UC Irvine
EECS 141B
Spring 2013

HOMEWORK 1
(Due 4/16/2013)

1. (a) A pair of signals $s_1(t)$ and $s_2(t)$ have a common duration T. Show that the inner product of this pair of signals is given by

$$\int_0^T s_i(t)s_k(t)dt = \mathbf{s}_i^T\mathbf{s}_k$$

where $\mathbf{s}_i$ and $\mathbf{s}_k$ are vector representations of $s_i(t)$ and $s_k(t)$, respectively.

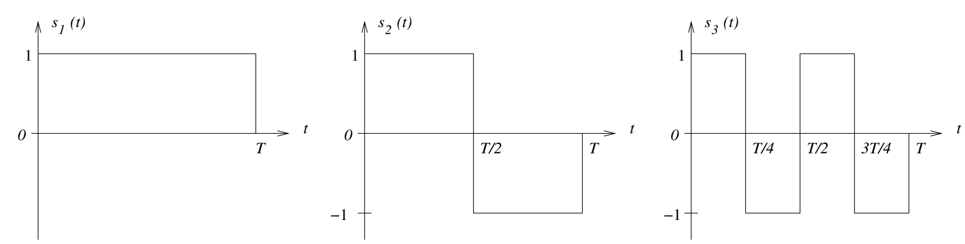
(b) As a followup to part (a), show that

$$\int_0^T (s_i(t) - s_k(t))^2 dt = \|\mathbf{s}_i - \mathbf{s}_k\|^2.$$

2. (a) Use the Gram-Schmidt orthonormalization procedure to calculate a set of basis functions for signals $s_1(t)$, $s_2(t)$, and $s_3(t)$ shown below.

(b) Draw the signal constellation corresponding to the signals $s_1(t)$, $s_2(t)$, $s_3(t)$.

(c) Could you have calculated the basis functions without going through the Gram-Schmidt orthonormalization procedure?



3. Let

$$s_i(t) = \begin{cases} \cos(2\pi(f_c + i\Delta f)t) & \text{for } 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

where $T = n/f_c$ for a positive integer n, and $i = 0, 1, \ldots, N-1$. In other words, T corresponds to an integer multiple of the period of $\cos(2\pi f_c t)$. What is the smallest value of Δf for which all $s_i(t)$ are orthogonal to each other? (*Hint:* Integration of $\cos(2\pi f_1 t)$, starting from $t = 0$ and over an integer multiple of *half* of its period $1/f_1$ yields 0 due to the cancellation of positive and negative quarters of a cycle.)

4. Let

$$s_1(t) = \begin{cases} \cos(2\pi f_c t) & \text{for } 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

and

$$s_2(t) = \begin{cases} \sin(2\pi f_c t) & \text{for } 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

where $T = n/f_c$ for a positive integer n.

(a) Use the Gram-Schmidt orthonormalization procedure to calculate a set of basis functions for signals $s_1(t)$ and $s_2(t)$.

(b) Draw the signal constellation corresponding to the signals $s_1(t)$ and $s_2(t)$.

(c) Could you have calculated the basis functions without going through the Gram-Schmidt orthonormalization procedure?

(d) What would your response to (c) above would be if T could take on any positive real value?

5. Suppose that $s(t)$ is a real-valued signal. We have a set of orthonormal basis functions $\{\phi_i(t)\}_{i=1}^{N}$ with which we wish to approximate $s(t)$ as

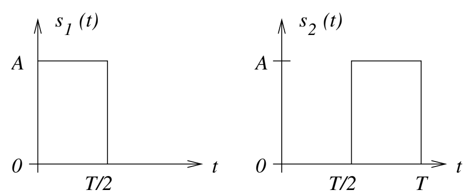
$$\hat{s}(t) = \sum_{i=1}^{N} s_i \phi_i(t)$$

such that the energy in the error signal $e(t) = s(t) - \hat{s}(t)$

$$E_e = \int_{-\infty}^{\infty} e^2(t)dt = \int_{-\infty}^{\infty} (s(t) - \hat{s}(t))^2 dt$$

is minimized. Calculate the optimum coefficients $\{s_i\}_{i=1}^{N}$ that minimize E_e.

6. Assume a transmitter transmits the following waveforms for messages m_1 and m_2, respectively.



(a) What is the dimensionality of the signal space?

(b) Draw the optimum receiver structure.

(c) Specify the optimum decision rule in its simplest form.

(d) Draw the signal constellation and show the decision regions.

(e) Assuming the channel adds white Gaussian noise with zero mean and power spectral density $N_0/2$ to the transmitted signal, calculate the probability of error that would be achieved by the optimum receiver.

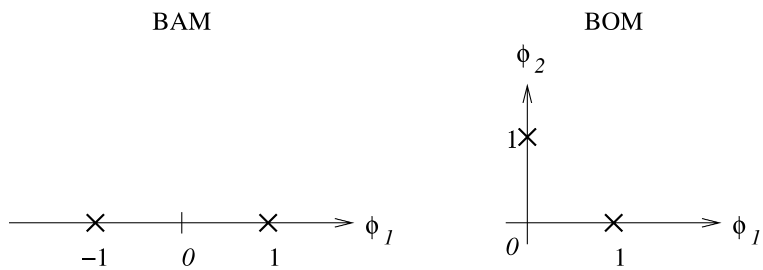
(f) Assume this system is used to transmit data at a rate of 4 Mb/s. During transmission, white Gaussian noise of zero mean and power spectral density 10^{-20} W/Hz is added to the signal. In the absence of noise, the amplitude of the received signal is 1 μV. Calculate the probability of error associated with this setup.

(g) Calculate the approximate probabilities of error if the transmission rate is doubled or halved of that in part (f).

7. (a) Consider the two constellations marked BAM (Binary Antipodal Modulation) and BOM (Binary Orthogonal Modulation) in the figure below. Assuming that these two signals go through the same zero-mean AWGN channel with power spectral density $N_0/2$, calculate the probability of error for BAM and BOM.

   (b) For probability of error performance, which one do you prefer and why?

   (c) Can you specify by how many decibels (dB) in signal-to-noise ratio the preferred one is better?



8. Let

$$s_i(t) = A_i \mathrm{rect}\left(\frac{t - T/2}{T}\right) = A_i \mathrm{rect}\left(\frac{t}{T} - \frac{1}{2}\right)$$

be used to transmit data during a period of T seconds where $A_i = \pm 1, \pm 3, \pm 5, \pm 7$ and $i = 1, 2, \ldots, 8$. The rectangular function $\mathrm{rect}(t)$ is defined as

$$\mathrm{rect}(t) = \begin{cases} 1 & |t| \le 1/2 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the following approximations to the symbol error probability

(a) The union bound,

(b) The nearest neighbor bound,

(c) The first order approximation based on $d_{\min}$.