The Effect of Antenna Correlation in Single-Carrier Massive MIMO Transmission

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***Abstract*— This work presents a single-carrier massive MIMO transmission system for the frequency selective Gaussian multi-user channel. It considers both cases of spatially uncorrelated and correlated channel and compares them in terms of the user sum-rate as well as the general performance. We consider a channel with M antennas at the base station which provides services for K single-antenna users. We develop a general expression for the achievable rate among users in the channel with a correlation among antennas at the base station. It has been shown that, when there is no correlation between base station antennas or users, the channel matched filter precoder outperforms any other precoder. In a highly-correlated massive MIMO channel however, the conventional channel matched filter precoder does not perform as expected. We show the failure of the channel matched filter in the presence of a correlation pattern among antennas at the base station with theoretical analysis and simulations.**

We apply three different precoders to enhance the achievable rate and their performances are determined by simulations. We show that the precoders outperform the channel matched filter in a highly-correlated channel with a large number of users. By increasing the number of users in the system, the conventional precoders show even better performance in terms of the user's sum-rates.

I. Introduction

The work in this paper is focused on single-carrier transmission for massive multiple-input multiple-output (MIMO) applications. This is motivated by a number of recent studies such as [1]. This work showed that a matched filter precoder is optimum for such systems in channels uncorrelated in space. However, unlike [1], we wish to determine the performance of such systems for correlated channels, such as in massive MIMO where the presence of correlation in the channel is expected to be high due to space limitation for the antenna elements.

In this paper, theoretical analysis and simulations have demonstrated the failure of the channel matched filter precoder, unlike its uncorrelated counterpart. Instead, three different precoders have been applied in a highly correlated massive MIMO system to enhance the achievable rate for a large number of users.

While many researchers and engineers are aware of the importance of correlation among antenna elements, proposals for system design are relatively limited. Two works that present results of precoder design for correlated MIMO channels under imperfect channel state information (CSI) are [2], [3].

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II. System Model

We consider a frequency-selective multi-user MIMO (MU-MIMO) downlink channel, with M base station antennas and K single-antenna users. The channel between the m-th transmit antenna and the k-th user can be modeled as a finite impulse response (FIR) filter with L taps. The L taps correspond to different delay components. As it is shown in [1], the l-th channel tap can be written as $\sqrt{d_l[k]}h_l^*[m,k]$ where $d_l[k]$ and $h_l^*[m,k]$ correspond to the slow-varying and fast-varying components of the channel, respectively. If the antenna elements (or delay components) are uncorrelated, then the matrix of the fast-varying components of the channel has independent and identically distributed (i.i.d.) elements (i.e., $h_l^*[m,k]$'s are i.i.d. and they are fixed during the transmission of N symbols). We define $\mathbf{y}[i] \triangleq [y_1[i], \cdots, y_K[i]]^T \in \mathbb{C}^K$ and $\mathbf{x}[i] \triangleq [x_1[i], \cdots, x_M[i]]^T \in \mathbb{C}^M$ as the vector of received signals at each user and the vector of transmitted signals from each antenna at the base station at time i, respectively. Let the noise vector $\mathbf{n}[i] \triangleq [n_1[i], \cdots, n_K[i]]^T$ be additive white Gaussian noise (AWGN) with i.i.d. components and complex Gaussian distribution. Let $s_k[i]$ be the information symbol to be communicated to the k-th user at time i. The vector form for information symbols is defined as $\mathbf{s}[i] \triangleq [s_1[i], \cdots, s_K[i]]^T$ and it is considered to have i.i.d. $\mathcal{CN}(0,1)$ components. We define $\mathbf{D}_l \triangleq \mathrm{diag}\{d_l[1], \cdots, d_l[K]\}$, and $\mathbf{H}_l \in \mathbb{C}^{M\times K}$ which is a matrix whose $[m,k]$-th element is $h_l[m,k]$.

To model the channel with correlated antenna elements, we consider a correlation pattern among the antennas at the base station. Using the correlation among antennas and expand it for all M antennas at the base station, we will have $\mathbf{A} \in \mathbb{C}^{M\times M}$ as the correlation matrix among base station antennas. To demonstrate the effect of matrix $\mathbf{A}$ on the channel, we consider a modified channel realization as $\mathbf{A}^{1/2}\mathbf{H}_l$. According to this modification, if a spatially uncorrelated channel is desired, we only need to change the correlation variables in order to set the correlation matrix $\mathbf{A} = \mathbf{I}_M$. We will employ two well-known correlation models in Section V. Note that in both cases $\mathbf{A}$ is a symmetric matrix.

We can write the received signal vector as

$$\mathbf{y}[i] = \sum_{l=0}^{L-1} \mathbf{D}_l^{1/2}\mathbf{H}_l^H\mathbf{A}^{1/2}\mathbf{x}[i-l] + \mathbf{n}[i], \tag{1}$$

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where $\mathbf{x}[i]$ is the transmitted signal vector using the channel matched filter precoder as in [1]

$$\mathbf{x}[i] = \sqrt{\frac{\rho_f}{MK}} \sum_{l=0}^{L-1} \mathbf{A}^{1/2}\mathbf{H}_l\mathbf{D}_l^{1/2}\mathbf{s}[i+l], \tag{2}$$

in which $\rho_f \triangleq \mathbb{E}[\|\mathbf{x}[i]\|^2]$ is the total average power transmitted by the base station antennas. Note that as it is shown in [1], the channel power delay profile (PDP) for each user is normalized such that

$$\sum_{l=0}^{L-1} d_l[k] = 1, \quad \forall k = 1, \ldots, K. \tag{3}$$

Also, we define the channel state matrix as $\mathbf{V}_l = \mathbf{A}^{1/2}\mathbf{H}_l\mathbf{D}_l^{1/2}$ and its vector format as $\mathbf{v}_l[k] = \mathbf{V}_l\mathbf{e}_k$, where $\mathbf{e}_k$ is a vector with all of its elements equal to 0 except the k-th element which is 1. Using this vector, we can rewrite the received signal of the k-th user at time i as separate terms of the desired signal and the effective noise as

$$\begin{aligned} y_k[i] = & \underbrace{\sqrt{\frac{\rho_f}{MK}}\Bigg(\sum_{l=0}^{L-1}\mathbb{E}\big[\mathbf{v}_l^H[k]\mathbf{v}_l[k]\big]\Bigg)s_k[i]}_{\text{Desired Signal Term}} \\ & + \underbrace{n'_k[i]}_{\text{Effective Noise Term}} . \end{aligned} \tag{4}$$

Using (1) and (2), one can obtain the effective noise term in the above equation as

$$\begin{aligned} n'_k[i] = & \sqrt{\frac{\rho_f}{MK}} \sum_{l=0}^{L-1} \Bigg(\mathbf{v}_l^H[k]\mathbf{v}_l[k] - \mathbb{E}[\mathbf{v}_l^H[k]\mathbf{v}_l[k]]\Bigg)s_k[i] \\ & + \sqrt{\frac{\rho_f}{MK}} \sum_{\substack{b=1-L \\ b\neq 0}}^{L-1} \sum_{l=L_1}^{L_2} \mathbf{v}_l^H[k]\mathbf{v}_{l-b}[k]s_k[i-b] \\ & + \sqrt{\frac{\rho_f}{MK}} \sum_{\substack{q=1 \\ q\neq k}}^{K} \sum_{b=1-L}^{L-1} \sum_{l=L_1}^{L_2} \mathbf{v}_l^H[k]\mathbf{v}_{l-b}[q]s_q[i-b] \\ & + n_k[i], \end{aligned} \tag{5}$$

where $L_1 = \max(b, 0)$, $L_2 = \min(L-1+b, L-1)$. Note that in the expression above, the first term is the additional interference (IF), the second term is the intersymbol interference (ISI), the third term is multiuser interference (MUI), and the last term is AWGN.

III. Capacity and Information Rate

The average power of the desired signal term in (4) can be shown as [1]

$$\begin{aligned} S_k = & \mathbb{E}_{s_k[i]}\Bigg[\Bigg|\sqrt{\frac{\rho_f}{MK}}\sum_{l=0}^{L-1}\mathbb{E}\big[\mathbf{v}_l^H[k]\mathbf{v}_l[k]\big]s_k[i]\Bigg|^2\Bigg] \\ = & \frac{M\rho_f}{K}. \end{aligned} \tag{6}$$

As one can see, the average power of the desired signal is independent of the correlation matrix. This means that the correlation among antennas at the base station has no effect on the desired signal power. Also, one can calculate the power of the effective noise in (5) as

$$\mathrm{Var}\big(n'_k[i]\big) = \frac{\mathrm{tr}(\mathbf{A}^2)}{M}\rho_f + 1, \tag{7}$$

where $\mathrm{tr}(\mathbf{A}^2)$ is the trace of square of the correlation matrix. Proposition 1 and Proposition 2 in the sequel show how the average power of the desired signal in (6) and effective noise power in (7) can be obtained from (4). In the case of an uncorrelated channel, one can simplify (7) to $\rho_f + 1$. Despite of the fact that the average power of the desired signal is independent of the correlation matrix, the effective noise power is affected by it.

When the correlated channel is desired, diagonal elements of matrix $\mathbf{A}$ remain the same as the uncorrelated case (for both models considered in Section V), but other elements have non-zero values and based on the correlation models that we consider in this work, these values cannot be negative. This means that when the channel is correlated, $\mathrm{tr}(\mathbf{A}^2) > M$ and therefore, $\frac{\mathrm{tr}(\mathbf{A}^2)}{M} > 1$. Thus, it can be seen that the power of the effective noise will increase due to the channel dependency. However, this dependency has no effect on the average power of the desired signal. Due to the correlation pattern, the information rate of the users decreases, but the capacity is still the same. This means, as the correlation parameter increases, there will be a huge gap between the information rate and the capacity.

Using the general formula in [1], one can obtain each user's information rate as $R_k = \log_2\big(1 + \frac{M^2\rho_f}{K\mathrm{tr}(\mathbf{A}^2)\rho_f + KM}\big)$. By considering the fact that each user's information rate is almost equal to the other users', we will have the sum-rate as

$$R_{sum}(\rho_f, M, K) = K\log_2\Bigg(1 + \frac{M\rho_f}{K\frac{\mathrm{tr}(\mathbf{A}^2)}{M}\rho_f + K}\Bigg). \tag{8}$$

Since the correlation has no effect on the average power of the desired signal, the cooperative sum-capacity is independent of the correlation pattern and it can be written as [1]

$$C_{coop}(\rho_f, M, K) \approx K\log_2\big(1 + \frac{M\rho_f}{K}\big). \tag{9}$$

Based on the effect of the correlation on the information rate and the sum-capacity, it can be seen that the channel matched filter precoder is not a good choice in a channel with a medium to high correlation among antennas at the base station. When the channel is uncorrelated, the white Gaussian noise dominates the effective noise power. Thus, the channel matched filter precoder seems to be an optimal candidate to send the information symbols through for the uncorrelated channel. As ρ_f increases, the information rate starts to saturate until the interference terms take over the effective noise power (as it will be seen in the results of the simulation as well). The process of saturation occurs sooner and faster as the correlation grows. This correlation between antennas at the base station causes the interference terms to be stronger and more effective than AWGN.

Proposition 1: The average power of the signal using the channel matched filter is given as in (15).

Proof: In order to obtain the average power of the desired signal, we need the following lemma.

Lemma 1: Let $\mathbf{q} \triangleq [q_1, q_2, \cdots, q_N]^T$ be an N-dimensional random vector. If $\mathbb{E}[\mathbf{q}] = \boldsymbol{\mu}$ and $\mathrm{Cov}[\mathbf{q}] = \boldsymbol{\Theta}$, then

$$\mathbb{E}[\mathbf{q}^H \mathbf{B} \mathbf{q}] = \mathrm{tr}(\mathbf{B}\boldsymbol{\Theta}) + \boldsymbol{\mu}^H \mathbf{B} \boldsymbol{\mu}, \tag{10}$$

in which, **B** is a real-valued and symmetric $N \times N$ matrix. The proof of Lemma 1 is provided in the Appendix.

Consider the average power of the desired signal in (4) as

$$\begin{aligned} S_k &= \frac{\rho_f}{MK} \times \mathbb{E}_{s_k[i]} \left[\left| \sum_{l=0}^{L-1} \mathbb{E}\big[\mathbf{v}_l^H[k]\mathbf{v}_l[k]\big] s_k[i] \right|^2 \right] \\ &= \frac{\rho_f}{MK} \times \mathbb{E}_{s_k[i]} \left[\sum_{l'=0}^{L-1} s_k^H[i] \mathbb{E}\big[\mathbf{v}_{l'}^H[k]\mathbf{v}_{l'}[k]\big] \right. \\ &\qquad \left. \times \sum_{l=0}^{L-1} \mathbb{E}\big[\mathbf{v}_l^H[k]\mathbf{v}_l[k]\big] s_k[i] \right]. \end{aligned} \tag{11}$$

We claim that the first multiplier can be written as

$$\begin{aligned} &\sum_{l'=0}^{L-1} s_k^H[i] \mathbb{E}\big[\mathbf{v}_{l'}^H[k]\mathbf{v}_{l'}[k]\big] = \\ &\qquad \sum_{l'=0}^{L-1} s_k^H[i] \mathbb{E}\big[\mathbf{e}_k^T \mathbf{D}_{l'}^{1/2} \mathbf{H}_{l'}^H \mathbf{A} \mathbf{H}_{l'} \mathbf{D}_{l'}^{1/2} \mathbf{e}_k\big]. \end{aligned} \tag{12}$$

Note that according to the structure chosen for the correlation of base station antennas, one can claim when $M \to \infty$, then $\mathbf{H}^H \mathbf{A} \mathbf{H} \to M \times \mathbf{I}_K$. Let's focus on the term $\mathbb{E}[\mathbf{v}_l^H[k]\mathbf{v}_l[k]]$. If we expand this term, it will be

$$\mathbb{E}[\mathbf{v}_l^H[k]\mathbf{v}_l[k]] = \mathbb{E}\big[\mathbf{e}_k^T \mathbf{D}_l^{1/2} \mathbf{H}_l^H \mathbf{A} \mathbf{H}_l \mathbf{D}_l^{1/2} \mathbf{e}_k\big]. \tag{13}$$

Note that using $\mathbf{D}_l^{1/2}\mathbf{e}_k = \sqrt{d_l[k]}\mathbf{e}_k$, we can rewrite the above equation as

$$\mathbb{E}[\mathbf{v}_l^H[k]\mathbf{v}_l[k]] = \sqrt{d_l[k]}\,\mathbb{E}\big[\mathbf{h}_l'[k]\mathbf{A}\mathbf{h}_l[k]\big]\sqrt{d_l[k]}, \tag{14}$$

where $\mathbf{h}_l[k] \triangleq \mathbf{H}_l\mathbf{e}_k$. Note that $\mathbb{E}[\mathbf{h}_l[k]] = \mathbb{E}[\mathbf{h}_l'[k]] = 0$ and $\mathrm{Cov}[\mathbf{h}_l[k]] = \mathrm{Cov}[\mathbf{h}_l'[k]] = \mathbf{I}_K$. By using Lemma 1, one can say $\mathbb{E}\big[\mathbf{h}_l'[k]\mathbf{A}\mathbf{h}_l[k]\big] = \mathrm{tr}(\mathbf{A})$ which is equal to M. Therefore

$$S_k = \frac{\rho_f}{MK} \mathbb{E}_{s_k[i]}\big[|s_k[i]|^2\big] \left(M \sum_{l=0}^{L-1} d_l[k] \right)^2 = \frac{M\rho_f}{K}. \tag{15}$$

Proposition 2: The effective noise power using the channel matched filter is given as in (23).

Proof: By taking a look at (5), we will see that different terms are independent from each other. Therefore, we can write the variance of the effective noise as

$$\begin{aligned} \mathrm{V}(n_k'[i]) &= \rho_1 \mathrm{V}\left[\sum_{l=0}^{L-1} \left(\mathbf{v}_l^H[k]\mathbf{v}_l[k] - \mathbb{E}[\mathbf{v}_l^H[k]\mathbf{v}_l[k]] \right) s_k[i] \right] \\ &\quad + \rho_1 \mathrm{V}\left[\sum_{\substack{b=1-L \\ b\neq 0}}^{L-1} \sum_{l=L_1}^{L_2} \mathbf{v}_l^H[k]\mathbf{v}_{l-b}[k] s_k[i_1] \right] \\ &\quad + \rho_1 \mathrm{V}\left[\sum_{\substack{q=1 \\ q\neq k}}^{K} \sum_{b=1-L}^{L-1} \sum_{l=L_1}^{L_2} \mathbf{v}_l^H[k]\mathbf{v}_{l-b}[q] s_q[i_1] \right] + 1 \end{aligned} \tag{16}$$

where we used V for Var, $\rho_1 = \rho_f/MK$, and $i_1 = i - b$. Note that the means of IF, ISI, and MUI terms are zero. Also, note that $\mathrm{Var}(s_k[i]) = 1$ and the information symbols are independent from all other terms. Therefore, we can rewrite the effective noise variance as

$$\begin{aligned} \mathrm{V}(n_k'[i]) &= \rho_1 \sum_{q=1}^{K} \mathbb{E}_{s_k}\left[s_q^H[i] \sum_{l=0}^{L-1} \mathbb{E}_{H_l}\big[|\mathbf{v}_l^H[k]\mathbf{v}_l[q]|^2\big] s_q[i] \right] \\ &\quad + \rho_1 \sum_{q=1}^{K}\sum_{b=1}^{L-1} \mathbb{E}_{s_k}\left[s_q^H[i_1] \sum_{l=b}^{L-1} \mathbb{E}_{H_l}\big[|\mathbf{v}_l^H[k]\mathbf{v}_{l-b}[q]|^2\big] s_q[i_1] \right] \\ &\quad + \rho_1 \sum_{q=1}^{K}\sum_{b=1}^{L-1} \mathbb{E}_{s_k}\left[s_q^H[i_2] \sum_{l=b}^{L-1} \mathbb{E}_{H_l}\big[|\mathbf{v}_{l-b}^H[k]\mathbf{v}_l[q]|^2\big] s_q[i_2] \right] \\ &\quad - \frac{\rho_f}{K} \sum_{l=0}^{L-1} \mathbb{E}\big[|s_k[i]|^2\big] \mathbb{E}\big[\mathbf{v}_l^H[k]\mathbf{v}_l[k]\big] + \frac{M\rho_f}{K} + 1. \end{aligned} \tag{17}$$

where $i_2 = i + b$. Let's focus on term $\mathbb{E}\big[|\mathbf{v}_l^H[k]\mathbf{v}_{l-b}[q]|^2\big]$. We can expand this term as

$$\mathbb{E}\big[|\mathbf{v}_l^H[k]\mathbf{v}_{l-b}[q]|^2\big] = \mathbb{E}\big[\mathbf{v}_{l'-b'}^H[q']\mathbf{v}_{l'}[k']\mathbf{v}_l^H[k]\mathbf{v}_{l-b}[q]\big]. \tag{18}$$

If $q \neq q'$, all terms inside the expectations in the above equation will be independent from each other and the final value will be zero. The result is the same for the cases that $k \neq k'$, $b \neq b'$, and $l \neq l'$. Thus, we have

$$\mathbb{E}\big[|\mathbf{v}_l^H[k]\mathbf{v}_{l-b}[q]|^2\big] = \mathbb{E}\big[\mathbf{v}_{l-b}^H[q]\mathbf{v}_l[k]\mathbf{v}_l^H[k]\mathbf{v}_{l-b}[q]\big]. \tag{19}$$

Note that in all the summations of the rewritten variance of the effective noise, $b \neq 0$. By using $\mathbf{D}_l^{1/2}\mathbf{e}_k = \sqrt{d_l[k]}\mathbf{e}_k$ and $\mathbf{h}_l[k] = \mathbf{H}_l\mathbf{e}_k$, we obtain

$$\begin{aligned} \mathbb{E}\big[|\mathbf{v}_l^H[k]\mathbf{v}_{l-b}[q]|^2\big] &= d_{l-b}[q] \\ &\quad \times \mathbb{E}\big[\mathbf{h}_{l-b}^H[q]\mathbf{A}\mathbf{h}_l[k]\mathbf{h}_l^H[k]\mathbf{A}\mathbf{h}_{l-b}[q]\big] d_l[k] \\ &= d_{l-b}[q] \\ &\quad \times \mathbb{E}_{H_{(l-b)}}\left[\mathbf{h}_{l-b}^H[q]\mathbf{A} \times \mathbb{E}_{H_l}\big[\mathbf{h}_l[k]\mathbf{h}_l^H[k]\big] \times \mathbf{A}\mathbf{h}_{l-b}[q] \right] d_l[k]. \end{aligned} \tag{20}$$

Note that $\mathbb{E}_{H_l}\big[\mathbf{h}_l[k]\mathbf{h}_l^H[k]\big] = 1$. Hence, we can rewrite the above equation using Lemma 1 as

$$\begin{aligned} \mathbb{E}\big[|\mathbf{v}_l^H[k]\mathbf{v}_{l-b}[q]|^2\big] &= d_{l-b}[q] \times \mathbb{E}\big[\mathbf{h}_{l-b}^H[q]\mathbf{A}^2\mathbf{h}_{l-b}[q]\big] \times d_l[k] \\ &= d_{l-b}[q] \times \mathrm{tr}(\mathbf{A}^2) \times d_l[k]. \end{aligned} \tag{21}$$

By using the above equation in all the summations of the rewritten variance of the effective noise, we obtain

$$\begin{aligned} \mathrm{V}(n_k'[i]) &= \frac{\mathrm{tr}(\mathbf{A}^2)\rho_f}{MK} \sum_{q=1}^{K}\sum_{b=1}^{L-1}\sum_{l=b}^{L-1} \big(d_{l-b}[q]d_l[k] + d_l[q]d_{l-b}[k] \big) \\ &\quad + \frac{\mathrm{tr}(\mathbf{A}^2)\rho_f}{MK} \sum_{q=1}^{K}\sum_{l=0}^{L-1} d_l[k]d_l[q] + 1. \end{aligned} \tag{22}$$

One can simplify the above equation as

$$\mathrm{V}(n_k'[i]) = \frac{\mathrm{tr}(\mathbf{A}^2)}{M}\rho_f + 1. \tag{23}$$

IV. Precoders For the Correlated Channel

In wireless communication technologies the precoding scheme has a significant role. We consider three precoders introduced in [4]. They will be given as functions of the channel state matrix $\mathbf{V}_l$. These precoders are defined as the following

- Maximum-Ratio: It aims to maximize the gain and the received power of the desired signal and is given by

$$\mathbf{W}_{MR}[f] = a_{MR}\mathbf{V}_f, \tag{24}$$

where $\mathbf{V}_f = \sum_{l=0}^{L-1} e^{-j2\pi fl/L}\mathbf{V}_l$ is the N-point Fourier transform of the channel state matrix, and a_{MR} is a power normalization factor.

- Zero-Forcing: It forces the system to eliminate the interference and is given by

$$\mathbf{W}_{ZF}[f] = a_{ZF}\mathbf{V}_f \times \left(\mathbf{V}_f^H\mathbf{V}_f\right)^{-1}, \tag{25}$$

where a_{ZF} is also a normalization factor for this precoder.

- Regularized Zero-Forcing: This precoder maximizes the power of the desired signal compared to the power of the noise and interference at the receiver. It is given by

$$\mathbf{W}_{RZF}[f] = a_{RZF}\mathbf{V}_f \times \left(\mathbf{V}_f^H\mathbf{V}_f + \beta\mathbf{I}_K\right)^{-1}, \tag{26}$$

where a_{RZF} is a power normalization factor and $\beta \in \mathbb{R}^+$ is a system parameter which depends on the SNRs and the path losses of the users.

A common theme among these precoders is the fact that they are defined in the frequency domain and are translated into the time domain. Simulations indicated that precoders defined in the time domain, such as those in [5], do not perform as well as these precoders.

Using these precoders instead of the channel matched filter precoder, one can generate a new system model. Let the channel be identified by its PDP matrix $\mathbf{D}_l$, realization matrix $\mathbf{H}_l$, and correlation matrix $\mathbf{A}$. Note that (1) is still valid and the received signal at user k using $\mathbf{v}_l[k] = \mathbf{V}_l\mathbf{e}_k$ can be obtained as

$$y_k[i] = \sum_{l=0}^{L-1} \mathbf{v}_l[k]^H\mathbf{x}[i-l] + n_k[i]. \tag{27}$$

In order to consider all of the conventional precoding scheme in obtaining the transmitted signal, let $\mathbf{W}[m]$ represent the precoding matrix in a general case. Note that in the definition of the precoders, we already consider the normalization factor. Therefore, the vector of the transmit signals is given by

$$\mathbf{x}[i] = \sum_{m=0}^{N-1} \mathbf{W}[m]\mathbf{s}[i-m]. \tag{28}$$

By using (28) in (27), the received signal at user k can be written as

$$y_k[i] = \sum_{m=0}^{N-1}\sum_{l=0}^{L-1} \mathbf{v}_l^H[k]\mathbf{W}[m]\mathbf{s}[i-l-m] + n_k[i]. \tag{29}$$

Note that (28) represents the cyclic convolution and all the indices of equation defining the transmit signal are taken modulo N (where $N > L$). Therefore, we can rewrite (29) as

$$y_k[i] = \sum_{m=1-N}^{0}\sum_{l=0}^{L-1} \mathbf{v}_l^H[k]\mathbf{W}[m]\mathbf{s}[i-l-m] + n_k[i]. \tag{30}$$

By changing the variable $m+l$ to b, defining $\mathbf{w}_k[m] \triangleq \mathbf{W}[m]\mathbf{e}_k$ as the vector of the precoding scheme related to user k at m-th sampled time, and considering the fact that $\mathbf{s}[i] = \sum_{q=1}^{K}\mathbf{e}_q s_q[i]$, we can rewrite (30) as

$$y_k[i] = \sum_{q=1}^{K}\sum_{b=1-N}^{L-1}\sum_{l=L_1}^{L_3} \mathbf{v}_l^H[k]\mathbf{w}_q[m]s_q[i-b] + n_k[i]. \tag{31}$$

where $L_3 = \min(N-1+b, L-1)$. Note that the desired signal is given by

$$S_k[i] = \sum_{l=0}^{L-1} \mathbb{E}\left[\mathbf{v}_l^H[k]\mathbf{w}_k[-l]\right]s_k[i]. \tag{32}$$

Using the equations of the desired and received signal, one can express the system model in terms of desired signal and effective noise of the channel. Thus, we can rewrite (31) as

$$y_k[i] = \sum_{l=0}^{L-1} \mathbb{E}\left[\mathbf{v}_l^H[k]\mathbf{w}_k[-l]\right]s_k[i] + n'_k[i], \tag{33}$$

where $n'_k[i]$ represents the effective noise and can be written as

$$\begin{aligned} n'_k[i] = &\sum_{l=0}^{L-1}\left(\mathbf{v}_l^H[k]\mathbf{w}_k[-l] - \mathbb{E}\left[\mathbf{v}_l^H[k]\mathbf{w}_k[-l]\right]\right)s_k[i] \\ &+ \sum_{\substack{b=1-N\\ b\neq 0}}^{L-1}\sum_{l=L_1}^{L_3} \mathbf{v}_l^H[k]\mathbf{w}_k[b-l]s_k[i-b] \\ &+ \sum_{\substack{q=1\\ q\neq k}}^{K}\sum_{b=1-N}^{L-1}\sum_{l=L_1}^{L_3} \mathbf{v}_l^H[k]\mathbf{w}_q[b-l]s_q[i-b] \\ &+ n_k[i], \end{aligned} \tag{34}$$

which again includes IF, ISI, MUI, and AWGN terms, respectively. The system model introduced in this section is used in simulations in the next section.

V. Simulation Results

A. Exponential Correlation Model

We will consider a linear antenna array for this correlation model. Assume antenna m_i is in position i and antenna m_j is in position j. It is reasonable to expect, and is evidenced in the literature, that the effect of these antennas on one another should be related to $|i-j|$ considering the dependency of the channel to be among the antennas. We introduce a basic correlation factor $0 < a < 1$ which shows the effects of antennas with respect to $|i-j|$ on each other, where a is a real number. One can obtain a correlation matrix based on this

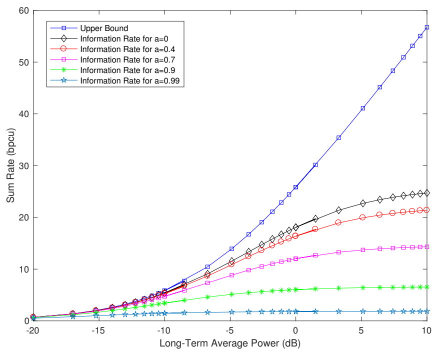

Fig. 1. Comparison between achievable rate of the channel matched filter in a channel with different correlation parameter $a \in \{0, 0.4, 0.7, 0.9, 0.99\}$. Note that based on the formulation for SNR in the channel, normalization is the same for all values of the correlation parameter. Therefore, only one upper bound is shown for all values of a.

model in which the elements are $[A]_{i,j} = a^{|i-j|}$. The matrix in this model is known as the exponential correlation matrix. This model is commonly employed when spatial correlation is considered for MIMO or spatial diversity channels [6], [7]. The correlation coefficient increases as the separation between antennas decreases. An S-parameter-based formulation shows that when only two monopoles are considered the coefficient varies from 0.8 to about 0.2 when the antenna separation is about 0.05 to 0.2 times the wavelength [8]. A measurement study shows that, depending on propagation conditions, spatial correlation can remain significantly higher than 0.9 for a wide range of antenna separation values [9]. A range of average a values from 0.4 to 0.7 for antenna separations at approximately 0.25 to 0.5 wavelength was reported in [10].

For the purpose of simulations, the PDP is considered to be exponential with $L = 4$ and $d_l[k] = \frac{e^{-\theta_k l}}{\sum_{i=0}^{3} e^{-\theta_k i}}$, $l = \{0, \cdots, 3\}$, where $\theta_k = \frac{K-1}{5}$, $k = \{1, \cdots, K\}$ as in [1]. Note that the achievable sum-rate is invariant of the channel PDP. Hence, any other PDP which satisfies (3) can also be considered. Fig. 1 of [1] shows the achievable sum-rate of the channel matched filter via theoretical analysis and simulation in an i.i.d. channel. In this case, the two results perfectly match. The analysis in this paper reduces to that in [1] for $\mathbf{A} = \mathbf{I}$. On the other hand, Fig. 2 of [1] provides typical scenarios in an i.i.d. channel, where the minimum required transmit power to achieve a fixed information rate per each user is shown for the channel matched filter and orthogonal frequency division multiplexing (OFDM) transmission.

Parameters used in simulations in this and the next subsection are as follows. Number of antennas at the base station $M = 50$, number of simultaneous users $K = 10$, channel length $L = 4$, block transmission length $T = 50$, DFT length $N = 20$, (N-point DFT). Simulations were run 40 times on the channel.

In Fig. 1 the performance of the channel matched filter precoder is shown and compared under the influence of the correlation parameter a. It can be seen that by increasing the correlation parameter a, the information rate of the channel

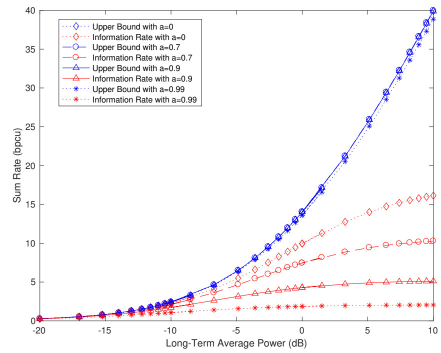

Fig. 2. Comparison between achievable rate of the maximum-ratio precoder in a channel with different correlation parameter $a \in \{0, 0.7, 0.9, 0.99\}$.

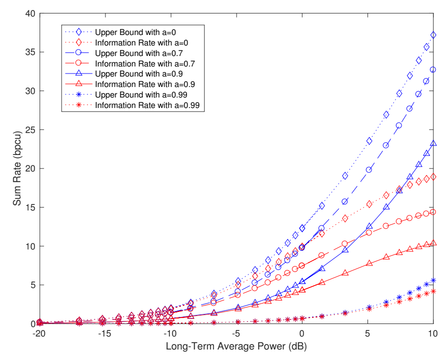

Fig. 3. Capacity and information rate for the zero-forcing precoder in a channel with correlation parameter $a \in \{0, 0.7, 0.9, 0.99\}$.

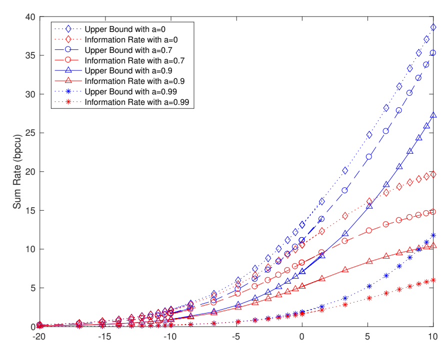

Fig. 4. Upper bound and achievable rate of a channel with correlation parameter $a \in \{0, 0.7, 0.9, 0.99\}$ using the regularized zero-forcing precoder.

matched filter precoder decreases dramatically. This figure proves that the channel matched filter precoder is not a good candidate in a correlated channel with a large correlation parameter, since there is a huge gap between the information rate for $a \in \{0.9, 0.99\}$ and the upper bound capacity of the channel. Note that the gap is still substantial for $a = 0.7$. Based on the simulation results, one can say that the loss in the achievable rate is negligible as long as the correlation parameter is less than 0.5 as it is also stated in [11].

One can see the performances of the three precoders from Section IV in Fig. 2 through Fig. 4. In order to run the simulations for the regularized zero-forcing precoder, we set the value for the system parameter β in order to maximize

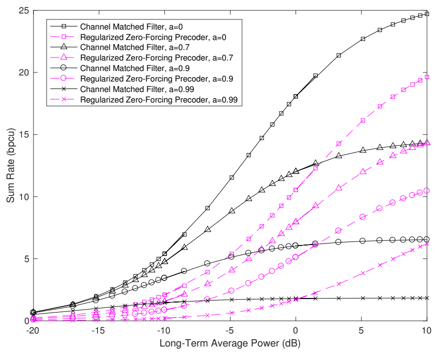

Fig. 5. Performances of the channel matched filter and the regularized zero-forcing precoder in terms of achievable rate in a channel with different correlation parameters.

the sum-rate of the users. How to find such optimal β is described in [4], [12]. In each figure, a comparison is made to show how the information rate changes as the correlation parameter a increases for different precoders. Note that in these figures, due to the dependency of the normalization factor on the correlation parameter, upper bound curves change as the correlation parameter changes, except for the maximum-ratio precoders which behave like the channel matched filter precoder.

As it can be seen, the maximum-ratio precoder starts to fail as the correlation parameter grows, which means this precoder works well for low-rate requirements, but it is limited by interference to below a certain rate. Also, a comparison between Fig. 1 and Fig. 2 shows that except for $a = 0.99$ where the maximum-ratio precoder shows slightly better performance, achievable rates using this precoder are worse than the corresponding ones using the channel matched filter. Due to the higher sensitivity of the maximum-ratio precoder to the correlation parameter, one can conclude that the other two precoders have superiority in highly-correlated channels. Also, it can be shown that the zero-forcing and regularized zero-forcing precoders perform equally well when the number of users is small as well [4]. Because of its ability to balance the resulting array gain and the amount of inter-user interference received by the users, the regularized zero-forcing precoder has an advantage over the zero-forcing precoder when it comes to a larger number of users.

Fig. 3 and Fig. 4 show that the performance of the zero-forcing and regularized zero-forcing precoders appear to be better than that of the channel matched filter for highly correlated channels (i.e., the correlation parameter $a = 0.7$ and higher).

Fig. 5 shows a comparison between the regularized zero-forcing precoder and the channel matched filter in terms of the information rate. As it can be seen, when the channel is correlation-free ($a = 0$), it is the channel matched filter that shows better performance and higher achievable data-rate and as SNR increases, the gap between information rate of the two increases. However, by increasing the correlation parameter in the channel, the regularized zero-forcing precoder shows its superiority in providing information rate in the channel. This behavior was also discussed in [5] as a general possibility.

Further investigation shows the same comparison when the number of users increase to $K = 15$ and the range of transmitted power increases to 40 dB, respectively. The information rate of the regularized zero-forcing precoder for higher SNRs passes that of the channel matched filter, even in an i.i.d. channel. In addition, saturation occurs faster and sooner for the channel matched filter.

B. Bessel Correlation Model

Another correlation model in MIMO systems is introduced in [13]. We refer to this model as the Bessel correlation model. The (i, j)-th element of the new correlation matrix can be obtained by

$$[A]_{i,j} = \frac{I_0\left(\sqrt{\eta^2 - 4\pi^2 d_{i,j}^2 + j4\pi\eta\sin(\mu)d_{i,j}}\right)}{I_0(\eta)}, \tag{35}$$

where $0 < \eta < \infty$ controls the width of the angle-of-arrival (AOA), $\mu \in [-\pi, \pi)$ is the mean direction of the AOA, I_0 is the zero-order modified Bessel function, and $d_{i,j}$ is the distance between j-th and i-th antenna normalized with respect to the wavelength [13]. We consider a linear array for the antenna elements at the base station in this model as well, therefore

$$d_{i,j} = |i - j|d, \tag{36}$$

where d is the distance between adjacent antenna elements, normalized with respect to the wavelength.

Based on the simulations, one can say that the parameter μ has no major effect on the achievable rate of the precoders or the channel matched filter. Therefore, it is taken as 0 in the simulations. The parameter η on the other hand, decreases the achievable rate of the system dramatically by changing from 0 (isotropic scattering) to ∞ (extremely non-isotropic). Fig. 6 shows the effect of the parameter η for a channel using the channel matched filter with $d = 0.5$. The isotropic scattering model, also known as the Clarke's model, corresponds to the uniform distribution for the angle of arrival (AOA). However, empirical measurements have shown that the AOA distribution of waves impinging on the user is more likely to be nonuniform [13]. It is shown in [14] that values of η show a very large variation, pointing out to the substantial change in achievable rate with this fact of propagation.

Fig. 7 shows the performance of the channel matched filter and the regularized zero-forcing precoder under the Bessel correlation model, where the distance between adjacent antenna elements in a linear antenna model is consider to be $d = \{0.125, 0.25, 0.5\}$, normalized with respect to the wavelength. Note that, unlike as in the exponential correlation model, the channel matched filter performs better than conventional precoders (regularized zero-forcing precoder performance is shown in Fig. 7) in the SNR range of [-20 dB, 15 dB] (except for $d = 0.125$). For SNRs higher than 15 dB, the regularized zero-forcing precoder has superior performance (except in the case that $d = 0.125$ for which it occurs at a higher SNR).

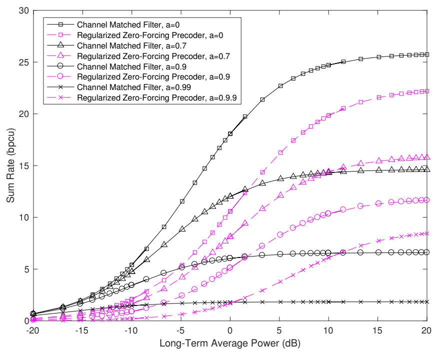

Fig. 6. The effect of the parameter η on the achievable rate for the channel matched filter in a channel with Bessel correlation model. The mean direction of the AOA is 0 and the distance between adjacent antenna elements is considered to be 0.5.

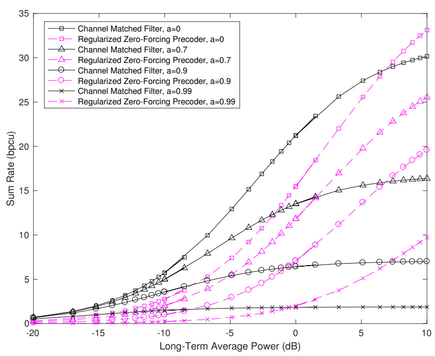

Fig. 7. Performance of the channel matched filter and the regularized zero-forcing precoder in a channel with Bessel correlation model and three different distances between adjacent antenna elements in a linear format.

VI. Conclusion

As a result of the analysis and simulations and analysis in this work, one can say that sum-rate of a channel is highly dependent on the nature of the channel. While for uncorrelated channels, the channel matched filter precoder has optimal performance, it does perform near the upper bound of the channel. However, the presence of correlation among the antennas at the base station leads to a decrease in the achievable sum-rate for the users in the system. Depending on the correlation model and the channel SNR, it is possible to employ another precoder to improve the sum-rate performance. In particular, the regularized zero-forcing precoder can outperform the matched-filter precoder.

Appendix

We will now prove Lemma 1. We can write

$$\begin{aligned}\mathbf{q}^H\mathbf{B}\mathbf{q} &= (\mathbf{q}-\boldsymbol{\mu})^H\mathbf{B}\mathbf{q} + \boldsymbol{\mu}^H\mathbf{B}\mathbf{q} \\ &= (\mathbf{q}-\boldsymbol{\mu})^H\mathbf{B}(\mathbf{q}-\boldsymbol{\mu}) + \boldsymbol{\mu}^H\mathbf{B}\mathbf{q} + (\mathbf{q}-\boldsymbol{\mu})^H\mathbf{B}\boldsymbol{\mu}.\end{aligned} \tag{37}$$

If we take expectations over the above equation, we obtain

$$\mathbb{E}[\mathbf{q}^H\mathbf{B}\mathbf{q}] = \mathbb{E}\big[(\mathbf{q}-\boldsymbol{\mu})^H\mathbf{B}(\mathbf{q}-\boldsymbol{\mu})\big] + \boldsymbol{\mu}^H\mathbf{B}\boldsymbol{\mu}. \tag{38}$$

Let $u_j \triangleq q_j - \mu_j$. Therefore $\mathbf{u} = \mathbf{q} - \boldsymbol{\mu}$ and we will have

$$\begin{aligned}\mathbb{E}\big[(\mathbf{q}-\boldsymbol{\mu})^H\mathbf{B}(\mathbf{q}-\boldsymbol{\mu})\big] &= \mathbb{E}[\mathbf{u}^H\mathbf{B}\mathbf{u}] = \sum_{i=1}^{N}\sum_{j=1}^{N}\mathbb{E}[u_iB_{i,j}u_j] \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}B_{i,j}[\mathrm{Cov}(\mathbf{u})]_{i,j} = \sum_{i=1}^{N}\sum_{j=1}^{N}B_{i,j}[\mathrm{Cov}(\mathbf{q}-\boldsymbol{\mu})]_{i,j} \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}B_{i,j}[\mathrm{Cov}(\mathbf{q})]_{i,j} = \sum_{i=1}^{N}\sum_{j=1}^{N}B_{i,j}\Theta_{i,j} \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}B_{i,j}\Theta_{j,i} = \sum_{i=1}^{N}[\mathbf{B}\boldsymbol{\Theta}]_{i,i} = \mathrm{tr}(\mathbf{B}\boldsymbol{\Theta}).\end{aligned} \tag{39}$$

Combining the two obtained results, we have

$$\mathbb{E}[\mathbf{q}^H\mathbf{B}\mathbf{q}] = \mathrm{tr}(\mathbf{B}\boldsymbol{\Theta}) + \boldsymbol{\mu}^H\mathbf{B}\boldsymbol{\mu}, \tag{40}$$

as desired.

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