UCI EECS
EECS141B
Spring 2008

FINAL EXAMINATION

Name:	
Student ID #:	

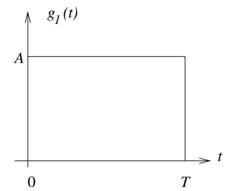
1	/15
2	/20
3	/20
4	/15
5	/15
6	/15
Total	

Midterm		
Final		
Homework Average		
Extra Credit		
Total		
Course Grade		

Useful identitites

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$
$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$
$$\sin(A)\cos(B) = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$$
$$\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$
$$\cos(A)\cos(B) = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$

1. *(15 points)*



Let $g_1(t)$ be as shown in the figure above and let

$$g_2(t) = \begin{cases} \sqrt{2}A\cos(2\pi f_c t) & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

where $T = n/f_c$, n is a positive integer. Consider the transmitted signal

$$s(t) = \sum_{k=-\infty}^{\infty} a_k g_1(t - kT) + b_k g_2(t - kT)$$

where $a_k \in \{\pm 1, \pm 3\}$ and $b_k \in \{\pm 1, \pm 3, \pm 5\}$. The signal $s(t)$ is corrupted by additive white Gaussian noise with zero mean and power spectral density $N_0/2$. Calculate the exact probability of symbol error for this system.

2. *(20 points)* Let $X_n = aW_{n-1} + W_n$ where W_n is a discrete-time zero-mean Gaussian process with autocorrelation function $E[W_n W_{n+k}] = \delta_k$ for $k = 0, \pm 1, \pm 2, \ldots$, and $0 < a < 1$ is a constant.

   (a) Calculate $E[X_n^2]$.

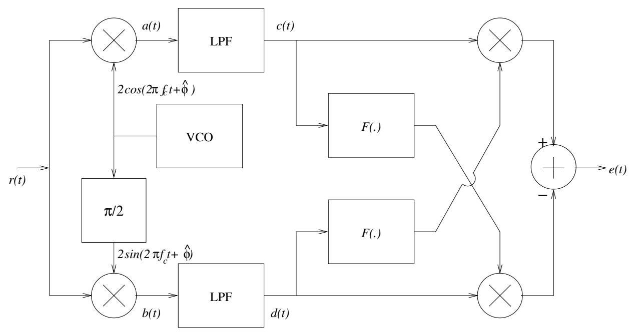
   (b) Calculate $E[X_n X_{n-1}]$.

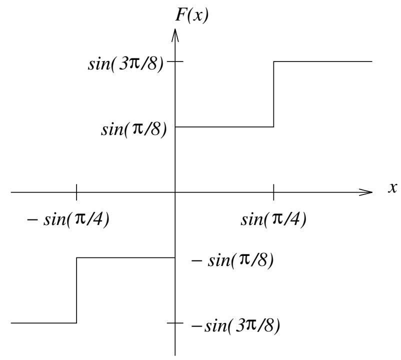
   (c) Let $d_n = X_{n+1}$. Calculate $E[d_n X_n]$ and $E[d_n X_{n-1}]$.

   (d) Calculate the coefficients of a one-dimensional Wiener filter to calculate d_n from X_n.

   (e) Calculate the coefficients of a two-dimensional Wiener filter to calculate d_n from X_n and X_{n-1}.

3. *(20 points)*





The circuit above is a Costas loop for an 8-PSK signal which is given as

$$\begin{aligned} s(t) &= \sum_k \sin(2\pi f_c t + \theta_k + \phi) \\ &= \sum_k a_k \cos(2\pi f_c t + \phi) + b_k \sin(2\pi f_c t + \phi). \end{aligned}$$

In this equation $a_k = \sin(\theta_k)$, $b_k = \cos(\theta_k)$, and $\theta_k = (2l_k - 1)\frac{\pi}{8}$ where $l_k \in \{1, 2, \ldots, 8\}$ is the message at time k. Note that, due to symmetry, both a_k and b_k take values from the set $\{\pm \sin(\pi/8), \pm \sin(3\pi/8)\}$ and $F(a_k \pm \delta) = a_k$ and $F(b_k \pm \delta) = b_k$ where δ is a small number (strictly speaking, $0 < \delta < \sin(\pi/4) - \sin(\pi/8)$).

(a) Calculate $a_k^2 + b_k^2$.

(b) Ignoring the effect of noise, during one symbol duration, the received signal is

$$r(t) = a_k \cos(2\pi f_c t + \phi) + b_k \sin(2\pi f_c t + \phi).$$

Calculate $a(t)$ and $b(t)$ based on $r(t)$ and local oscillator signals as shown.

(c) Calculate $c(t)$ and $d(t)$.

(d) Assuming $\Delta\phi = \phi - \hat{\phi}$ is small, and considering the property of $F(.)$ above, calculate $F(c(t))$ and $F(d(t))$.

(e) Now calculate $e(t)$.

(f) Can $e(t)$ be used to drive the VCO? Why?

4. *(15 points)* Let $\mathcal{S}$ be a discrete memoryless source with the following symbols and the associated probability distribution.

s_k	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
p_k	0.30	0.20	0.16	0.10	0.08	0.05	0.04	0.04	0.02	0.01

(a) Calculate the entropy $H(\mathcal{S})$ of the source $\mathcal{S}$ in bits.

(b) Calculate the Huffman code for this source and list the codewords associated with each source symbol.

(c) Calculate the average codeword length $\bar{L}$ for this code.

(d) Calculate the efficiency $\eta = H(\mathcal{S})/\bar{L}$ of this code.

5. *(15 points)* Recall that the polynomial $X^{15} + 1$ can be factored as

$$(X^{15} + 1) = (1 + X)(1 + X + X^2)(1 + X + X^4)(1 + X^3 + X^4)(1 + X + X^2 + X^3 + X^4).$$

You will design a (15,3) code based on the generator polynomial

$$g(X) = (1 + X + X^4)(1 + X^3 + X^4)(1 + X + X^2 + X^3 + X^4)$$

(a) Calculate a nonsystematic generator matrix based on $g(X)$.

(b) Convert the nonsystematic generator matrix of part (a) into a systematic generator matrix **G**.

(c) Calculate all codewords generated by **G** and their Hamming weights.

(d) How many bit errors can this code correct?

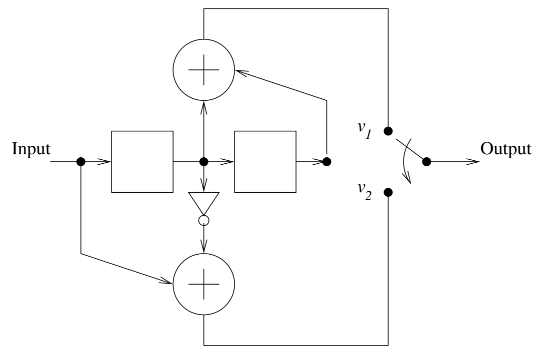
(e) Calculate the parity-check matrix **H** corresponding to **G**.

(f) Calculate the syndrome values associated with all single bit errors.

(g) Draw the encoder circuit associated with $g(X)$.

(h) Draw the syndrome circuit associated with $g(X)$.

6. *(15 points)*



Consider the convolutional code in the figure above.

(a) Draw the state transition diagram of the convolutional encoder as a finite-state machine.

(b) Draw a section of the trellis assocated with the convolutional encoder.

(c) Assume that the original state of the convolutional encoder is 00. Let the received sequence be 0010101101. Using the Viterbi algorithm, calculate the optimum decision for the input sequence in the maximum likelihood sense. (*Hint:* If two path metrics entering into the same state are of equal value, you can pick one of the paths randomly.)