UC Irvine
EECS 141B
Spring 2010

FINAL EXAMINATION

Name:	
Student ID #:	

1	/20
2	/20
3	/16
4	/16
5	/16
6	/12
Total	

Midterm		
Final		
Homework Average		
Extra Credit		
Total		
Course Grade		

Useful identitites

$$\cos(A)\cos(B) = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$

$$\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$

$$\sin(A)\cos(B) = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$$

1. *(20 points)* Consider the message signal

$$s_k(t) = \begin{cases} A_c \cos\left(2\pi f_{c,1} t + \frac{\pi}{2} i\right) + \dfrac{A_c}{\sqrt{2}} \cos(2\pi f_{c,2} t + \pi j) & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

where $i \in \{0, 1, 2, 3\}$, $j \in \{0, 1\}$, $k = i + 4j \in \{0, 1, \ldots, 7\}$, $f_{c,1} = \frac{m}{T}$, $f_{c,2} = \frac{n}{T}$, m and n integers, $m \neq n$.

(a) Specify a minimum set of orthonormal basis functions for $s_k(t)$, $k = 0, 1, \ldots, 7$.

(b) Draw the optimum receiver.

(c) Draw the signal constellation and specify the optimum decision regions assuming the 8 messages are equally likely.

(d) Calculate the probability of symbol error.

(e) In your constellation in part (c) above, show a bit mapping that minimizes the probability of bit error.

2. *(20 points)* Let

$$X_n = aW_n + W_{n-1}$$

where $0 < a < 1$ and W_n is a white random process, $E[W_n] = 0$, $E[W_n^2] = 1$. The desired response is $d_n = X_{n+1}$.

(a) Calculate the optimum predictor filter of order $M = 1$.

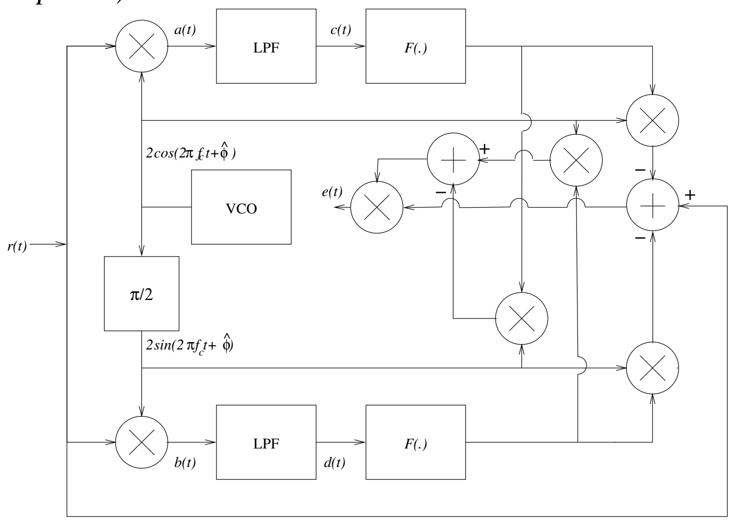
(b) Calculate the optimum predictor filter of order $M = 2$.

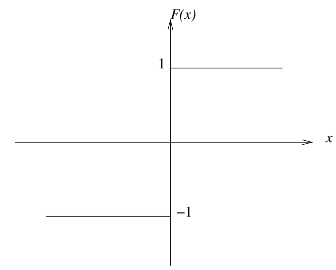
(c) Calculate the mean square error for the optimum predictor filter of order $M = 1$, $J_{\min}(1)$.

(d) Calculate the mean square error for the optimum predictor filter of order $M = 2$, $J_{\min}(2)$.

(e) What can you say about $J_{\min}(1)$ and $J_{\min}(2)$? Why?

3. *(16 points)*





The circuit above is a carrier phase tracking loop proposed for wideband QAM signals. We will assume the operation is for 4-QAM, but other constellations can also be supported by changing the function $F(x)$. The signal is given as

$$s(t) = \sum_k a_k \cos(2\pi f_c t + \phi) + b_k \sin(2\pi f_c t + \phi)$$

where $a_k, b_k \in \{\pm 1\}$ specify the message at time k.

(a) Ignoring the effect of noise, during one symbol duration, the received signal is

$$r(t) = a_k \cos(2\pi f_c t + \phi) + b_k \sin(2\pi f_c t + \phi).$$

Calculate $a(t)$ and $b(t)$ based on $r(t)$ and local oscillator signals as shown.

(b) Calculate $c(t)$ and $d(t)$.

(c) Assuming $\Delta\phi = \phi - \hat{\phi}$ is small, and considering the property of $F(.)$ above, calculate $F(c(t))$ and $F(d(t))$.

(d) Now calculate $e(t)$.

(e) Can $e(t)$ be used to drive the VCO? Why?

4. *(16 points)* Let $\mathcal{S}$ be a discrete memoryless source with the following symbols and the associated probability distribution.

s_k	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
p_k	0.29	0.19	0.12	0.09	0.08	0.07	0.06	0.055	0.035	0.01

(a) Calculate the entropy $H(\mathcal{S})$ of the source $\mathcal{S}$ in bits.

(b) Calculate the Huffman code for this source and list the codewords associated with each source symbol.

(c) Calculate the average codeword length $\bar{L}$ for this code.

(d) Calculate the efficiency $\eta = H(\mathcal{S})/\bar{L}$ of this code.

5. *(16 points)* Recall that the polynomial $X^{15} + 1$ can be factored as

$$(X^{15} + 1) = (1 + X)(1 + X + X^2)(1 + X + X^4)(1 + X^3 + X^4)(1 + X + X^2 + X^3 + X^4).$$

You will design a (15,4) code based on the generator polynomial

$$g(X) = (1 + X)(1 + X + X^2)(1 + X + X^4)(1 + X + X^2 + X^3 + X^4)$$

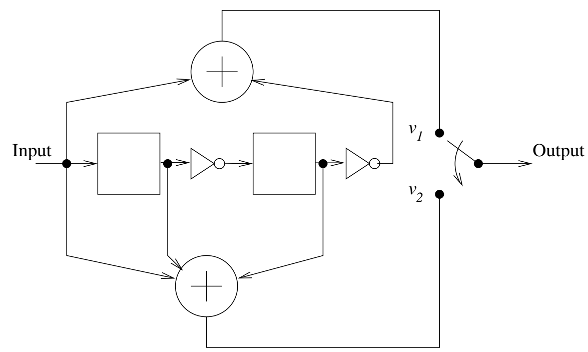
(a) Calculate a nonsystematic generator matrix based on $g(X)$.

(b) Convert the nonsystematic generator matrix of part (a) into a systematic generator matrix **G**.

(c) Draw the encoder circuit associated with $g(X)$.

(d) Draw the syndrome circuit associated with $g(X)$.

6. *(12 points)*



Consider the convolutional code in the figure above.

(a) Draw the state transition diagram of the convolutional encoder as a finite-state machine.

(b) Draw a section of the trellis assocated with the convolutional encoder.

(c) Assume that the original state of the convolutional encoder is 00. Let the received sequence be 0111001011. Using the Viterbi algorithm, calculate the optimum decision for the input sequence in the maximum likelihood sense.