R_k(\mathbf{x})\right). \tag{14}$$

This pricing function, $\theta_s(\mathbf{x}_s)$, captures the aggregate marginal decrease in the energy efficiency of the other sectors. Note that the terms in the summation are assumed to be constant during each time instant. The same assumption is made in [9]–[11]. The first constraint in (13) enforces the minimum rate requirements of users associated with Sector s to be satisfied. When we consider the minimum rate constraints due to the different QoS levels of users, the formulation in (13) enables us to take into account the users with strict GBR requirements as well as the best effort users by assigning a low rate requirement. The second constraint in (13) corresponds to the boundary conditions of the power control parameters, ε_s and β_s.

Thus, the Lagrangian of the problem in (13) can be written as

$$\mathcal{L}(\mathbf{x}_s,\boldsymbol{\lambda}) =\eta_s(\mathbf{x}_s) - \underbrace{\theta_s(\mathbf{x}_s)}_{\substack{\text{Interference}\\ \text{Prices}}} - \underbrace{\sum_{k\in\mathcal{K}_s}\lambda_k c_k}_{\text{Dual prices}} + \mathcal{F}_s(\mathbf{x}_s,\nu_s,\tau_s,\rho_s). \tag{15}$$

Note that the KKT conditions of the problems (8) and (13) are equivalent. Therefore, any optimal solution satisfying the KKT conditions of (13) is at least a local maximum of the problem (8) [17].

A. Proposed Solution

We first set the cell-center radius which identifies the cell center-region boundaries. Second we assign the RBs to users. Then, we determine the power levels of each subband for MeNBs that maximize the energy efficiency and satisfy minimum rate requirement of each MUEs simultaneously.

1) Setting Cell Center Radius: The cell-center radius is an important design parameter for multi-tier networks that determines the region of users in the network. In this paper, we use the cell-center selection algorithm which is described in [4]. This algorithm sets the cell-center radius so that the number of users in the cell-center and cell-edge region will be proportional with the ratio of the number of RBs assigned to these regions. This algorithm evenly distributes the resources between MUEs.

2) Frequency Assignment: In this paper, we solve the frequency assignment problem by adopting the scheduler that has been described in [18]. The scheduler in [18] is proposed to maximize the minimum throughput of the users with constant power allocations. By maximizing the minimum throughput, MeNBs can reduce their transmission levels even more while satisfying minimum rate constraints. This scheduler first assigns a subcarrier to a user with the best channel, then removes the user and the carrier from the set. The algorithm continues until either the user or the subcarrier set is empty. If there are more subcarriers than the number of users, then the scheduler assigns remaining subcarriers to users by starting from the user whose data rate is the lowest. This process continues until all subcarriers are assigned to a user. In our adopted version, due to the fact that the smallest granularity in LTE standards is per RB which contains 12 subcarriers, we run this scheduler over the RBs instead of subcarriers. Note that the cell-center and cell-edge MUEs associated with the same MeNB are scheduled separately during the scheduling process. The reason is that these users do not compete for the same

RBs. Therefore, each MeNB runs this algorithm twice, one for the cell-center users and one for the cell-edge users.

3) Power Allocation: In order to calculate power allocations, first each MeNB should calculate the interference prices and distribute these values to other MeNBs through the X2 interface. In this paper, we adopt the same approach that has been described in [9] to calculate interference pricing. The difference is that, in this paper, the transmission powers of the MeNBs are dependent on two parameters: ε and β. Therefore, the interference prices are calculated over these parameters. We solve the problem in (13) using the Levenberg-Marquardt method which has quadratic rate of convergence. At iteration l, we update the power control parameters including the minimum rate constraints as

$$\mathbf{x}_s^{(l+1)} = \mathbf{x}_s^{(l)} - \mu_s^{(l)} \left(\nabla^2_{\mathbf{x}} \mathcal{L}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)}) - \varsigma \mathbf{I}\right)^{-1} \nabla_{\mathbf{x}} \mathcal{L}\left(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)}\right), \tag{16}$$

where $\mathbf{x}_s^{(l)}$ and $\mathbf{x}_s^{(l+1)}$ denote the power control parameters at iterations l and $l+1$, respectively. The gradient and the Hessian of the Lagrangian function in (16) are given by $\nabla_{\mathbf{x}} \mathcal{L}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)})$ and $\nabla^2_{\mathbf{x}} \mathcal{L}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)})$, respectively. The step size, $\mu_s^{(l)}$, is determined by the Armijo rule which guarantees monotonic increase in the utility [17]. The identity matrix with dimension 2×2 is denoted by $\mathbf{I}$. The constant offset, ς, is to enforce the Hessian to be negative definite. The Lagrange multiplier corresponding to the minimum rate constraints of user k at iteration l is updated by

$$\lambda_k^{(l+1)} = \left[\lambda_k^{(l)} + \alpha_k c_k^{(l)}\right]^+, \tag{17}$$

where α_k is a sufficiently small step-size and the operator $[x]^+$ is $\max(0, x)$. If the throughput of user k at iteration l, $R_k(\mathbf{x}_s^{(l)})$, is less than $R_{\min,k}$, that is $c_k^{(l)}$ is negative, the corresponding Lagrange multiplier is increased. If the throughput of user is greater or equal to $R_{\min,k}$, then the Lagrange multiplier is reduced or remains unchanged for the next iteration.

The proposed power control algorithm with the minimum rate constraints is summarized under the heading Algorithm 1. The operator eig($\mathbf{X}$) finds the eigenvalues of the matrix $\mathbf{X}$ and δ is a small offset parameter. The proposed algorithm consists of successive power control parameter and dual updates. To update the power control parameters, we employ the Lagrangian penalty function. As this function is differentiable, it enables us to use derivative-based line search methods. With the Levenberg-Marquardt method which has a quadratic convergence rate, the power control parameters are updated using both the gradient and the Hessian of the Lagrangian penalty function. We first take steps towards the primal problem maximum while keeping the dual variables constant. In the next step, we update the dual solution for a given optimal primal solution. This process is called as dual decomposition, see, [19, Chapter 4], [20], and [17]. There are several stopping criteria proposed in the literature to terminate the derivative-based line search algorithms, see, e.g., [21]. These include exiting the loop if the absolute value of the gradient

Algorithm 1 Distributed Network Energy Efficiency Maximization with Minimum Rate Constraints

1: **Initialize:** $\mathbf{x}_s^{(0)} = (\varepsilon_0 \;\; \beta_0)^T$, $\boldsymbol{\lambda}_s^{(0)} = \mathbf{0}$,
2: *Power Update:* At time t, each sector solves (13) using the following steps:
3: **for** $l := 1$ to $l_{\max}$ **do**
4:     **if** $\lambda_{\max} = \max(\text{eig}(\nabla^2_{\mathbf{x}} \mathcal{L}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)}))) < 0$ **then**
5:         $\varsigma = 0$.
6:     **else**
7:         $\varsigma = \lambda_{\max} + \delta$.
8:     **end if**
9:     $\mathbf{D}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)}) = (\nabla^2_{\mathbf{x}} \mathcal{L}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)}) - \varsigma \mathbf{I})^{-1}$.
10:     Update the power control parameters, $\mathbf{x}_s^{(l+1)}$, using
$$\mathbf{x}_s^{(l+1)} = \mathbf{x}_s^{(l)} - \mu_s^{(l)} \mathbf{D}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)}) \nabla \mathcal{L}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)})$$
11:     Update the Lagrange multiplier, $\boldsymbol{\lambda}_s^{(l+1)}$, using $\lambda_k^{(l+1)} = \left[\lambda_k^{(l)} + \alpha_k c_k^{(l)}\right]^+$.
12:     **if** $\left|\nabla \mathcal{L}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)})^T \mathbf{D}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)}) \nabla \mathcal{L}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)})\right| \leq \epsilon$ **then**
13:         **Break**
14:     **end if**
15: **end for**
16: *Price Update:* Each user calculates interference prices and feds these values back to its base station.
17: Interference prices are distributed among base stations.
18: Go to Step 2 and repeat for $t \leftarrow t+1$.

falls below a predetermined threshold or if the normalized increase in the utility function cannot be improved more than a threshold. The criterion in Step 12 uses the negative-definiteness of the Lagrangian function. If the norm of the expression $\nabla \boldsymbol{\eta}_s(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)})^T \mathbf{D}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)}) \nabla \boldsymbol{\eta}_s^{(l)}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)})$ is below a sufficiently small threshold denoted by ϵ, then the stopping criterion is satisfied and the algorithm terminates. In our simulations, we take $\epsilon = 10^{-4}$. Note that the expression $\sqrt{\nabla \boldsymbol{\eta}_s^{(l)}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)})^T \mathbf{D}(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)}) \nabla \boldsymbol{\eta}_s(\mathbf{x}_s^{(l)}, \boldsymbol{\lambda}_s^{(l)})}$ is also referred to as the Newton increment [21].

Our approach to introduce the minimum rate constraints to the energy efficiency maximization problem can establish the fairness in the system such that each user can meet its minimum rate constraints. In particular, this approach is beneficial in applications which have strict GBR requirements. For example, several applications have been identified in LTE systems such as Voice-over-IP, video call, streaming, and real-time gaming applications with strict QoS requirements [22]. Notice that imposing GBR requirements comes at the expense of a reduction in the energy efficiency. It is important to characterize this loss. Using the optimization tools, we characterize this loss as the following

$$\eta_s^*(\mathbf{0}) - \eta_s^*(\mathbf{R}_{\min}) \leq \boldsymbol{\lambda}^T \mathbf{R}_{\min}, \tag{18}$$

where $\eta_s^*(\mathbf{0})$ denotes the optimal energy efficiency solution without the minimum rate constraints. Note that the energy efficiency loss imposed by the minimum capacity constraints,

Table II
SIMULATION PARAMETERS

Parameter	Setting
Channel bandwidth	10 MHz
Total number of RBs	50 RBs
Freq. selective channel model (CM)	Extended Typical Urban CM
UE to MeNB PL model	$128.1 + 37.6 \log_{10}(d)$
UE to pico-eNB PL model	$140.7 + 36.7 \log_{10}(d)$
Effective thermal noise power, N_0	-174 dBm/Hz
UE noise figure	9 dB
MeNB and pico-eNB antenna gain	14 dBi and 5 dBi
UE antenna gain	0 dBi
Antenna horizontal pattern, $A(\theta)$	$-\min(12(\theta/\theta_{3dB})^2, A_m)$
Penetration loss, A_m and θ_{3dB}	20 dB, 20 dB, and 70°
Macro- and picocell shadowing	8 dB and 10 dB
Inter-site distance	500 m
Traffic model	Full buffer

that is $\eta_s^*(\mathbf{0}) - \eta_s^*(\mathbf{R}_{\min})$, is at most $\boldsymbol{\lambda}^T \mathbf{R}_{\min}$. Each Lagrange multiplier λ_k, is analogous to the dual price interpretation [23], [24]. These multipliers represent the energy efficiency loss for a unit increase in $\mathbf{R}_{\min}$. If user k exceeds its minimum rate constraint, then the corresponding Lagrange multiplier, λ_k needs to be zero to satisfy the complementary slackness condition. If the minimum rate requirement is not satisfied, then λ_k is updated using (17).

IV. NUMERICAL RESULTS

In this section, we study the performance of the proposed algorithm. First, we present the simulation scenario and then discuss the simulation results.

We consider a 19-cell uniform hexagonal grid layout that is depicted in Fig. 1. Three-sector antennas are employed at MeNBs and omnidirectional antennas at the pico-eNBs. In each sector, we generate 30 users and two pico-eNBs. To observe clustering effects, we consider Configuration 4a of [25]. This scenario assumes that two users per pico-eNB are randomly generated within an initial radius of 40 meters and the remaining users are uniformly distributed within the sector. The simulation models and parameters are presented in Table II, which follow the standards in [25] for baseline simulations for HetNets. We employ the wrap-around technique to mitigate boundary effects. The full-buffer traffic model captures continuous traffic and non-varying interference characteristics [25]. In the FFR scheme, we consider $N_A = 20$, $N_B = 10$, $N_C = 10$, and $N_D = 10$. For simplicity, we consider the same target rate for all users, that is $R_{\min,k} = R$, $\forall k \in \mathcal{K}_m$. However, in real applications, different traffic mixtures of GBR and best-effort (BE) users are also possible [22]. In such cases, the proposed algorithm can still be employed by assigning respective GBR requirements in $\mathbf{R}_{\min}$. We consider strict GBR constraints of 64, 128, and 256 kbits/sec (kbps) as in [22]. In the proposed algorithm, dual prices are updated right after each power control iteration and interference prices are distributed at the end of each time instant. Simulations ran for ten time instants. We compare the performance of the proposed algorithm with the non-cooperative resource allocation scheme which solves (13) without any interference price exchange. These two schemes are denoted by *Interference Pricing* and *Non-cooperative*, respectively.

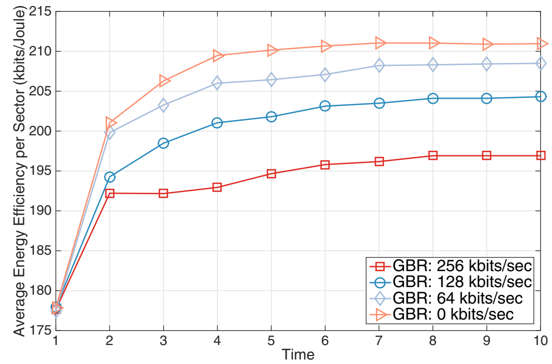

Figure 2. The average energy efficiency per sector for various GBR requirements of MUEs.

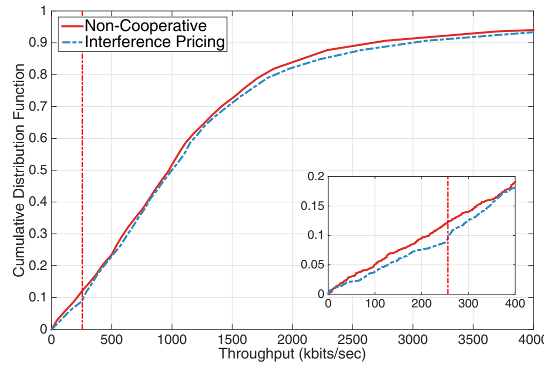

Figure 3. The cumulative distribution function of user rates at the first and tenth time instants for GBR requirement of 256 kbps. The inset shows the region where the user rates are below 400 kbps.

Fig. 2 illustrates the performance of the proposed algorithm for strict GBR requirements of MUEs ranging from 64 kpbs up to 256 kbps. We observe that as we enforce more strict GBR requirements, the average energy efficiency per sector decreases more to satisfy the rate constraints. As we discussed in Section III, this loss is upper bounded by (18). The energy efficiency difference between the first and tenth time instants demonstrates the interference pricing gain. If there are no GBR requirements (GBR = 0 kbps), interference pricing provides 18.6% gain in energy efficiency in Fig. 2. Notice that both of these schemes compared employ the proposed power control algorithm. The only difference is the interference pricing distribution. For the GBR requirements of 64, 128, and 256 kbps, the interference pricing gains are 17.4%, 14.8%, and 10.8%, respectively.

Fig. 3 presents the cumulative distribution function of the user rates at the first and tenth time instants comparing the interference pricing and non-cooperative schemes. We observe that, compared to the non-cooperative scheme, the percentage of users that do not satisfy the GBR requirements is decreased with the interference pricing. The inset shows that the outage is reduced from 12.2% to 9.1%. We need to emphasize that along

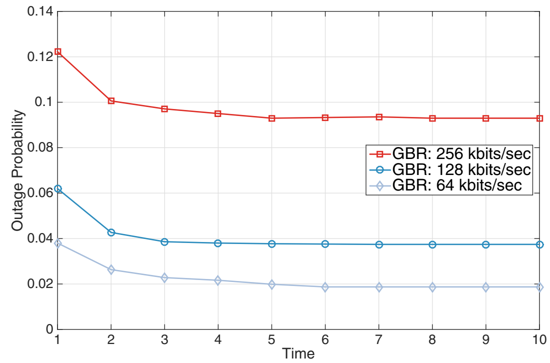

Figure 4. The outage probability of users for different GBR requirements.

with this improvement, the proposed algorithm increases the sum rate by 12%, 10.35%, and 7% for the GBR requirements of 64, 128, and 256 kbps, respectively.

In cases where the user rate requirements are not satisfied, user outages occur. In Fig. 4, we study the outage probability of users for different GBR requirements. As expected, the outage probability increases with the increase in the GBR requirement. We observe that as interference prices are distributed and dual prices are updated, the outage probability significantly reduces. For example, when we compare the proposed algorithm with the non-cooperative scheme for the GBR requirements of 128 and 64 kbps, the outage probabilities decrease from 6.2% to 3.7% and 3.8% to 1.8%, respectively, in ten time instants.

V. Conclusion

An energy-efficient resource allocation problem with QoS constraints for HetNets is investigated in this paper. To increase energy efficiency, power control is employed at MeNBs. The minimum rate constraints of MUEs are included by using the Lagrange relaxation method. To reduce the effect of the inter-sector interference, the FFR scheme and interference pricing mechanism are employed. The interference pricing also limits the selfish transmission power increases of the MeNBs. The simulation results demonstrate that the proposed algorithm significantly increases the energy efficiency of the network while satisfying the minimum rate constraints of users.

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