A Generalized Framework on Beamformer Design and CSI Acquisition for Single-Carrier Massive MIMO Systems in Millimeter Wave Channels

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***Abstract*—In this paper, we establish a general framework on the reduced dimensional channel state information (CSI) estimation and pre-beamformer design for frequency-selective massive multiple-input multiple-output (MIMO) systems employing single-carrier (SC) modulation in time division duplex (TDD) mode by exploiting the joint angle-delay domain channel sparsity in millimeter (mm) wave frequencies. First, based on a generic subspace projection taking the joint angle-delay power profile and user-grouping into account, the reduced-rank minimum mean square error (RR-MMSE) instantaneous CSI estimator is derived for spatially correlated wideband MIMO channels. Second, the statistical pre-beamformer design is considered for frequency-selective SC massive MIMO channels. We examine the dimension reduction problem and subspace (beamspace) construction on which the RR-MMSE estimation can be realized as accurately as possible. It is observed that the proposed techniques show remarkable robustness to the pilot interference (or contamination) with a significant reduction in pilot overhead.**

I. Introduction

It is anticipated that massive MIMO systems in the mm wave range form an important part of 5G systems expected to support much larger (e.g., 1000 times faster) data rates than the currently deployed standards [1]. In practice, CSI is typically obtained with the assistance of the periodically inserted pilot signals [2]. This brings the pilot overhead consumed by the training data to be proportional to the number of active users in the system for uplink training, and the number of BS antennas for downlink training respectively [3]. We assume CSI at the BS can be acquired by means of uplink training in time division duplex (TDD) mode, where the uplink pilots provide the BS with downlink as well as uplink channel estimates simultaneously via leveraging channel reciprocity [2], [3].

The processing of the signals with very large dimensionality, the pilot interference, and the pilot overhead are thought to be limiting factors for an accurate channel acquisition and throughput of massive MIMO transmission in mm wave especially in high mobility or in applications requiring low latency and short-packet duration. Recently, the *two-stage beamforming* concept under the name of Joint Spatial Division and Multiplexing (JSDM) [4], [5] has been proposed to reduce the dimension of the MIMO channel effectively, and to enable massive MIMO gains and simplified system operations [6]. Even though JSDM is suggested as an effective reduced-complexity two-stage downlink precoding scheme for flat-fading multi-user MIMO systems in frequency division duplex (FDD) mode initially, the idea of *two-stage beamforming* (in [4]–[6]) can be applied to both downlink and uplink transmission in TDD. The key idea lies in *user-grouping*, i.e., partitioning the user population supported by the serving BS into multiple groups each with approximately the same channel covariance eigenspaces. Then, one can decompose the MIMO beamformer at the BS into two steps via the use of a spatial *pre-beamformer*, which distinguishes *intra-group* signals from other groups by suppressing the *inter-group interference* while reducing the signaling dimension. The major complexity reduction comes from the approach that the *pre-beamformer* is properly designed based only on the second-order statistics of the channel, and not on the instantaneous CSI (which varies on a much higher rate).

The wideband massive MIMO channel is expected to be sparse both in the angle and time (delay) domains. The *channel sparsity* [7], [8], which becomes particularly relevant at mm wave frequencies, is observed in practical cellular systems, where the channels are often characterized with limited scattering and hence correlated in the spatial domain; the BS sees the incoming multi-path components (MPCs) under a very constrained angular range, i.e., angle of arrival (AoA) support, and the MPCs occur in clusters in the *angle-delay* plane [7].

In this paper, a general framework on the reduced-dimensional CSI estimation and the *pre-beamformer* design based on the statistical *user-grouping* idea (used in the JSDM scheme) is established for frequency-selective massive MIMO systems employing single-carrier (SC) modulation in TDD mode. The statistical pre-beamformer was designed to reduce dimensionality and pilot overhead while mitigating *inter-group interference* leading to pilot contamination by exploiting the *channel sparsity* indicated by the joint *angle-delay* domain power profile.

II. System Model

We consider a cellular system based on massive MIMO transmission operating at mm wave bands in the TDD mode employing SC in which a BS, having N antennas, serves K single-antenna UTs. In order to reduce the overhead while acquiring the instantaneous CSI associated with massive MIMO, *two-stage beamforming* [4]–[6] is adopted throughout this paper. It is assumed that K users are partitioned into G groups,

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where the K_g users in group g have statistically independent but identically distributed (*i.i.d.*) channels [4], [5].

At the beginning of every coherence interval, all users of the intended group g transmit training sequences with length T. We assume a linear modulation (e.g., PSK or QAM) and a transmission over frequency-selective channel for all UTs with a slow evolution in time relative to the signaling interval (symbol duration). Under such conditions, the baseband equivalent received signal samples, taken at symbol rate (W) after pulse matched filtering, are expressed as[1]

$$\mathbf{y}_n = \underbrace{\sum_{\{k=1,\ g_k\in\Omega_g\}}^{K_g} \sum_{l=0}^{L_g-1} \mathbf{h}_l^{(g_k)} x_{n-l}^{(g_k)}}_{\text{Intra-Group Signal}} + \underbrace{\sum_{\{\forall g'_k\in\Omega_{g'}\ |g'\neq g\}} \left(\sum_{k=1}^{K_{g'}} \sum_{l=0}^{L_{g'}-1} \mathbf{h}_l^{(g'_k)} x_{n-l}^{(g'_k)} \right) + \mathbf{n}_n}_{\boldsymbol{\eta}_n^{(g)}:\text{Inter-Group Interference + AWGN}} \quad (1)$$

for $n = 0, \ldots, T-1$, where $\mathbf{h}_l^{(g_k)}$ is $N \times 1$ multi-path channel vector, namely, the array impulse response of the serving BS stemming from the l^{th} multi-path component (MPC) of k^{th} user in group g. Here, $\left\{x_n^{(g_k)};\ -L_g+1 \leq n \leq T-1\right\}$ are the training symbols for the k^{th} user in group g[2], L_g is the channel memory of group g multi-path channels, Ω_g is the set of all UTs belonging to group g with cardinality $|\Omega_g| = K_g$, and $\{g_k\}_{k=1}^{K_g}$ are UT indices forming Ω_g. The $L_g - 1$ symbols at the start of the preamble, prior to the first observation at $n = 0$, are the precursors. Training symbols are selected from a signal constellation $S \in \mathbb{C}$ and $\mathbb{E}\left\{|x_n^{(g_k)}|^2\right\}$ is set to E_s for all g_k. In (1), $\mathbf{n}_n$ are the additive white Gaussian noise (AWGN) vectors during uplink pilot segment with spatially and temporarily *i.i.d.* as $\mathcal{CN}\left(\mathbf{0}, N_0\mathbf{I}_N\right)$, and N_0 is the noise power. The first term of (1) is the transmitted signal of the intended group g, named as the *intra-group* signal of group g users. The second term, $\boldsymbol{\eta}_n^{(g)}$, namely the *inter-group interference*, comprises of all the interfering signals, which stem from all inner or outer cell users belonging to different groups other than g. In (1), we assume users come in groups, either by nature or by the application of proper *user grouping* algorithms in [5], [7], which are out of scope of this work. Finally, the average received signal-to-noise ratio (*snr*) can be defined as $snr \triangleq \frac{E_s}{N_0}$[3].

A. Fundamental Assumptions on Signal and Channel Model

Each resolvable MPC of the users, belonging to any group g, is assumed to span some particular angular sector in azimuth-elevation plane, capturing local scattering around the corresponding UTs' angle of arrival (AoA) (with respect to the BS). Then, their corresponding cross-covariance matrices can be expressed in the form of

$$\mathbb{E}\left\{\mathbf{h}_l^{(g_k)} \left(\mathbf{h}_{l'}^{(g'_{k'})}\right)^H\right\} = \rho_l^{(g)} \mathbf{R}_l^{(g)} \delta_{gg'}\delta_{kk'}\delta_{ll'}, \quad (2)$$

for $\sum_{l=0}^{L_g-1} \rho_l^{(g)} = 1$ and $\mathrm{Tr}\left\{\mathbf{R}_l^{(g)}\right\} = 1$ by using the uncorrelated local scattering model where all MPCs are assumed to be mutually independent according to the well-known wide sense stationary uncorrelated scattering (WSSUS) model [7], [8], the multi-path channel vectors are uncorrelated with respect to l, and also mutually uncorrelated with that of the different users (independent of whether in the same group or not). In (2), $\rho_l^{(g)}$ is the power delay profile (pdp) of the group g multi-path channels, showing the average channel strength at each delay, and the auto-covariance of each MPCs in group g is given by

$$\mathbf{R}_l^{(g)} = \mathbf{U}_l^{(g)} \boldsymbol{\Lambda}_l^{(g)} \left(\mathbf{U}_l^{(g)}\right)^H,\ l = 0, \ldots, L_g - 1, \quad (3)$$

where $\mathbf{U}_l^{(g)}$ is the $N \times r_{g,l}$ matrix of the eigenvectors corresponding to the $r_{g,l}$ non-zero or dominant eigenvalues of $\mathbf{R}_l^{(g)}$, given as the diagonal elements of the diagonal $r_{g,l} \times r_{g,l}$ matrix $\boldsymbol{\Lambda}_l^{(g)}$ in (3). In (2), $\mathbf{R}_l^{(g)}$ can be considered as the common spatial covariance matrix of group g UTs at l^{th} delay. In this model, each antenna element at a BS is assumed to see the incoming MPCs at the same common support on the *angle-delay* plane (similar to the one in [7]). Also, the effective rank of $\mathbf{R}_l^{(g)}$, namely, $r_{g,l}$ is expected to be much smaller than the number of array elements, N, due to the *channel sparsity* pronounced at mm wave [7], [9].

When Rayleigh-correlated channel coefficients are assumed such as $\mathbf{h}_l^{(g_k)} \sim \mathcal{CN}\left(\mathbf{0}, \rho_l^{(g)}\mathbf{R}_l^{(g)}\right)$, mutually independent across the users for all g_k, the *Karhunen-Loeve* representation [10] of the multi-path channel vector belonging to the k^{th} user in group g is given as the following by using (3)

$$\mathbf{h}_l^{(g_k)} = \left(\rho_l^{(g)}\right)^{1/2} \mathbf{U}_l^{(g)} \left(\boldsymbol{\Lambda}_l^{(g)}\right)^{1/2} \mathbf{c}_l^{(g_k)},\ l = 0, \ldots, L_g - 1, \quad (4)$$

where the entries of $\mathbf{c}_l^{(g_k)} \in \mathbb{C}^{r_{g,l}\times 1} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_{r_{g,l}}\right)$.

The channel auto-covariance of each group in (3) is slowly varying in time as the AoA of each user signal evolves depending on the user mobility, variation rate of the scattering environment characteristics, etc. [7], [11], [12]. Thus, their rate of change is significantly lower than that of the small-scale fading (instantaneous CSI), and they can be estimated with guaranteed accuracy for all intended groups in practice.

Spatio-temporal covariance matrix of the *inter-group interference* in (1) can be calculated by taking *long-term* expectation over all MPCs $\mathbf{h}_{l'}^{(g'_{k'})}$s other than the ones belonging to group (g) in spatial domain, and transmitted symbols $x_{n'}^{(g'_{k'})}$s in temporal domain. Considering the mutual independence across multi-path channel vectors (due to the WSSUS model) given by (2), and considering that the transmitted symbols of different users are uncorrelated (including the data transmission

[1] Only the UTs, belonging to same group, are assumed to be synchronized for coherent uplink SC transmission.

[2] Training sequences are assumed to be non-orthogonal for synchronized *intra-group* users for SC transmission in general.

[3] It shows the maximum achievable *snr* after beamforming when the beam is steered towards a point, i.e., angular location by assuming that the channel is normalized so that $\frac{1}{N}\frac{E_s}{N_0}$ can be seen as the average received *snr* at each antenna element before beamforming.

period), i.e., $\mathbb{E}\left\{x_n^{(g_k)}\left(x_{n'}^{(g'_{k'})}\right)^H\right\}=\gamma^{(g)}E_s\delta_{nn'}\delta_{gg'}\delta_{kk'}$, the following is obtained: $\mathbb{E}\left\{\boldsymbol{\eta}_n^{(g)}\left(\boldsymbol{\eta}_{n'}^{(g)}\right)^H\right\}=\mathbf{R}_{\boldsymbol{\eta}}^{(g)}\delta_{nn'}$, where

$$\mathbf{R}_{\boldsymbol{\eta}}^{(g)} \triangleq E_s\left(\sum_{g'\neq g}\gamma^{(g')}K_{g'}\sum_{l=0}^{L_{g'}-1}\rho_l^{(g')}\mathbf{R}_l^{(g')}\right)+N_0\mathbf{I}_N, \quad (5)$$

and $\gamma^{(g')}$ for $g'\neq g$ can be regarded as the relative average received power at BS of *inter-group* users normalized with that of the group g users. In (5), $\gamma^{(g')}$s are accountable for the *near-far* effect stemming from the fact that average received signal strength of different UTs may differ significantly depending on their distance to the BS. Moreover, It is important to note that $N\times N$ covariance matrix of the *inter-group interference* $\mathbf{R}_{\boldsymbol{\eta}}^{(g)}$ in (5) consists of all the statistical information of the CSI in spatial domain (i.e., AoA support) for all inner and outer cell users interfering with group g users.

B. Spatio-Temporal Domain Vector Definitions

Before elaborating on the details of the estimation technique, we give the following matrix and vector definitions that will be useful in the subsequent sections. First, the training matrix (or convolution matrix), comprising of the transmitted pilots with the precursors for k^{th} user in group g, is defined as $\mathbf{X}_k^{(g)} \triangleq \left[x_{i-j}^{(g_k)}\right]_{T\times L_g}$. The extended multi-path channel vector of the k^{th} user, belonging to the intended group g, and its corresponding expansion coefficients after *Karhunen-Loeve* Transform (KLT) in (4) are given as $\mathbf{f}_k^{(g)} \triangleq \left[\mathbf{h}_{i-1}^{(g_k)}\right]_{NL_g\times 1}$ and $\mathbf{b}_k^{(g)} \triangleq \left[\mathbf{c}_{i-1}^{(g_k)}\right]_{\left(\sum_{l=0}^{L_g-1}r_{g,l}\right)\times 1}$ by concatenating all MPCs of the k^{th} user in group g. Then, by using the vectors given in (1) and the previously defined ones, it will be useful to construct the following vectors that represents the whole received vector of signals at BS (in space-time domain) during training phase, the concatenated channel vector and its KLT coefficients (that include the channel parameters of all users in group g to be estimated) respectively:

$$\mathbf{y} \triangleq \mathrm{vec}\left\{\left[\begin{array}{cccc}\mathbf{y}_0 & \mathbf{y}_1 & \cdots & \mathbf{y}_{T-1}\end{array}\right]_{N\times T}\right\} \quad (6)$$

$$\mathbf{h}^{(g)} \triangleq \mathrm{vec}\left\{\left[\begin{array}{cccc}\mathbf{f}_1^{(g)} & \mathbf{f}_2^{(g)} & \cdots & \mathbf{f}_{K_g}^{(g)}\end{array}\right]_{NL_g\times K_g}\right\} \quad (7)$$

$$\mathbf{c}^{(g)} \triangleq \mathrm{vec}\left\{\left[\begin{array}{cccc}\mathbf{b}_1^{(g)} & \mathbf{b}_2^{(g)} & \cdots & \mathbf{b}_{K_g}^{(g)}\end{array}\right]_{\left(\sum_{l=0}^{L_g-1}r_{g,l}\right)\times K_g}\right\}.$$

In a similar way, the *inter-group interference* matrix with respect to group g in space-time domain can be defined as

$$\boldsymbol{\xi}^{(g)} \triangleq \mathrm{vec}\left\{\left[\begin{array}{cccc}\boldsymbol{\eta}_0^{(g)} & \boldsymbol{\eta}_1^{(g)} & \cdots & \boldsymbol{\eta}_{T-1}^{(g)}\end{array}\right]_{N\times T}\right\} \quad (8)$$

with the covariance matrix

$$\mathbf{R}_{\boldsymbol{\xi}}^{(g)} \triangleq \mathbb{E}\left\{\boldsymbol{\xi}^{(g)}\left(\boldsymbol{\xi}^{(g)}\right)^H\right\}=\mathbf{I}_T\otimes\mathbf{R}_{\boldsymbol{\eta}}^{(g)}. \quad (9)$$

Finally, the complete training matrix that consists of the training data of all users in group g during the signaling interval T is given by

$$\mathbf{X}^{(g)} \triangleq \left[\begin{array}{cccc}\mathbf{X}_1^{(g)} & \mathbf{X}_2^{(g)} & \cdots & \mathbf{X}_{K_g}^{(g)}\end{array}\right]_{T\times K_gL_g}. \quad (10)$$

The extended multi-path channel vector of group g in (7), carrying the complete CSI of all UTs in g, can be expressed in terms of KLT coefficients (small-scale fading) given in (4) as

$$\mathbf{h}^{(g)}=\underbrace{\left(\mathbf{I}_{K_g}\otimes\mathbf{V}\right)}_{\triangleq\boldsymbol{\Upsilon}_U^{(g)}}\mathbf{c}^{(g)} \text{ where} \quad (11)$$

$$\mathbf{V} \triangleq \mathrm{bdiag}\left[\left\{\left(\rho_l^{(g)}\right)^{1/2}\mathbf{U}_l^{(g)}\left(\boldsymbol{\Lambda}_l^{(g)}\right)^{1/2}\right\}_{l=0}^{L_g-1}\right]. \quad (12)$$

In (11), $\boldsymbol{\Upsilon}_U^{(g)} \triangleq \mathbf{I}_{K_g}\otimes\mathbf{V}$ is a $NK_gL_g\times K_g\left(\sum_{l=0}^{L_g-1}r_{g,l}\right)$ transform matrix (in spatio-temporal domain) constructed by the eigenbasis of group g at each delay.

The pre-beamforming is applied in order to distinguish *intra-group* signal of group g users from other groups by suppressing the *inter-group interference* while reducing the signaling dimension of $\mathbf{y}$ in (7). At the pre-beamforming stage, a DT-dimensional space-time vector $\mathbf{y}^{(g)}$ can be formed by using (1) for all *intra-cell* groups by a linear transformation through a matrix $\left(\boldsymbol{\Upsilon}_S^{(g)}\right)^H \triangleq \left(\mathbf{I}_T\otimes\left[\mathbf{S}_D^{(g)}\right]^H\right)$ as

$$\begin{aligned}\mathbf{y}^{(g)} &\triangleq \left(\boldsymbol{\Upsilon}_S^{(g)}\right)^H\mathbf{y},\ g=1,\ldots,G\\ &=\left(\mathbf{X}^{(g)}\otimes\left[\mathbf{S}_D^{(g)}\right]^H\right)\mathbf{h}^{(g)}+\left(\boldsymbol{\Upsilon}_S^{(g)}\right)^H\boldsymbol{\xi}^{(g)}\end{aligned} \quad (13)$$

where $\mathbf{S}_D^{(g)}$ is an $N\times D$ statistical pre-beamforming matrix that projects the N-dimensional received signal samples $\{\mathbf{y}_n\}_{n=0}^{T-1}$ in (1) on a suitable D-dimensional subspace in spatial domain.

III. Covariance-Based Reduced Rank Channel Estimation

In this section, based on the model in (13), the reduced dimensional linear minimum mean square error (LMMSE) channel estimator is derived while the side information lying in the second order statistics of the MPCs of each group is utilized. As it is well-known, the LMMSE channel estimator is often referred to as the *Wiener filter* [10]. The reduced rank MMSE (RR-MMSE) estimate of the instantaneous CSI can be expressed in the following general form:

$$\begin{aligned}\hat{\mathbf{h}}^{(g)} &= \boldsymbol{\Upsilon}_U^{(g)}\hat{\mathbf{c}}^{(g)}=\boldsymbol{\Upsilon}_U^{(g)}\left(\mathbf{W}_{mmse,D}^{(g)}\right)^H\mathbf{y}^{(g)}\\ &=\underbrace{\boldsymbol{\Upsilon}_U^{(g)}\left(\mathbf{W}_{mmse,D}^{(g)}\right)^H\left(\boldsymbol{\Upsilon}_S^{(g)}\right)^H}_{\text{Reduced Rank Wiener Filter}}\mathbf{y}\end{aligned} \quad (14)$$

where the RR-MMSE estimate of $\mathbf{h}^{(g)}$, namely, $\hat{\mathbf{h}}^{(g)}$ is written in terms of the RR-MMSE estimate of $\mathbf{c}^{(g)}$, namely, $\hat{\mathbf{c}}^{(g)}$ (small-scale fading) by the KLT through the $\boldsymbol{\Upsilon}_U^{(g)}$ matrix given in (11). In (14), first, $\hat{\mathbf{c}}^{(g)}$ is formed by a reduced dimensional linear *Wiener filter* (or MMSE) in spatio-temporal domain through $\left(\mathbf{W}_{mmse,D}^{(g)}\right)^H$ matrix for group g users after projecting (reducing the dimension in Kernel space) full dimensional observation $\mathbf{y}$ in (6) onto a suitable subspace represented by $\left(\boldsymbol{\Upsilon}_S^{(g)}\right)^H$ (pre-beamforming) in (13). Then, the LMMSE estimate of the full-dimensional multi-path channel vector of group g, $\hat{\mathbf{h}}^{(g)}$, is constructed by transforming $\hat{\mathbf{c}}^{(g)}$ back to the original space by KLT through $\boldsymbol{\Upsilon}_U^{(g)}$.

A. Joint Angle-Delay Domain RR-MMSE Estimator

The subsequent downlink or uplink processing, preceded by the pre-beamformer, can access and utilize only the following *effective* multi-path channel vector of each group user: $\mathbf{h}_{l,eff}^{(g_k)} \triangleq \left[\mathbf{S}_D^{(g)}\right]^H \mathbf{h}_l^{(g_k)}$. Then, from the definition of extended multi-path channel for group g in (7), the *effective* extended multi-path channel seen after pre-beamforming can be expressed as

$$\mathbf{h}_{eff}^{(g)} \triangleq \left(\mathbf{I}_{K_g L_g} \otimes \left[\mathbf{S}_D^{(g)}\right]^H\right) \mathbf{h}^{(g)} \tag{15}$$

Based on (14), the RR-MMSE estimate of the *effective* channel $\mathbf{h}_{eff}^{(g)}$ in (15) is constructed as

$$\begin{aligned}\hat{\mathbf{h}}_{eff}^{(g)} &\triangleq \left(\mathbf{I}_{K_g L_g} \otimes \left[\mathbf{S}_D^{(g)}\right]^H\right) \hat{\mathbf{h}}^{(g)} \qquad (16)\\ &= \left(\sum_{l=0}^{L_g-1} \left(\mathbf{X}^{(g)}\left[\mathbf{I}_{K_g} \otimes \mathbf{E}_{L_g,l}\right]\right) \otimes \mathbf{SNR}_{mimo}^{(g)}(l)\right)^H \\ &\quad \left(\sum_{l=0}^{L_g-1} \mathbf{R}_{code}^{(g)}(l) \otimes \left[\mathbf{SNR}_{mimo}^{(g)}(l)\right]^H + \mathbf{I}_{TD}\right)^{-1} \mathbf{y}^{(g)}\end{aligned}$$

where $\hat{\mathbf{h}}^{(g)}$ is obtained by using (14) after substituting the reduced rank *Wiener filter* $\left(\mathbf{W}_{mmse,D}^{(g)}\right)^H$ of group g in its position and

$$\begin{aligned}\mathbf{SNR}_{mimo}^{(g)}(l) &\triangleq \qquad (17)\\ &\rho_l^{(g)}\left(\left[\mathbf{S}_D^{(g)}\right]^H \mathbf{R}_{\boldsymbol{\eta}}^{(g)} \mathbf{S}_D^{(g)}\right)^{-1}\left(\left[\mathbf{S}_D^{(g)}\right]^H \mathbf{R}_l^{(g)} \mathbf{S}_D^{(g)}\right), \\ \mathbf{R}_{code}^{(g)}(l) &\triangleq \left(\mathbf{X}^{(g)}\left[\mathbf{I}_{K_g} \otimes \mathbf{E}_{L_g,l}\right]\left[\mathbf{X}^{(g)}\right]^H\right). \qquad (18)\end{aligned}$$

In (16), $\mathbf{E}_{L_g,l}$ is an $L_g \times L_g$ elementary diagonal matrix where all the entries except the $(l+1)^{th}$ diagonal one are zero. The details of the mathematical derivation of this RR-MMSE estimator can be found in our extended work [13] where the solution of Wiener-Hopf equation [10], based on (13) defined in spatio-temporal domain, was provided.

Different than the previous low-rank LMMSE approaches in [12], [14], the RR-MMSE estimator in (16) can be interpreted as the *reduced rank approximation* of the full-dimensional spatio-temporal *Wiener filter* by using two different transform basis sets, namely, the dimension reducing subspace projection (pre-beamformer) and the KLT characterizing the joint *angle-delay* channel sparsity.

The RR-MMSE shows remarkable robustness to the pilot interference or contamination due to the use of non-orthogonal pilots among *inter-cell* users or pilot reuse among *intra-cell* users since the *inter-group interference* is mitigated by the statistical pre-beamformer in the spatial domain. In addition to the considerable reduction in pilot interference, the pilot overhead is also reduced when the pilot length T is kept small by allowing non-orthogonal training sequences among *intra-group* or *inter-group* users.

B. Angle Domain RR-MMSE Estimator

One can consider the following approximation of (16) by assuming that the MPCs of each group have the same AoA support (common angular sector) with the following covariance matrix: $\mathbf{R}_{sum}^{(g)} \triangleq \sum_{l=0}^{L_g-1} \rho_l^{(g)} \mathbf{R}_l^{(g)}$. This corresponds to the use of *angular* information only when all the AoA supports of each MPC, belonging to the same group, are unified. In (17), by replacing $\rho_l^{(g)} \mathbf{R}_l^{(g)}$ with $\mathbf{R}_{sum}^{(g)}$, one can get the following approximation for the effective channel estimate in (16):

$$\begin{aligned}\hat{\mathbf{h}}_{eff,2}^{(g)} &= \left(\mathbf{X}^{(g)} \otimes \mathbf{SNR}_{mimo}^{total,(g)}\right)^H \\ &\quad \left(\mathbf{R}_{code}^{(g)} \otimes \mathbf{SNR}_{mimo}^{total,(g)} + \mathbf{I}_{TD}\right)^{-1} \mathbf{y}^{(g)} \qquad (19)\end{aligned}$$

where the following matrices are defined as

$$\begin{aligned}\mathbf{SNR}_{mimo}^{total,(g)} &\triangleq \qquad (20)\\ &\left(\left[\mathbf{S}_D^{(g)}\right]^H \mathbf{R}_{\boldsymbol{\eta}}^{(g)} \mathbf{S}_D^{(g)}\right)^{-1}\left(\left[\mathbf{S}_D^{(g)}\right]^H \mathbf{R}_{sum}^{(g)} \mathbf{S}_D^{(g)}\right), \\ \mathbf{R}_{code}^{(g)} &\triangleq \mathbf{X}^{(g)}\left[\mathbf{X}^{(g)}\right]^H. \qquad (21)\end{aligned}$$

This estimator is called *angle* domain RR-MMSE estimator, which will be used in pre-beamformer design and performance comparison in the sequel.

IV. Nearly Optimal Beamformer Design

In this section, our goal is to find a good subspace (spanned by the columns of $\mathbf{S}_D^{(g)}$ matrix) on which the reduced dimensional instantaneous channel estimation can be realized as accurately as possible, so that a minimal performance compromise in the subsequent statistical signal processing operations after beamforming is provided.

A. Beamformer Design Criteria

We examine the dimension reduction problem from three different viewpoints based on the instantaneous CSI estimation accuracy. These criteria result in an equivalent optimization problem yielding the optimal dimension-reducing subspace.

1) Reconstruction Error Minimizing Subspace

The reduced rank Wiener filtering in (14) can be seen as the data reconstruction process from noisy observations after dimension reduction. A legitimate goal is the minimization of the reconstruction error according to a criterion. If we denote the reconstruction error vector with $\mathbf{e}^{(g)} \triangleq \mathbf{h}^{(g)} - \hat{\mathbf{h}}^{(g)}$ for group g channels, the covariance matrix of error $\mathbf{R_e}^{mmse} \triangleq \mathbb{E}\left\{\left(\mathbf{h}^{(g)} - \hat{\mathbf{h}}^{(g)}\right)\left(\mathbf{h}^{(g)} - \hat{\mathbf{h}}^{(g)}\right)^H\right\}$ can be calculated as follows:

$$\mathbf{R_e}^{mmse} = \mathbf{R}_{full}^{(g)} - \mathbf{R}_{full}^{(g)} \mathbf{F}_s^{(g)} \left(\mathbf{R}_{\mathbf{y}}^{(g)}\right)^{-1} \left(\mathbf{F}_s^{(g)}\right)^H \mathbf{R}_{full}^{(g)} \tag{22}$$

where

$$\mathbf{R}_{\mathbf{y}}^{(g)} = \left(\mathbf{F}_s^{(g)}\right)^H \mathbf{R}_{full}^{(g)} \mathbf{F}_s^{(g)} + \left(\boldsymbol{\Upsilon}_S^{(g)}\right)^H \mathbf{R}_{\boldsymbol{\xi}}^{(g)} \boldsymbol{\Upsilon}_S^{(g)}$$

$$\mathbf{R}_{full}^{(g)} \triangleq \mathbb{E}\left\{\mathbf{h}^{(g)}\left(\mathbf{h}^{(g)}\right)^H\right\} = \sum_{l=0}^{L_g-1}\left[\mathbf{I}_{K_g} \otimes \mathbf{E}_{L_g,l}\right] \otimes \rho_l \mathbf{R}_l^{(g)}$$

$$\mathbf{F}_s^{(g)} \triangleq \left(\left[\mathbf{X}^{(g)}\right]^H \otimes \mathbf{S}_D^{(g)}\right) \tag{23}$$

It is important to note that the inverse of $\mathbf{R_e}^{mmse}$ in (22) is actually the Fisher information matrix [10]. The details of the mathematical steps in obtaining (22), as well as the results in (24), (25), and (26) below can be found in [13].

Error Volume: The minimization of the estimation error volume, namely, the determinant of $\mathbf{R_e}^{mmse}$ in (22), can be regarded as one of the important objectives on which D-dimensional subspace $\mathbf{S}_D^{(g)}$ is optimized. If one takes the determinant of both parts in (22), the following expression is obtained

$$\mathcal{V} = \frac{\det\left(\mathbf{R}_{full}^{(g)}\right)}{\det\left(\mathbf{I}_{TD} + \sum_{l=0}^{L_g-1}\mathbf{R}_{code}^{(g)}(l) \otimes \mathbf{SNR}_{mimo}^{(g)}(l)\right)} \tag{24}$$

where $\mathcal{V} \triangleq \det\left(\mathbf{R_e}^{mmse}\right)$ and $\mathbf{R}_{full}^{(g)}$ defined in (23) can be seen as a priori error volume of $\mathbf{h}^{(g)}$ before the training period.

Normalized Mean Square Error: The normalized mean square error (*nMSE*) covariance can be defined as the estimation error covariance matrix of KLT coefficients $\mathbf{c}^{(g)}$ in (11) as $\mathbf{nMSE}^{(g)} \triangleq \mathbb{E}\left\{\left(\mathbf{c}^{(g)} - \hat{\mathbf{c}}^{(g)}\right)\left(\mathbf{c}^{(g)} - \hat{\mathbf{c}}^{(g)}\right)^H\right\}$. Then, the trace of $\mathbf{nMSE}^{(g)}$, as an alternative objective function, can be obtained in the following compact form as

$$\begin{aligned}\mathrm{Tr}\left\{\mathbf{nMSE}^{(g)}\right\} &= \left(K_g\left(\sum_{l=0}^{L_g-1} r_{g,l}\right) - TD\right) + \\ &\mathrm{Tr}\left\{\left(\sum_{l=0}^{L_g-1}\mathbf{R}_{code}^{(g)}(l) \otimes \mathbf{SNR}_{mimo}^{(g)}(l) + \mathbf{I}_{TD}\right)^{-1}\right\}.\end{aligned} \tag{25}$$

2) Mutual Information Preserving Subspace

The mutual information between $\mathbf{h}^{(g)}$ and $\mathbf{y}^{(g)}$ in (13) can be compactly obtained by assuming that both $\mathbf{h}^{(g)}$ and $\mathbf{y}^{(g)}$ are jointly Gaussian, and using the covariance matrix of *inter-group interference* $\mathbf{R}_{\boldsymbol{\xi}}^{(g)}$ in (9) as follows:

$$\begin{aligned}&I\left(\mathbf{h}^{(g)}; \mathbf{y}^{(g)}\right) = I\left(\mathbf{c}^{(g)}; \mathbf{y}^{(g)}\right) \\ &= \log\left\{\det\left(\mathbf{I}_{TD} + \sum_{l=0}^{L_g-1}\mathbf{R}_{code}^{(g)}(l) \otimes \mathbf{SNR}_{mimo}^{(g)}(l)\right)\right\}.\end{aligned} \tag{26}$$

B. Nearly Optimal Solution: Generalized Eigenvector Space

All three criteria in Section IV-A, namely, the minimization of (24) or (25) and the maximization of (26) can be shown to result in equivalent optimization problems yielding the same optimal pre-beamformer.

Joint Angle-Delay Domain: It is possible to simplify this optimization problem where the optimal pre-beamformer depends on the training pattern $\mathbf{X}^{(g)}$ in general. In mm wave massive MIMO channels, *channel sparsity* implies that eigenspaces of each MPC are nearly orthogonal [7], [8]. In light of this near-orthogonality assumption, the pre-beamformer of group g can be constructed as

$$\mathbf{S}_D^{(g)} \triangleq \left[\begin{array}{cccc}\mathbf{S}_D^{(g)}(0) & \mathbf{S}_D^{(g)}(1) & \cdots & \mathbf{S}_D^{(g)}(L_g-1)\end{array}\right]_{N \times D}$$

where the $N \times d_l$ matrix $\mathbf{S}_D^{(g)}(l)$ can be seen as the sub-beamformer that allows l^{th} resolvable MPC of group g to pass while suppressing the *inter-group interference* in the spatial domain, and $\sum_{l=0}^{L_g-1} d_l = D$. Due to the apparent near-orthogonality among the different MPCs (especially for mm wave frequencies), $\mathbf{S}_D^{(g)}(l)$ is also expected to reject each MPC of group g other than the one at l^{th} delay. As shown in [13], this orthogonality assumption leads to the following approximation of the optimization criterion in (25):

$$\mathrm{Tr}\left\{\mathbf{nMSE}^{(g)}\right\} \approx \sum_{l=0}^{L_g-1}\sum_{m=1}^{R_l}\sum_{n=1}^{d_l}\frac{1}{\beta_m^l \lambda_n^l + 1} + \mathcal{E} \tag{27}$$

where β_m^l and λ_n^l are the eigenvalues of $\mathbf{R}_{code}^{(g)}(l)$ and $\mathbf{SNR}_{mimo}^{(g)}(l)$ for $l = 0, \ldots, L_g - 1$ respectively, and $\mathcal{E} \triangleq \left(K_g \sum_{l=0}^{L_g-1} r_{g,l} - \sum_{l=0}^{L_g-1} R_l d_l\right)$. In (27), R_l is the rank of $\mathbf{R}_{code}^{(g)}(l)$ matrix with $R_l \leq \min\{T, K_g\}$ from (18). For a given dimension of the pre-beamformer $\mathbf{S}_D^{(g)}(l)$ in (27) with $\sum_l d_l = D$, and the training pattern determining β_m^l, it can be noted that the minimum value of the cost function $\mathrm{Tr}\left\{\mathbf{nMSE}^{(g)}\right\}$ in (27), under the constraint that $\mathbf{S}_D^{(g)}$ is a full column rank matrix is achieved by the first d_l dominant *generalized eigenvectors* of $\mathbf{R}_l^{(g)}$ and $\mathbf{R}_{\boldsymbol{\eta}}^{(g)}$ [13]. The minimum value of (27) is attained by choosing λ_n^l as the n^{th} dominant *generalized eigenvalue* of $\mathbf{R}_l^{(g)}$ and $\mathbf{R}_{\boldsymbol{\eta}}^{(g)}$. One can also determine the optimal d_l values among the possible alternatives satisfying $\sum_l d_l = D$ by using the *generalized eigenvalues* λ_n^l minimizing (27).

It is observed that the pilot overhead is significantly reduced since even for $T = K_g$ (independent of N), $\mathrm{Tr}\left\{\mathbf{nMSE}^{(g)}\right\}$ approaches zero when $d_l = r_{g,l}$ and $R_l = K_g$.

Angle-Only Domain: By unification of different MPCs in angular domain corresponding to the use of $\mathbf{R}_{sum}^{(g)}$ in (20) as the common AoA support of all MPCs in group g, the optimization criterion in (25) can be simplified as $\mathrm{Tr}\left\{\left(\mathbf{R}_{code}^{(g)} \otimes \mathbf{SNR}_{mimo}^{total,(g)} + \mathbf{I}_{TD}\right)^{-1}\right\}$ from Section III-B. In this case, the minimum cost is achieved by choosing the first D dominant *generalized eigenvectors* of $\mathbf{R}_{sum}^{(g)}$ and $\mathbf{R}_{\boldsymbol{\eta}}^{(g)}$ as the columns of $\mathbf{S}_D^{(g)}$.

V. Numerical Results and Discussion

In this section, we provide some numerical results to evaluate the performance of the reduced rank channel estimators and examine the efficiency of the *generalized eigenvector beamspace* (GEB) in Section IV-B for the reduced dimensional processing. Throughout the demonstrations, we consider a massive MIMO system with uplink training in TDD mode where a BS is equipped with a uniform linear array (ULA) of $N = 100$ antenna elements along the y-axis[4], and each of K users has a single receive antenna.

In the studied scenario, K users were clustered into eight groups ($G = 8$), and each UT is assumed to be located at a specific azimuth angle θ along the ring centered at the origin in the x-y plane. The channel covariance matrix of each group is specified with the center azimuth angle θ (AoA), and can be calculated in a similar way to the ones in [4], [6]. In the simulations, our focus is on the channel estimation accuracy of the intended group g with 3 MPCs, i.e., $L_g = 3$. The first two MPCs of group g stem from a azimuth angular sector $[-1°, 1°]$ for delays at $l = 0, 1$, and the angular sector of the last MPC at $l = 2$ of g is given as $[5°, 7°]$ in azimuth. We assume two users served simultaneously for group g, i.e., $K_g = 2$. Each of the other 7 groups (interfering with the intended one) consists of three users, i.e., $K_{g'} = 3, \ g' \neq g$ and these users have 3 MPCs whose angular sectors have same supports of AoA ($L_{g'} = 3, \ g' \neq g$) given by $[-29, -26]$, $[-21, -19]$, $[-12, -9]$, $[-5.5, -3.5]$, $[9.5, 12.5]$, $[15, 17]$, $[24, 27]$ in azimuth respectively. The channel vector for each user is independently generated according to the model (4). The noise power is set as $N_0 = 1$ so that all dB power values are relative to 1. *Intra-group* users (of the intended group) use non-orthogonal training waveforms composed of 6 chips ($T = 6$), and these are obtained by truncating length-63 *Kasami* codes by simply choosing the first T chips of last K_g *Kasami* sequences without any optimization.

The trace of the estimation error covariance matrix (for the extended channel vector of group g users in (7) given by $\mathbf{R_e} \triangleq \mathbb{E}\left\{\left(\mathbf{h}^{(g)} - \hat{\mathbf{h}}^{(g)}\right)\left(\mathbf{h}^{(g)} - \hat{\mathbf{h}}^{(g)}\right)^H\right\}$ is evaluated analytically by using the matrix model (13) and (14) to compare the performance of different reduced rank estimators based on different pre-beamformers. The covariance matrix of the *inter-group interference* is evaluated by (5) when the angular sector of each group is provided.

The GEB is constructed in a similar way to the one in Section IV-B by taking the number of resolvable MPCs in the angular domain as 2, and setting $d_0 = d_1 = \frac{D}{2}$. In this paper, we compare the performance of dimension reduction based on GEB (shown to be nearly optimal under some realistic assumptions) with that of the conventional subspace composed of the first D dominant eigenvectors of $\mathbf{R}_{sum}^{(g)}$ in (20). We call this conventional beamspace as discrete Fourier transform (DFT) beamspace because the eigenvectors of the spatial correlation matrix of the ULA channel are well-approximated by the columns of the $N \times N$ unitary DFT matrix whose indices correspond to the support of the Fourier transform of the spatial correlation function (owing to the Szegö's asymptotic theory) [5] depending on the angular sector of group g UTs. This conventional beamspace is known to be information preserving for spatially white interference case, and thus, widely used in practical *hybrid beamforming* applications, where the beamforming in RF analog domain can be implemented by simple phase shifters [15].

In Figure 1, the beam patterns created by the GEB and DFT beamspaces are depicted for $D = 6$ at $snr = 30$ (dB). The GEB is designed based on the AoA support of the intended group g for $l = 0, 1, 2$ while taking the angular locations of the interfering groups into account. The *inter-group* users signal are assumed to have same power level with that of the intended group. As can be seen from the figure, the GEB tries to create deep nulls at the angular locations of interfering UTs, whereas the conventional pre-beamformer only tries to maximize the captured power of the intended group MPCs for a given dimension. It is expected that as the number of BS antennas increases, the eigenspaces of each group are approximately orthogonal. However, the number of transmit antennas is finite in practice, and there always exists some overlap among the virtual angular sectors of each group which leads to a leakage to the intended group signal. Therefore, as it will be shown later, the accuracy of the channel estimation realized on the reduced dimensional subspace, spanned by the conventional DFT beamspace, is considerably lost due to the residual *inter-group interference* after pre-beamforming. On the other hand, the GEB suppresses the *inter-group interference* while allowing the MPCs of the intended group to pass with a negligible distortion.

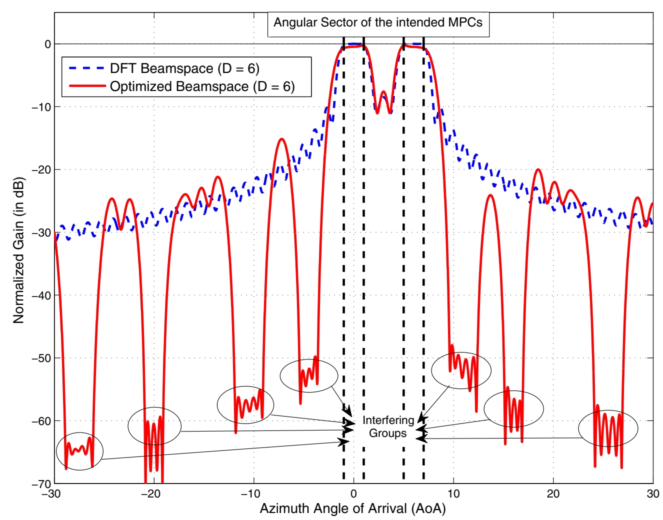

Fig. 1. Beam pattern of different pre-beamformers

In Figure 2, the average mean square error (MSE) values given by $\mathrm{Tr}\left\{\mathbf{R_e}\right\}/K_g$ as a function of the dimension of the spatial domain pre-beamformer (D) are depicted for both joint *angle-delay* domain and *angle* domain RR-MMSE estimators given in (14) at $snr = 30$ dB. For joint *angle-delay*, the exact knowledge of the covariance for each MPC is used, whereas for *angle* domain, a common angular region (obtained by the unification of each delay) is assumed for each MPC (of group g) and used instead of $\mathbf{R}_l^{(g)}$ in the channel acquisition. Also, the performance of full dimensional *Wiener* estimator ($D = N$) is demonstrated when there is no interfering groups. It is clear that the *angle* domain RR-MMSE estimator is

[4]Although the proposed estimators are valid for an arbitrary array structure in this paper, ULA is considered for the ease of exposition only.

inferior to the joint *angle-delay* domain estimator due to the inefficient use of the training and noise enhancement. Moreover, it is seen that there is a remarkable performance gap between the performances of RR-MMSE estimators based on two different pre-beamformers (the GEB and the conventional one) especially at lower dimensions. Also, it can be concluded that the RR-MMSE estimator based on the GEB achieves a very close performance to that of the full-dimensional estimator even for $D = 7$ (for group g), that is roughly 15-fold dimension reduction. On the other hand, with the conventional beamspace, in spite of the optimal *Wiener filtering* after dimension reduction, the MSE performance is not satisfactory for dimensions below 14.

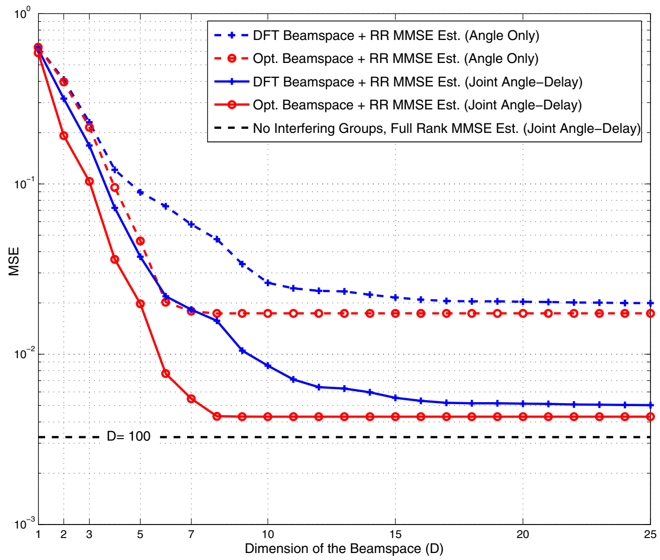

Fig. 2. MSE values of various RR-MMSE estimators versus dimension

In Figure 3, the MSE values of the RR-MMSE estimators in (14) as a function of the *snr* (after beamforming) are depicted for various dimension values. It is observed that DFT beamspace based processing needs much larger dimensions to obtain the same accuracy level with that of the estimators based on the GEB.

VI. Conclusions

In this paper, we provided a general description for massive MIMO transmission employing SC in frequency-selective fading, and examined the dimension reduction problem from several different viewpoints related to the instantaneous CSI estimation accuracy. Adaptive learning of *long-term parameters* (such as AoA supports and delays), and adaptive subspace construction (or tracking) with *user-grouping*, under the proposed beamformer design and CSI acquisition framework, can be topics of future studies.

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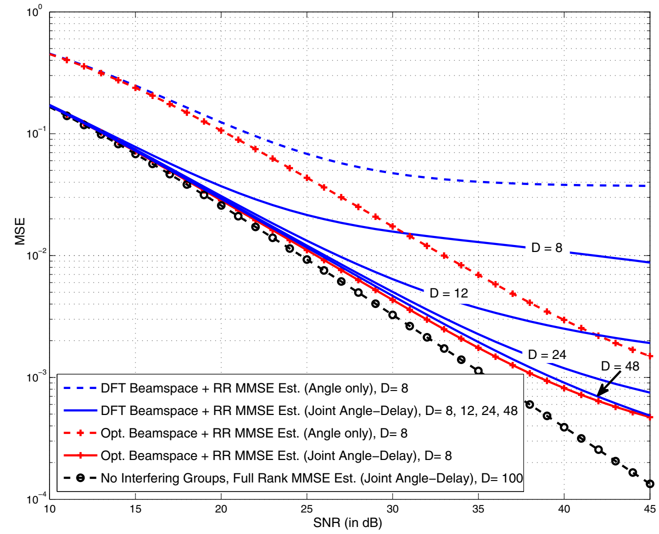

Fig. 3. MSE values for the various RR-MMSE estimators versus *snr*

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