

HOMEWORK 4
(due 6/4/2013)

1. An analog signal with 4 kHz bandwidth is sampled at 1.25 times the Nyquist rate, and each sample is quantized into one of 256 equally likely levels. The successive samples are statistically independent.
 - (a) After sampling and quantization, how many bits per second does the source generate?
 - (b) Can the output of this source be transmitted without error over an AWGN channel with a bandwidth of 10 kHz and an SNR of 20 dB?
 - (c) Find the SNR required for error-free transmission for part (b).
 - (d) Find the bandwidth required for an AWGN channel for error-free transmission of the output of this source if the SNR ratio is 20 dB.
2. Consider the (8, 4) linear systematic code with the generator matrix

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- (a) List all the codewords generated by this code together with their Hamming weight.
 - (b) How many bit errors can this code correct?
 - (c) Calculate the parity-check matrix for this code.
 - (d) Calculate the syndrome values associated with all bit errors in your response to (b) above.
 - (e) Assume $\mathbf{r} = (0\ 1\ 0\ 1\ 1\ 1\ 0\ 1)$ is received. Calculate the syndrome.
 - (f) What is the corresponding error pattern?
 - (g) What is the corrected codeword?
 - (h) What is the transmitted message?
3. In a *repetition code* every binary symbol is repeated n times, where $n = 2m + 1$ is an odd integer. There are only two codewords in the repetition code, an all-0 codeword and an all-1 codeword. Consider a repetition code with $n = 5$.
 - (a) Construct the generator matrix \mathbf{G} for this (5, 1) block code.
 - (b) Using \mathbf{G} find all all codewords and their Hamming weights.
 - (c) How many bit errors can this code correct?

- (d) Find the parity-check matrix \mathbf{H} for this code.
- (e) Calculate the syndrome values associated with all bit errors in your response in (c) above.
- (f) Assume $\mathbf{r} = (1\ 0\ 1\ 0\ 0)$ is received. What is the syndrome, corresponding error pattern, corrected codeword, and transmitted message?
- (g) The code has a special structure due to repetition. Can you propose a decoding technique simpler than syndrome decoding, employing this special structure?
- (h) Assume that every bit transmitted to the channel has a probability of being in error equal to p . Show that the probability of the message being in error is equal to

$$P_e = \sum_{i=3}^5 \binom{5}{i} p^i (1-p)^{5-i}.$$

- (i) Let $p = 0.01$, calculate P_e .

4. The polynomial $X^7 + 1$ can be factored as

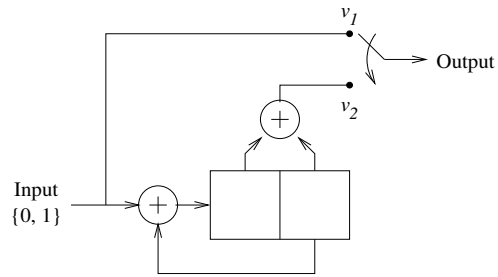
$$X^7 + 1 = (1 + X)(1 + X^2 + X^3)(1 + X + X^3).$$

We will design a (7,3) code based on the generator polynomial

$$g(X) = (1 + X)(1 + X^2 + X^3).$$

- (a) Calculate a nonsystematic generator matrix based on $g(X)$.
- (b) Convert the nonsystematic generator matrix of part (a) into a systematic generator matrix \mathbf{G} .
- (c) Calculate all codewords generated by \mathbf{G} and their Hamming weights.
- (d) How many bit errors can this code correct?
- (e) Calculate the parity-check matrix \mathbf{H} corresponding to \mathbf{G} .
- (f) Calculate the syndrome values associated with all bit errors in your response to (d) above.
- (g) Draw the encoder circuit associated with $g(X)$.
- (h) Assume the message $\mathbf{m} = (1\ 0\ 1)$ is fed into the encoder circuit and calculate its output.
- (i) Draw the syndrome circuit associated with $g(X)$.
- (j) Assume $\mathbf{r} = (0\ 0\ 1\ 1\ 0\ 0\ 1)$ is fed into the syndrome circuit and calculate its output.
- (k) Based on this syndrome, what is the error pattern?
- (l) Calculate the corrected codeword.
- (m) What is the transmitted message \mathbf{m} ?

5. Consider the convolutional encoder in the figure below.



- Draw the state transition diagram for the convolutional encoder.
- Draw a section of the trellis for the convolutional encoder.
- Assuming that the initial state of the convolutional encoder is 00, and assuming a channel bit sequence 1101010011 is received, calculate the message sequence optimum in the maximum likelihood sense.