UC Irvine
EECS 141B
Spring 2011

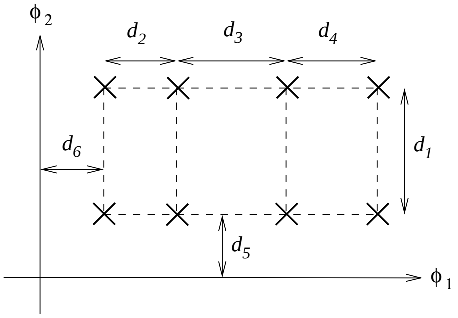
FINAL EXAMINATION

Name:	
Student ID #:	

1	/20
2	/12
3	/18
4	/18
5	/20
6	/12
Total	

Midterm		
Final		
Homework Average		
Extra Credit		
Total		
Course Grade		

1. *(20 points)*



Consider the constellation in the figure above. It is known that each of the eight signals in the constellation is equally likely. The signal in each dimension is corrupted by additive white Gaussian noise with zero mean and variance σ^2. For this constellation, calculate the *exact* probability of symbol error incurred by the optimum receiver.

2. *(12 points)* Due to the limited dynamic range of the implementation, the LMS algorithm can present a problem if the weight vector $\mathbf{w}(kT)$ becomes very large. To prevent this possibility, the algorithm can be modified to keep the "length" of the vector $\mathbf{w}(kT)$ from growing without bound. Also, averages of other even powers of $e(kT)$ than its square are sometimes considered.

   (a) Note that the square of the "length" of a vector $\mathbf{w}$ is given as $\sum_{j=0}^{N-1} w_j^2$ where $\mathbf{w} = (w_0, w_1, \ldots, w_{N-1})^T$. We denote it as $\|\mathbf{w}\|^2 = \mathbf{w}^T\mathbf{w} = \sum_{j=0}^{N-1} w_j^2$. Calculate

   $$\frac{\partial \|\mathbf{w}\|^2}{\partial w_i} = \frac{\partial}{\partial w_i} \sum_{j=0}^{N-1} w_j^2.$$

   for $i = 0, 1, \ldots, N-1$, and specify $\nabla_{\mathbf{w}} \|\mathbf{w}\|^2$.

   (b) We derived in class that if $\mathcal{E} = \sum_k e^2(kT)$, then

   $$\frac{\partial \mathcal{E}}{\partial w_i} = 2\sum_k e(kT)\frac{\partial e(kT)}{\partial w_i} = 2\sum_k e(kT)\frac{\partial y(kT)}{\partial w_i} = 2\sum_k e(kT)x(kT - iT)$$

   for $i = 0, 1, \ldots, N-1$, or

   $$\nabla_{\mathbf{w}}\mathcal{E} = \left(\frac{\partial \mathcal{E}}{\partial w_0}, \frac{\partial \mathcal{E}}{\partial w_1}, \ldots, \frac{\partial \mathcal{E}}{\partial w_{N-1}}\right)^T = 2\sum_k e(kT)\mathbf{x}(kT).$$

   Define a cost function

   $$J = \alpha\|\mathbf{w}\|^2 + \sum_k e^4(kT)$$

   where $0 < \alpha < 1$ is a constant. Calculate $\nabla_{\mathbf{w}} J$.

   (c) Recall that the steepest descent algorithm to minimize J is given as

   $$\mathbf{w}(kT + T) = \mathbf{w}(kT) - \frac{1}{2}\mu\nabla_{\mathbf{w}} J.$$

   By substituting your result from part (c) above in the steepest descent formula above, derive the "true gradient" algorithm to minimize J.

   (d) Substitute the true gradient $\nabla_{\mathbf{w}} J$ with the "stochastic gradient" $\hat{\nabla}_{\mathbf{w}} J$ to calculate an algorithm to minimize J recursively. Recall that in the case of the LMS algorithm, the true gradient is $2\sum_k e(kT)\mathbf{x}(kT)$, whereas the stochastic gradient is an approximation of it, equal to $2e(kT)\mathbf{x}(kT)$. The algorithm you derive should be similar to the LMS algorithm, with a slight difference.

3. *(18 points)* Consider the linearized model of the PLL as discussed in class with loop filter $G(s)$. Let $e(t) = \phi(t) - \hat{\phi}(t)$ be the error in phase. We define the steady-state of the error as $\lim_{t \to \infty} e(t)$.

   (a) Consider the phase transfer function defined as $H(s) = \hat{\Phi}(s)/\Phi(s)$. The linear loop filter $G(s)$ is in the form of the ratio of two polynomials $N(s)$ and $D(s)$, $G(s) = N(s)/D(s)$ where the degree of $N(s)$ is less than or equal to the degree of $D(s)$. The *order* of a PLL is defined as the largest power of s in the denominator of $H(s)$. Show that this number is equal to one plus the highest power of s in $D(s)$.

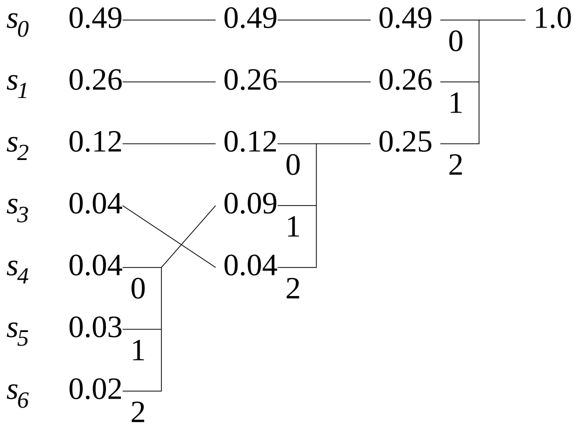
   (b) Assume the input is $\phi(t) = \phi_0 \, u(t)$ (phase step). What is the condition on $G(0)$ that will ensure that the steady-state error is zero?

   (c) Assume the input is $\phi(t) = 2\pi\Delta f \, t \, u(t)$ (frequency step). What is the condition on $G(0)$ that will ensure that the steady-state error is zero?

   (d) Fill in the following table.

$G(s)$	$G(0)$	PLL Order	Steady-State Error for Phase Step	Steady-State Error for Frequency Step
$K_0 s$				
K_1				
$\frac{K_1}{s+K_1}$				
$K_1 + \frac{K_2}{s}$				
$\frac{K_1+s}{K_2+s}$				

4. *(18 points)*



s_0	0
s_1	1
s_2	20
s_3	22
s_4	210
s_5	211
s_6	212

Consider the fixed finite alphabet $\mathcal{S} = \{s_0, s_1, \ldots, s_6\}$ with the corresponding probabilities

p_0	p_1	p_2	p_3	p_4	p_5	p_6
0.49	0.26	0.12	0.04	0.04	0.03	0.02

.

For this source, a ternary Huffman tree (with symbols 0, 1, and 2) and the corresponding ternary Huffman code is given in the figure above. The average length of this code is

$$\bar{L} = 1 \times (0.49 + 0.26) + 2 \times (0.12 + 0.04) + 3 \times (0.04 + 0.03 + 0.02) = 1.34 \text{ ternary symbols},$$

and the entropy $H_3(\mathcal{S})$ is

$$H_3(\mathcal{S}) = -\sum_{k=0}^{6} p_k \log_3(p_k) = 1.27 \text{ ternary symbols.}$$

(a) We will use quaternary symbols 0, 1, 2, and 3 to compress tis source. Draw the quaternary Huffman tree and calculate the quaternary Huffman code for each of the source symbols.

(b) What is the average length of the quaternary Huffman code you calculated in part (a) above? Compare with the quaternary entropy $H_4(\mathcal{S}) = -\sum_{k=0}^{6} p_k \log_4(p_k)$?

(c) We will use binary symbols 0 and 1 to compress tis source. Draw the binary Huffman tree and calculate the binary Huffman code for each of the source symbols.

(d) What is the average length of the binary Huffman code you calculated in part (c) above? Compare with the binary entropy $H_2(\mathcal{S}) = -\sum_{k=0}^{6} p_k \log_2(p_k)$?

(e) Comment on your results.

5. *(20 points)* Consider a $(6, 3)$ systematic linear block code with the generator matrix

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

(a) We want to employ this code to correct all single bit errors. Calculate and tabulate the standard array to serve this purpose. In your array, show the syndrome value for each coset.

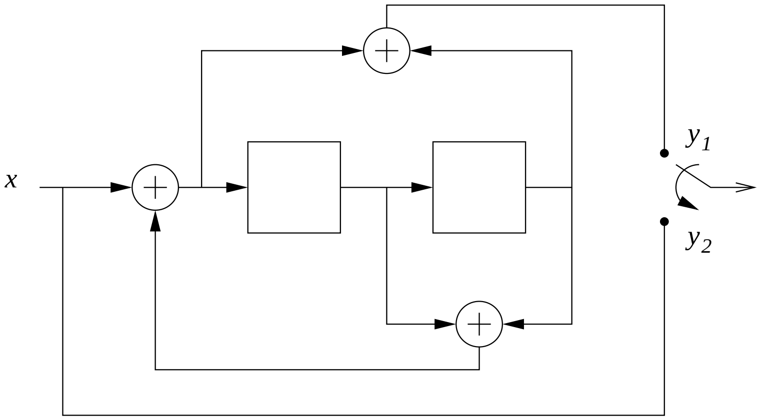
(b) Fill in the following table where **r** is the received 6-tuple, **s** is the syndrome, **e** is the error pattern, **c** is the transmitted 6-tuple, and **m** is the transmitted 3-bit message.

r	**s**	**e**	**c**	**m**
101100				
000010				
000011				

(c) How many more cosets can the standard array accommodate?

(d) *(Bonus)* Can you determine the remaining cosets? If yes, list them on the standard array, together with their syndrome values.

6. *(12 points)*



Consider the convolutional code in the figure above.

(a) Draw its state transition diagram. Label the edges in terms of the input and output variables as x/y_1y_2.

(b) Draw one section of the trellis for this convolutional code. Clearly label edges in terms of the input and output symbols as x/y_1y_2.

(c) Assume that the both of the memory elements of this convolutional code contain the value 0 at time 0. Assume that the receivver receives the sequence 00 10 01 00 10. Determine the channel sequence that must have been transmitted, optimum in the maximum likelihood sense.

(d) What is the optimum input sequence corresponding to the channel sequence you calculated in part (c) above?