UCI EECS
EECS141B
Spring 2007

FINAL EXAMINATION

Name:	
Student ID #:	

1	/20
2	/10
3	/20
4	/15
5	/20
6	/15
Total	

Midterm		
Final		
Homework Average		
Extra Credit		
Total		
Course Grade		

Useful identitites

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

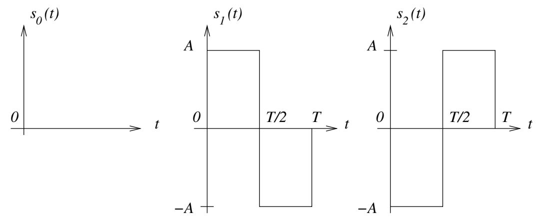
$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\sin(A)\cos(B) = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$$

$$\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$

$$\cos(A)\cos(B) = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$

1. *(20 points)*



Assume the signals $s_0(t)$, $s_1(t)$, and $s_2(t)$ shown above are used to convey one of three messages with a priori probabilities p_0, p_1, and p_2, respectively. Assume the message goes through a zero-mean additive white Gaussian noise channel with power spectral density $N_0/2$ W/Hz. Let $p_1 = p_2$.

(a) Draw the optimum receiver.

(b) Draw and label the signal constellation.

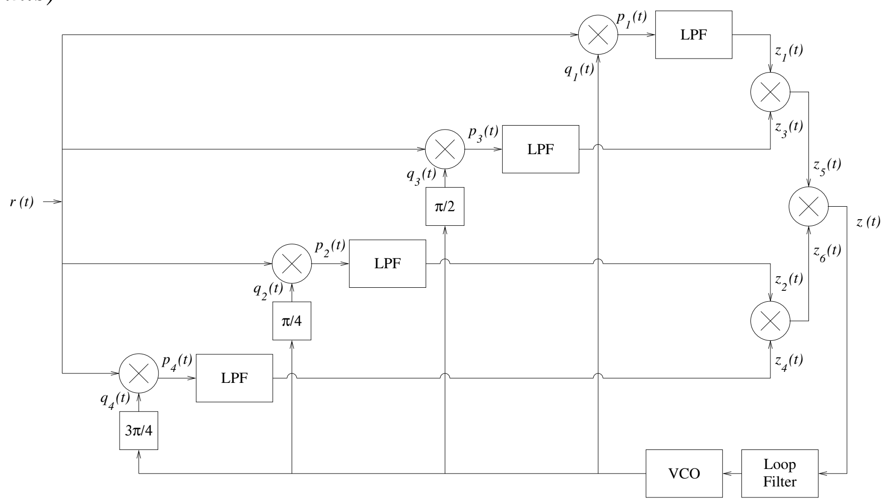
(c) Determine the optimum decision rule.

(d) Calculate the probability of error associated with the optimum receiver.

(e) Assume p_0, p_1, p_2 are not known at the receiver. Based on this condition, calculate the probability of error associated with the optimum receiver.

2. *(10 points)* Let $X_n = aX_{n-1} + W_n$ where W_n is a discrete-time zero-mean Gaussian process with autocorrelation function $E[W_n W_{n+k}] = \delta_k$ for $k = 0, \pm 1, \pm 2, \ldots$, and $0 < a < 1$ is a constant. Calculate the coefficients of a Wiener filter of order M such that the desired response is $d_n = X_{n+2}$.

3. *(20 points)*



Consider the Costas loop for QPSK shown in the figure above. In the absence of noise, let

$$r(t) = \cos(2\pi f_c t + \theta_m(t) + \phi)$$

where the message is carried by $\theta_m(t) \in \{0, \pi/2, \pi, 3\pi/2\}$ which changes every symbol period T, and ϕ is the transmitter clock phase. As usual, assume $T = n/f_c$ where n is a positive integer. As shown in the figure above, let

$$q_i(t) = \sin\left(2\pi f_c t + \hat{\phi} + (i-1)\frac{\pi}{4}\right) \qquad i = 1, 2, 3, 4$$

where $\hat{\phi}$ is the receiver clock phase.

(a) Calculate $p_1(t)$ and $p_3(t)$, and then $z_1(t)$ and $z_3(t)$.

(b) Calculate $z_5(t) = z_1(t)z_3(t)$.

(c) Calculate $p_2(t)$ and $p_4(t)$, and then $z_2(t)$ and $z_4(t)$.

(d) Calculate $z_6(t) = z_2(t)z_4(t)$.

(e) Calculate $z(t) = z_5(t)z_6(t)$.

(f) Explain if $z(t)$ can be used to drive the VCO so that $\hat{\phi}$ tracks ϕ.

4. *(15 points)* Let $\mathcal{S}$ be a discrete memoryless source with the following symbols and the associated probability distribution.

s_k	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
p_k	0.20	0.18	0.16	0.15	0.11	0.08	0.05	0.04	0.03

(a) Calculate the entropy $H(\mathcal{S})$ of the source $\mathcal{S}$.

(b) Calculate the Huffman code for this source and list the codewords associated with each source symbol.

(c) Calculate the average codeword length $\bar{L}$ for this code.

(d) Calculate the efficiency η of this code.

5. *(20 points)* Recall that the polynomial $X^7 + 1$ can be factored as

$$(X^7 + 1) = (1 + X)(1 + X^2 + X^3)(1 + X + X^3).$$

You will design a (7,3) code based on the generator polynomial

$$g(X) = (1 + X)(1 + X + X^3)$$

(a) Calculate a nonsystematic generator matrix based on $g(X)$.

(b) Convert the nonsystematic generator matrix of part (a) into a systematic generator matrix **G**.

(c) Calculate all codewords generated by **G** and their Hamming weights.

(d) How many bit errors can this code correct?

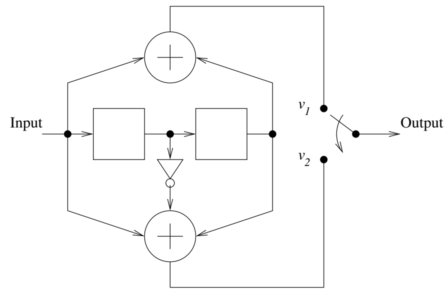
(e) Calculate the parity-check matrix **H** corresponding to **G**.

(f) Calculate the syndrome values associated with all bit errors in your response to (d) above.

(g) Draw the encoder circuit associated with $g(X)$.

(h) Draw the syndrome circuit associated with $g(X)$.

6. *(15 points)*



Consider the convolutional code in the figure above.

(a) Draw the state transition diagram of the convolutional encoder as a finite-state machine.

(b) Draw a section of the trellis assocated with the convolutional encoder.

(c) Assume that the original state of the convolutional encoder is 00. Let the received sequence be 0110100000. Using the Viterbi algorithm, calculate the optimum decision for the input sequence in the maximum likelihood sense. (*Hint:* If two path metrics entering into the same state are of equal value, you can pick one of the paths randomly.)